Preliminary Test of the Penman-Monteith Equation for Estimating Daily Reference Evapotranspiration in Botswana

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Abstract: Many consider the Penman-Monteith equation conceptually more general and realistic and more universally applicable relative to other Penman-based estimators of reference canopy evapotranspiration (PET). They have recommended its worldwide adoption. However, its practical advantage over currently-used PET estimators still needs to be demonstrated especially in the semi-arid tropics. The question addressed here was whether Penman-Monteith estimates would better match the only existing set of thirty six, 5-day averaged, lysimeter observations of reference PET in semi-arid Botswana than the currently-used Penman-based estimator and, if not why not. In this preliminary test, both equations showed the same pattern of estimated PET, but the correspondence between measured values and estimates depended on the season. The Penman-Monteith estimates were closer to measured values during the fall and winter while the Botswana Department of Meteorological Services (DMS) equation better estimated the observations during the spring and summer. Sensitivity analyses showed that small and justifiable adjustments of the windspeed coefficient in the aerodynamic term of the DMS equation would enable better correspondence between estimated and observed values. PET estimates using the Penman-Monteith equation were comparatively much less sensitive to variations in the two resistance input parameters ($r_s$ and $r_t$) needed for its application. It would require unjustifiable adjustment of these parameters to improve the correspondence between the Penman-Monteith estimates and the observed summer PET in Botswana vis-a-vis the existing DMS Penman-based equation. Overall, the findings underline the need for further long-term, multi-locational lysimeter measurements of reference crop PET in Botswana to more fully evaluate the applicability of the Penman-Monteith equation.

Keywords: Semi-arid tropics, water management, reference canopy evapotranspiration

Introduction

Estimating either natural evaporation rates from open water surfaces ($E_w$) or from well-watered plant canopies (potential evapotranspiration or PET) require quantifying the diabatic and adiabatic components of the latent heat transfer from the atmosphere to the evaporating surface at a given temperature and pressure. The former component depends on the net solar energy available for evaporation at the surface and the latter on the state and movement of moisture-unsaturated air above the surface. Over 50 years ago, Howard Penman (Penman, 1948, 1956) analyzed and combined these two thermodynamic processes into a single equation. This equation (commonly called the Penman...
combination equation) integrated the effects of net solar radiation, air temperature, air humidity and wind speed conditions at a given location to obtain a semi-empirical estimate of the evaporative demand of the atmosphere above open water surfaces. However, in contrast to \( E_p \), PET also depends on soil and plant canopy characteristics. To circumvent this difficulty, the PET estimated by Penman-based equations was defined as applicable to an idealized reference well-watered plant canopy actively growing under optimal conditions. The PET of any given canopy is then estimated as the product of this reference PET and an empirically determined crop coefficient. Although any reference canopy could be chosen, the most widely used have been grass or alfalfa maintained at a specified height.

Since its introduction, continuous effort has been made to refine Penman-based estimation of reference PET (Doorenbos and Pruitt, 1977; Monteith, 1965; Monteith and Unsworth, 1990; Allen et al., 1998). The primary scientific motivation underpinning this effort was (and still is) the need to more fully understand and realistically quantify the thermodynamics of evapotranspiration. As a result, there is a current consensus that Penman-based approaches provide reference PET estimates that best match experimental observations. More recently, the many variants of the Penman-based approach have converged into the Penman-Monteith equation to estimate reference PET (Monteith, 1965; Monteith and Unsworth, 1990; Allen et al., 1998; Itenfisu et al., 2000). Currently, many consider the Penman-Monteith equation to be a more standardized and universally applicable PET estimator. Its adoption has been recommended worldwide (Walter et al., 2001; ASCE-EWRI, 2004). The United Nations Food and Agricultural Organization (FAO) has recommended the Penman-Monteith equation as their sole standard method (Allen et al., 1998). This report (Allen et al., 1998) states that "... it (the Penman-Monteith equation) is a method with the strong likelihood of correctly predicting reference PET in a wide range of locations..." and that "... it overcomes shortcomings of the previous FAO Penman method and provides values more consistent with actual crop water use data worldwide...". In some areas where extensive lysimeter observations of reference PET were available, the tendency has been to adopt the Penman-Monteith PET estimator. Yet relatively more areas continue to use variants of the traditional Penman method. Applicability of the Penman-Monteith equation still needs to be demonstrated in these areas.

