Comparison of Different Models for Estimating Cumulative Infiltration

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ABSTRACT

Infiltration process is one of the most important components of the hydrologic cycle. The ability to quantify infiltration is of great importance in watershed management. Prediction of flooding, erosion and pollutant transport all depend on the rate of runoff which is directly affected, by the rate of infiltration. Quantification of infiltration is also necessary to determine the availability of water for crop growth and to estimate the amount of additional water needed for irrigation. Thus, an accurate model is required to estimate infiltration process. In this study, the ability of seven different infiltration models (i.e., Philip (PH), Soil Conservation Service (SCS), Kostiakov (KO), Horton (HO), Swartzendruber (SW), Modified Kostiakov (MK) and Revised Modified Kostiakov (RMK) models) to fit infiltration data were evaluated. For this purpose, 95 sets of infiltration data with four-texture classes were utilized. Comparison criteria including Coefficient of Determination ($R^2$), Mean Root Mean Square Error (MRMSE), Root Mean Square Error (RMSE) were used to determine the optimum model. The greatest amounts of $R^2$ values were obtained with RMK, MK and Swartzendruber models. The SCS model with two parameters yielded to the lowest $R^2$. According to the results obtained from mean of RMSE (MRMSE) values, the MK model provided the lowest values, indicating that infiltration was well described by this model. The results of ranking models according to two criteria: RMSE, and MRMSE, indicated that based on RMSE the goodness of cumulative infiltration can be estimated by the RMK, MK, Kostiakov, Swartzendruber, Horton, Philip, SCS models, respectively. But according to the MRMSE statistics cumulative infiltration can be estimated by the MK, RMK, Swartzendruber, Philip, Kostiakov, SCS and Horton models, respectively. Based on the results of ranking model the CSC model obtained the lowest ranking between the all of the models.

Key words: Infiltration models, mean root mean square error, root mean square error, coefficient of determination

INTRODUCTION

Infiltration has an important role in land-surface and sub-surface hydrology, runoff generation, soil erosion, irrigation rate. In addition, the infiltration rate of the soil is influence by various factors depending on the condition of soil surface, its chemical and physical properties (Siyal et al., 2002). Also water infiltration is an index of soil compaction (Younesi Alamouti and Navabzadeh, 2007). Hence, infiltration process modeling have been considered through the past century (Kostiakov, 1932; Smith, 1972; Philip, 1957; Mein and Larson, 1973; Kao and Hunt, 1998; Argyrokastritis and Kerkides, 2003) and there are a large number of models for its computation.
An accurate infiltration model, predicting the real infiltration correctly, is required to estimate the runoff initiation time, planning of irrigation systems and management of water resources.

Through the past century, several infiltration models have been developed and categorized as physically-based, semi-empirical and empirical (Mishra et al., 1999).

**Philip model:** Philip (1957) developed an infinite-series solution to solve the non-linear partial differential Richards’ equation which describes transient fluid flow in a porous medium. For cumulative infiltration, the general form of the Philip model is expressed in powers of the square-root of time as:

\[ I = St^{0.5} + At \]  

(1)

where, \( I \) is the cumulative infiltration (L), \( S \) is the sorptivity (LT\(^{-0.5}\)), \( t \) is the time of infiltration (T), and \( A \) is a parameter with dimension of the saturated hydraulic conductivity (LT\(^{-1}\)).

**Swartzendruber model:** Swartzendruber (1987) presented an infiltration model:

\[ I = f(t) = \frac{c}{d} \left[ 1 - \exp \left( -d^{b} t^{c} \right) \right] \]  

(2)

where, \( f \) is the final infiltration rate (LT\(^{-1}\)), \( c \) and \( d \) are empirical constants.

**Horton model:** Horton (1940) presented a three-parameter semi-empirical infiltration model expressed as:

\[ I = Ct + m(1 - e^{-a}) \]  

(3)

where, \( C \) is the final infiltration rate (LT\(^{-1}\)). Parameters \( c \), \( m \) (LT\(^{-1}\)), and \( a \) (T\(^{-1}\)) must be evaluated using observed infiltration data.

**Kostiakov model:** A simple and general form of infiltration model presented by Kostiakov (1932) is:

\[ I = \alpha_{1}t^{b} \]  

(4)

where, \( \alpha_{1} \) and \( \beta_{1} \) are constants and evaluated using the observed infiltration data.

**Modified Kostiakov model (MK):** Kostiakov was modified by adding the term of ultimate infiltration capacity (\( \alpha_{3} \)) by Smith (1972), as follow:

\[ I = \alpha_{1}t^{b} + \alpha_{3} \]  

(5)

where, \( \alpha_{3} \) is the final infiltration rate (LT\(^{-1}\)), and \( \alpha_{2} \) and \( \beta_{2} \) are the same as \( \alpha_{1} \) and \( \beta_{1} \) at the Kostiakov model.
Revised modified Kostiakov model (RMK): Recently, Parhi et al. (2007) revised Modified Kostiakov model and obtained a four parameters model as:

\[ I = \frac{\alpha_4}{\beta_3 + 1} \cdot e^{\beta_3} + \frac{\alpha_4}{1 - \beta_3} \]  

(6)

where, \( \alpha_4 \), \( \beta_3 \), \( \alpha_6 \) and \( \beta_4 \) are parameters to be determined empirically, using measured infiltration data.

SCS model: Experts of US Department of Agriculture, Natural Resources and Conservation Service (1974) found that the Kostiakov model did not apply at long time. They have done many experiments and concluded that coefficient of 0.6985 to be added to this model which would work well at all times:

\[ I = a^b + 0.6985 \]  

(7)

where, \( a \) and \( b \) are constants and evaluated using the observed infiltration data.

There are several approaches for selection of a suitable model. One of the simplest approaches is minimizing the difference between observed and predicted data to find the best model. For example, a model with higher \( R^2 \) may be preferred more than one with smaller \( R^2 \). Gifford (1976) and Machiwal et al. (2006) used the coefficient of determination (\( R^2 \)) to compare infiltration models. Mishra et al. (2003) examined the suitability of the infiltration models with coefficient of efficiency. Turner (2003) and Dashtaki et al. (2009) used both the coefficient of determination (\( R^2 \)) and the Mean Root Mean Square Error (MRMSE) to select the best infiltration model. Abdel-Nasser et al. (2007) used correlation coefficient to compared Kostiakov and Philip models.

The objectives of this study were to test a variety of models with different underlying assumptions to determine which model represents best the soil infiltration. To achieve these objectives, we compared seven models described above. Three comparison techniques were considered to define the best models: the coefficient of determination (\( R^2 \)), Mean Root Mean Square Error (MRMSE), Root Mean Square Error (RMSE statistic).

MATERIALS AND METHODS

The infiltration data were obtained by Double Rings method from 95 locations in 5 different provinces with different climates in Iran. The soil of these regions are classified as Mollisols, Aridisols, Inceptisols and Entisols in soil taxonomy (Soil Survey Staff, 2010). The infiltration experiments were conducted until infiltration rate reached a constant value for each soil. However, the minimum required time for infiltration measurement was 270 min. Each infiltration measurement was replicated three times, using double ring apparatus with outer and inner diameters of 70 and 30 cm, respectively. The soil texture class of surface horizons were determined as clay loam, silty loam, loam and silty clay loam.

In this study, the RMK model with four fitting parameters was used as a referenced model to compare with results from the other six models (i.e., Philip, SCS and Kostiakov with two parameters; Horton, Swartzendruber and MK with three parameters).
The RMSE statistic is an index of the correspondence between measured and predicted data and has frequently been used as a means of evaluating the accuracy of models. The mean RMSE (MRMSE) values and mean R\(^2\) values of all soils for each model were calculated. The model having the smallest MRMSE value and highest mean R\(^2\) was selected as best model. RMSE and R\(^2\) statistics were calculated as follow:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} (I_o - I_p)^2}{N}}
\]

\[
R^2 = \left[ \frac{N(\sum I_o)(\sum I_p) - (\sum I_o)(\sum I_p)}{\sqrt{[N\sum I_o^2 - (\sum I_o)^2][N\sum I_p^2 - (\sum I_p)^2]}} \right]^2
\]

where, I\(_o\), I\(_p\), and N are observed, predicted and number of observed data values, respectively.

All models were fitted to experimental infiltration data using an iterative nonlinear regression procedure which finds the values of the fitting parameters that give the best fit between the model and the data. This procedure was done using the Matlab 7.11 software.

RESULTS AND DISCUSSION

A statistic of the estimated parameter values (minimum, maximum and mean) of the infiltration models is given in Table 1.