Monteith and Unsworth (1990) took Penman's original concept to a higher level of abstraction and formulated an equation (the Penman-Monteith equation) that can be utilized for the direct calculation of evapotranspiration from any canopy. He did this by incorporating air resistances in series within and above the canopy that controlled the evaporative fluxes the under water vapor and energy gradients that drive evapotranspiration rates. The first of these resistances termed as the bulk surface resistance \( r_b \) lumped the effect of specific soil and canopy characteristics on the fluxes within the canopy into a single parameter. The second, termed as the aerodynamic resistance \( r_a \), was used to characterize the effect of air movements assuming neutral vertical stability conditions of the atmospheric boundary layer above the canopy. Although conceptually more general and realistic, these resistance parameters change with canopy type and development and soil wetness over time. Quantifying them over time require sophisticated instrumentation. Consequently, the Penman-Monteith equation may offer little practical advantage over the traditional use of crop coefficients to estimate PET for different canopy types. However, there is evidence (Walter et al., 2001; ASCE-EWRI, 2004) showing that it is more effective for estimating reference PET where the canopy type and soil wetness conditions are fixed. The resistances \( r_b \) and \( r_a \) have been reasonably well defined for a well watered grass reference crop with an assumed height of 0.12 m having an albedo \( \tau \) of 0.23 as \( r_b = 0.07 \) m s\(^{-1}\) and \( r_a = (208/u) \) s m\(^{-1}\) where \( u \) is the measured windspeed in m s\(^{-1}\) at 2 m above the ground.

Botswana is a landlocked country in Southern Africa between 22° and 26° south latitude and between 21° and 28° east longitude. It falls in the semi-arid climatic category (BSH climate according to the Koppen-Geiger-Pohl climatic classification system). The Kalahari semi-desert covers about
two-thirds of Botswana. Most of the population lives along the desert’s fringe where the mono-modal summer rainfall is sufficiently reliable to sustain crop and livestock production (Campbell, 1980). Evapotranspiration is a large component of the soil water balance and therefore its accurate estimation is more important in such low rainfall semi-arid climates than in humid regions. Reasonably accurate estimates of E, and PET are essential for quantifying water balances needed for effective, science-based hydrological engineering and water management in Botswana. However, applicability of the Penman-Monteith equation has not been evaluated in Botswana although a small set of lysimeter observations were recorded in 1989 for the agro-meteorological station located at the DMS headquarters in Gaborone (24° 40′ S, 25° 55′ E, elevation 983 m). The question addressed here was whether the Penman-Monteith equation would better match these existing lysimeter observations of reference PET in Botswana than their currently-used Penman-based estimator and, if not why not.

Materials and Methods

Penman-based equation for Botswana: The form of the Penman-based equation currently used by the Department of Meteorological Services (DMS) in Gaborone, Botswana is tailored to the type of instrumentation and data collected at the few fully equipped agrometeorological stations in the country. The Penman-based DMS equation is:

\[
\text{PET} = \frac{\Delta}{\Delta + \gamma} R_s (1 - \gamma) \left( \frac{a_1 + a_2 n}{N} \right) - \frac{\Delta}{\Delta + \gamma} \sigma T^4 \left( a_3 - a_4 \sqrt{e} \right) \left( a_5 + a_6 \frac{n}{N} \right) + \frac{\gamma}{\Delta + \gamma} a_7 (a_8 + a_9 u) (e_s - e)
\]

(1)

where PET = mean daily potential evapotranspiration in mm water; \(\Delta\) = slope of the saturation vapor pressure versus temperature curve in mb/K at the value for the mean daily air temperature \(T^\circ K\); \(\gamma = 0.66\) mb/K is the wet and dry bulb psychrometer constant (Rose, 1966). It represents the value of \(aP = C_p P / (0.622 L)\) in the wet-and-dry-bulb equation. Under well-ventilated conditions at sea level \((P = 1013\) mb) with \(C_p\) (specific heat at constant pressure of air) = 1005 J kg\(^{-1}\)K\(^{-1}\) and L (latent heat of vaporization of water at 298 K) = 2.453×10\(^6\) J kg\(^{-1}\), the value of \(aP\) is 0.667 mb/K; \(R_s\) = mean daily Angot's value for a given location in mm water; \(r =\) reflection coefficient or albedo for a particular surface such as open water or fresh green vegetation; \(n =\) mean daily hours bright sunshine recorded on the Stokes-Campbell bright sunshine recorder; \(N =\) mean day length in hours for a given location and is calculated from astronomical formulae also given by Iqbal (1983) and Gommes (1983); \(\sigma =\) a pseudo-constant = Stefan-Boltzmann constant divided by L the latent heat of vaporization of water in cal g\(^{-1}\)K\(^{-1}\); \(T =\) mean daily temperature of the particular surface in\(^\circ K\); \(e_s(T) =\) saturation vapor pressure in mb at temperature \(T (\circ K)\).

The first two terms estimate the potential evaporative demand due to the net incident solar radiation (incident short wave minus the net outgoing long-wave radiation) on the reference canopy. The third term represents the evaporative loss caused by advection of air at a given level of dryness and is usually termed as the aerodynamic component. The relative importance of these two components depends on climatic conditions. The temperature dependent ratio \(\Delta (\Delta + \gamma)\) in effect weights the relative importance of the energy and aerodynamic components since \(\Delta (\Delta + \gamma) / \gamma (\Delta + \gamma) = 1\).

Equation 1 indicates that the psychrometric constant (\(\gamma\)) is independent of barometric pressure change due to station elevation. However, DMS indicated that in their algorithm the value of \(\gamma\) was corrected for pressure (P) decrease with station elevation (z) as recommended by McCollough (1965). The DMS algorithm used the hypsometric equation for a neutral standard atmosphere (i.e., with a
temperature profile corresponding to the dry adiabatic lapse rate of 0.0065°K m⁻¹) i.e., \( P(z) = P_m \left[ 1 - (0.0065 \frac{z}{T_s}) \right]^{1.24} \) where \( z \) is in meters, \( P_m = \) atmospheric pressure of 1013 mb at mean sea level and \( T_s \) is the surface temperature taken as equal to the air temperature.

The coefficients \( a_i \) and \( b_i \) in the DMS equation represent the semi-empirical regression coefficients in the Angstrom equation \( R/R_s = a_i + b_i \cdot N/N \) relating observed solar radiation and hours of bright sunshine. They were selected by DMS based on the study results of Andringa (1987) and Persaud et al. (1997) which supported earlier studies for similar climates by Martínez-Lezana et al. (1984). The DMS algorithm uses the coefficients in the second term \( (a_i, a_n, a_s, a_{an}) \) and in the third term \( (a_n, a_s, a_{an}) \) of Penman's equation as recommended by Doorenbos and Pruitt (1977). As reported by Vossen (1988). The value of \( r = 0.25 \) is generally accepted for short green grass in the middle and high latitudes.

Values of the saturation vapor pressure values for \( e_s(T) \) needed to obtain vapor pressure from relative humidity data were calculated using the following standard Goff-Gratch equation (Goff and Gratch, 1946; Goff, 1957) as modified by Gomes (1983).

The values of day length \( N \) on a given date for the DMS headquarters station in Gaborone was calculated as \( N = (D/2) \cdot (24/h) \) with \( D/2 \) (radians) = \( \arccos \left( \left[ 1 - 0.01673 \cdot \cos \left( 0.017214(J-1) \right) \right] \right) \), \( \phi \) (radians) is the latitude (+ve for North, -ve for South) and \( \delta \) (radians) is the declination of the sun on a given date. \( \sin \delta = \sin \epsilon \cdot \sin \lambda \) where \( \epsilon = 23.45° = 0.4093 \) radians is the angle between the earth's equator and ecliptic and \( \lambda \) (radians) is the celestial longitude measured anti-clockwise from the earth's position at the vernal equinox. The Angot value \( (R_n, \text{in mm H}_2\text{O}) \) received per unit area of the earth's surface at a given location at this latitude was calculated as (Iqbal, 1983; Gomes, 1983):

\[
R_n = 7.76 \frac{D \sin \phi \sin \delta + 2 \cos \phi \cos \delta \sin \frac{D}{2}}{\left[ 1 - 0.01673 \cdot \cos \left( 0.017214(J-1) \right) \right]^2} \tag{2}
\]

Where \( J \) is the day counted from January 1.

The Penman-Monteith equation: The Penman-Monteith equation (Allen et al., 1998) is:

\[
\text{PET} = \frac{L}{ \Delta \left( R_n - G \right) + K_n \rho_v C_p \frac{e_s-e}{T_s} } \tag{3}
\]

where \( \text{PET} \) = the reference evapotranspiration in mm day⁻¹; \( L \) = latent heat of vaporization in MJ kg⁻¹; \( R_n \) and \( G \) = the net radiation and soil heat flux, respectively in MJ m⁻² d⁻¹; \( e_s-e \) = the vapor pressure deficit of the air in mb, \( \Delta = \frac{\Delta e}{\Delta T} \) and \( \gamma = 0.67 \) = the psychrometric constant in mb°K⁻¹ (or more appropriate in this context in J m⁻²°K⁻¹); \( K_n \) = the number of seconds in the period over which PET values are calculated = 86,400 seconds for daily values; \( \rho_v \) = mean natural air density in kg m⁻³ at atmospheric pressure; \( C_p \) is the specific heat of the air in MJ kg⁻¹°C⁻¹; \( T_s \) and \( r \) is the aerodynamic and bulk surface resistance, respectively in s m⁻¹.

\( L \) (the latent heat of evaporation in J kg⁻¹) was assumed fixed and equal to 2.453 MJ kg⁻¹ as for the DMS equation. For daily periods \( G \) is effectively zero (Allen et al., 1998). Values of \( e_s(T) \), \( e \) and \( \Delta \) in mb were calculated as described above for the DMS equation. Since direct measurements of \( R_n \) were unavailable anywhere in Botswana, its value (already in mm day⁻¹) was taken as the same as that estimated for the DMS equation using Angot's value and hours bright sunshine as:

\[
R_n \text{ in mm day}^{-1} = R_s \left( 1 - r \right) \left( a_s + a_n \frac{n}{N} \right) - \sigma T^4 \left( a_s - a_n \sqrt{n} \right) (a_s + a_n \frac{n}{N}) \tag{4}
\]
The density of moist air was approximated as that of dry air since the latter is only slightly greater. Using the ideal gas law \( \rho_i = P_i/(R_i T) \), where \( R_i \) is the specific gas constant for dry air = 287 J kg\(^{-1}\) K\(^{-1}\). As explained in the introduction, \( \gamma \) in mm H\(_2\)O K\(^{-1}\) at \( P_{a,0} = C_1 P_{a,0}/(0.622 \text{ L}) \) and therefore \( (K, \rho_i, C_1)/(R_i L) = 86,400 \left( 0.622 \gamma / (R_i T r_i) \right) \left( \text{mm H}_2\text{O K}^{-1}\text{day}^{-1} \times 100 \right) \). Substituting the appropriate numerical values, \( (K, \rho_i, C_1)/(R_i L) - 100 \left( 86,400 \left( 0.622 \gamma / (287 T) \right) \right) \left( 1/208 \right) = 90 \gamma T \left( \text{mm H}_2\text{O K}^{-1}\text{day}^{-1} \right) \). Also, substituting the numerical values for \( r_i \) and \( r_s \) gives \( 1 + r_s/r_i = 1 + 0.34 \). Consequently, the Penman-Monteith equation used for computational purposes in this study was:

\[
\text{PET in mm day}^{-1} = \frac{\Delta R_n + 90 \gamma T (e_s - e)}{\Delta + \gamma (1 + 0.34 u)}
\]

Comparison of estimated and lysimeter PET values. Information on the set of PET values for grass was contained in a report by Sakamoto et al. (1990). They were measured in two free drainage lysimeters constructed in 1986 at the meteorological station at the located at the DMS headquarters in Gaborone. Records of thirty-six five-day mean lysimeter values of PET under grass cover were available during the period March 21, 1989 through 20 March, 1990. As stated in the report, the 5-day average was considered more realistic since it was not practical to maintain ideal steady-state, unsaturated water flow conditions in these lysimeters and there were periods when they were not maintained (Sakamoto et al., 1990). The 36 values were distributed as follows: fall 9, winter 8, spring 10, summer 9. These observed values were compared to corresponding estimates using the DMS Penman-based equation and the Penman-Monteith equation.

**Results**

Figure 1 presents the measured 5-day lysimeter means and corresponding values using the DMS and Penman equations over the year starting March 21, 1989. Both equations show the same pattern of estimated PET, but it appears that the correspondence between measured values and estimates depended on the season. The Penman-Monteith estimates were closer to measured values during the fall and winter of 1989-1990 while the DMS equation better estimated the observations during the
Table 1: Mean sum of squares of deviations between measured lysimeter values (5-day averages) and corresponding values using DMS and Penman-Monteith estimators over 4 seasons in Botswana starting March 21, 1989

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Fall</th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>All data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n = 9</td>
<td>n = 8</td>
<td>n = 10</td>
<td>n = 9</td>
<td>n = 36</td>
</tr>
<tr>
<td>DMS</td>
<td>0.83</td>
<td>1.11</td>
<td>2.91</td>
<td>1.74</td>
<td>1.70</td>
</tr>
<tr>
<td>Penman-Monteith</td>
<td>0.81</td>
<td>0.54</td>
<td>3.12</td>
<td>3.74</td>
<td>2.12</td>
</tr>
</tbody>
</table>

*DMS = Department Meteorological Services, Gaborone, Botswana

Table 2: Long-term maximum and minimum monthly averages of variables used for estimating sensitivity of reference PET estimates in Botswana to coefficients in the Penman-based DMS formula

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Ratio</th>
<th>Rn</th>
<th>e</th>
<th>e(T)</th>
<th>u</th>
<th>Δ</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rn (mm H2O)</td>
<td>n/N</td>
<td>°K</td>
<td>mb</td>
<td>mb</td>
<td>km day⁻¹</td>
<td>mb K⁻¹</td>
<td>mb K⁻¹</td>
</tr>
<tr>
<td>Maximum</td>
<td>17.77</td>
<td>0.87</td>
<td>299.4</td>
<td>18.6</td>
<td>33.9</td>
<td>218</td>
<td>2.01</td>
<td>0.594</td>
</tr>
<tr>
<td>Minimum</td>
<td>8.99</td>
<td>0.68</td>
<td>286.6</td>
<td>7.0</td>
<td>15.0</td>
<td>155</td>
<td>0.97</td>
<td>0.592</td>
</tr>
</tbody>
</table>

*DMS = Department Meteorological Services, Gaborone, Botswana

spring and summer. In order to better quantify this finding, the mean sum of squares of deviations between measured lysimeter 5-day averages and corresponding values using the DMS and Penman-Monteith equations were calculated for each of the seasons in Botswana starting March 21, 1989. These results (Table 1) confirm the inference from Fig. 1. Table 1 also shows that for the entire year the DMS equation gave a lower mean sum of squares of deviations.

The same meteorological variables were used went into both equations for calculation of Rn. Therefore the differences in the estimates (Fig. 1) were likely due to the coefficients a, a, a, and r and r, of the Penman-Monteith equation. The question that needed answering was how sensitive were the PET estimates to these parameters.

In general the change ΔF in the value of a function F in response to small changes in parameters a, a, a, a, of the function is:

\[ \Delta F = \sum_{i=1}^{n} (\frac{\partial F}{\partial a_i}) \Delta a_i \]  

The coefficients in the DMS equation appear as linear relations and therefore the estimates of PET will respond linearly to step changes in these coefficients. Thus using sets of maximum and minimum values of the climatic variables associated with these coefficients, it was possible to estimate a maximum or minimum possible PET response to an upward or downward step change in each of the coefficients.

The long-term maximum and minimum values for the appropriate meteorological variables were estimated from DMS mean monthly records for Gaborone and are given in Table 2. As already discussed, the values of γ in this table were corrected for the elevation (983 m) of the DMS headquarters station in Gaborone. Substitution of these values into the appropriate derivatives of the DMS equation showed that the response of the PET estimates were most sensitive to changes a (Table 3). This suggested that the DMS value of 0.01 for a may be too high for the winter months. Lowering its value would tend to give better correspondence with the observed lysimeter values. The value of the parameter a in Penman-based PET estimators has been contentious. In the original Penman equation for open water a was set = 1/100 for u in miles day⁻¹ or 1/161 for u in km day⁻¹. Doorenbos and Pruitt (1977) used a = 1/100 for u in km day⁻¹ stating that the similarity to the value for a in the original Penman equation was purely coincidental. Frère and Popov (1979) suggested that in and semiarid conditions, whenever the mean monthly temperature exceeds 5°C, a should be
Table 3: Response of PET estimates to step increases in the coefficients \( a_0, a_1, a_2 \) for the DMS equation using maximum and minimum values of the input variables

<table>
<thead>
<tr>
<th>Coefficient and base value</th>
<th>Step increase</th>
<th>Partial derivative with respect to coefficient</th>
<th>Response (mm day(^{-1})) to step increase in coefficient using maximum(^1)</th>
<th>Minimum(^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 = 0.27 )</td>
<td>0.01</td>
<td>( \frac{\gamma}{\Delta + \gamma} (a_0 + a_1 u) (e_s - e) )</td>
<td>0.112</td>
<td>0.064</td>
</tr>
<tr>
<td>( a_1 = 1.0 )</td>
<td>0.01</td>
<td>( \frac{\gamma}{\Delta + \gamma} (e_s - e) )</td>
<td>0.009</td>
<td>0.008</td>
</tr>
<tr>
<td>( a_2 = 0.01 )</td>
<td>0.001</td>
<td>( \frac{\gamma}{\Delta + \gamma} a_2 (e_s - e) u )</td>
<td>0.207</td>
<td>0.092</td>
</tr>
</tbody>
</table>

\(^1\)Maximum and minimum values of variables given in Table 2

Fig. 2: Response of PET estimates to unit step increases (1 s m\(^{-1}\)) over the range of possible values in Botswana for the parameter \( r_s \) in the Penman-Monteith equation using maximum and minimum values of the input variables with \( r_s = 70 \text{ s m}^{-1} \)

taken as a function of \( \Delta T \) the difference between the maximum and minimum air temperature otherwise the value of \( a_0 \) should be set equal to 1/161. Indeed, the long term mean monthly temperature records for Gaborone showed that the mean temperature is above 5\(^\circ\)C throughout the year. Clearly, there is a need for experiments to better quantify the behavior of the wind coefficient (\( a_0 \)) in the aerodynamic term of the Penman-based DMS equation.

Unlike the partial derivatives for the DMS equation, those for the Penman-Monteith equation with respect to \( r_s \) and \( r_e \) are non-linear. This means \( \partial(\text{PET})/\partial r_s = f(r_s, r_e) \) and \( \partial(\text{PET})/\partial r_e = f(r_s, r_e) \). Since \( r_s = 208/u \), using values for \( u \) in Table 2, would give a possible values for \( r_s = 82.4 \text{ s m}^{-1} \) for \( u_{ave} = 218 \text{ km day}^{-1} \) (2.525 s m\(^{-1}\)) and 156.3 s m\(^{-1} \) for \( u_{ave} = 115 \text{ km day}^{-1} \) (1.331 s m\(^{-1}\)). In terms of \( r_s \) and \( r_e \), Eq. 5 above can be rewritten as PET (mm day\(^{-1}\)) = \( w(r_s, r_e) (v(r_s, r_e)) \) with \( u = \Delta Rn + [187.25 \gamma (e_s - e)] / (T r_e) \) and \( v = \Delta + \gamma [1 + (e_s - e) / (\gamma e_s)] \). Based on the values for \( n, N, e_s \) and \( e \) in Table 2, the maximum and minimum values of \( Rn \) using Eq. 4 were, respectively 7.30 and 2.00 mm H\(_2\)O day\(^{-1}\). Values of \( \Delta(\text{PET}) \) for unit step increases over the possible range of \( r_s \) can be obtained using Eq. 6 for the maximum and minimum variable values given in Table 2. Similarly \( \Delta(\text{PET}) \) can be obtained for unit step increases of \( r_e = 70 \pm 20 \text{ s m}^{-1} \) (i.e., in a reasonable neighborhood about the recommended value for \( r_e = 70 \text{ s m}^{-1} \)). These results (Fig. 2 and 3) show that the PET estimates using the Penman-Monteith
Fig. 3: Response of PET estimates to unit step increases (1 s m\(^{-1}\)) over a range of possible values for the parameter \( r_s \) in the Penman-Monteith equation using maximum and minimum values for the input variables and \( r_s \) in Botswana.

...equation were comparatively much less sensitive to variations in \( r_s \) and \( r_s \) than the DMS equation was to variation in \( a_s \). Therefore it would mean unrealistic lowering of \( r_s \) and \( r_s \) to adjust for the mismatch of the Penman-Monteith estimates of summer PET in Botswana. On the other hand, adjusting \( a_s \) in the existing DMS Penman-based equation is justifiable.

Conclusions

The findings in their entirety show that the ability of both the DMS and Penman-Monteith equations to estimate the observed reference PET depended on the season. Estimated and observed values can be better matched by small adjustments in the wind speed coefficient in the aerodynamic term of the DMS equation. On the other hand, the appreciable lowering of the two resistance input parameters (\( r_s \) and \( r_s \)) needed to improve the correspondence between the Penman-Monteith estimates and the observed summer PET in Botswana would be unjustifiable. It is clear that further long-term, multi-locational lysimeter measurements of reference crop PET in Botswana are needed to more fully evaluate the applicability of the Penman-Monteith equation.

References