R\(^2\) ranged from 0.77 to 1.00 among all of the soils and all of the models (Table 1, Fig. 1). The greatest amounts of R\(^2\) values were obtained with RMK, MK and Swartzendruber models. The SCS

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**Fig. 1:** Box plot for R\(^2\) percentiles as the goodness of fit of seven models for all soils. PH = Philip, KO = Kostiakov, HO = Horton, SW = Swartzendruber, MK = Modified Kostiakov and RMK = Revised modified Kostiakov models
Table 1: Statistics of optimized parameters of the infiltration models for all soils

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameters</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Philip</td>
<td>A</td>
<td>0.000</td>
<td>1.00</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>0.000</td>
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<td></td>
<td>R¹</td>
<td>0.73</td>
<td>1.00</td>
<td>0.992</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.000</td>
<td>4.52</td>
<td>0.384</td>
</tr>
<tr>
<td>Swartzendruber</td>
<td>F¹</td>
<td>0.000</td>
<td>1.00</td>
<td>0.960</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>0.140</td>
<td>3.38</td>
<td>0.920</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>0.000</td>
<td>0.68</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>R¹</td>
<td>0.940</td>
<td>1.00</td>
<td>0.998</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.004</td>
<td>4.52</td>
<td>0.321</td>
</tr>
<tr>
<td>Horton</td>
<td>a</td>
<td>0.000</td>
<td>1.00</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>0.004</td>
<td>34.8</td>
<td>2.200</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.001</td>
<td>58.5</td>
<td>5.250</td>
</tr>
<tr>
<td></td>
<td>R¹</td>
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<tr>
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<td>6.16</td>
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<td>b</td>
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<td>1.20</td>
<td>0.762</td>
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<tr>
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<td>R¹</td>
<td>0.880</td>
<td>1.00</td>
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<tr>
<td></td>
<td>RMSE</td>
<td>0.131</td>
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<td>Kostiakov</td>
<td>α₁</td>
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<td>3.11</td>
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<tr>
<td></td>
<td>RMSE</td>
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<td>4.20</td>
<td>0.427</td>
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<tr>
<td>Modified Kostiakov (MK)</td>
<td>α₂</td>
<td>0.000</td>
<td>1.00</td>
<td>0.668</td>
</tr>
<tr>
<td></td>
<td>β₂</td>
<td>0.000</td>
<td>0.84</td>
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<tr>
<td></td>
<td>α₃</td>
<td>0.000</td>
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<tr>
<td></td>
<td>R¹</td>
<td>0.920</td>
<td>1.00</td>
<td>0.998</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.000</td>
<td>4.20</td>
<td>0.276</td>
</tr>
<tr>
<td>Revised modified Kostiakov (RMK)</td>
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<td>0.000</td>
<td>0.81</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>β₃</td>
<td>0.987</td>
<td>3.39</td>
<td>0.435</td>
</tr>
<tr>
<td></td>
<td>α₅</td>
<td>0.000</td>
<td>1.60</td>
<td>0.437</td>
</tr>
<tr>
<td></td>
<td>β₄</td>
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<td>0.445</td>
</tr>
<tr>
<td></td>
<td>R¹</td>
<td>0.910</td>
<td>1.00</td>
<td>0.998</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.002</td>
<td>4.81</td>
<td>0.281</td>
</tr>
</tbody>
</table>

model with two parameters yielded the lowest $R^2$. Among the models with three parameters, MK and Swartzendruber, had mean $R^2$ higher than that of Horton model. Among the models with two parameters, mean $R^2$ was higher for the Kostiakov model than that for Philip and SCS models. The range of $R^2$ values of each model obtained by curve-fitting is presented in Fig. 1. Considering mean $R^2$, RMK, MK and Swartzendruber models were selected as the best models for all soils.

According to the results obtained from mean of RMSE (MRMSE) values (Table 1), the MK model provided the lowest values, indicating that infiltration was well described by this model. Parlange and Haverkamp (1989), Dashtaki et al. (2009) and Araghi et al. (2010) reported that the MK model is the best model for quantifying the infiltration process compared to the other infiltration models.
A comparison model was accepted for the soil if its RMSE was smaller than RMSE of reference model. Considering RMSE criterion, the RMK model was the best in 87 out of 95 soils (70.6%) compared with the MK and higher number of soils for other models.

The results of ranking models according to MRMSE and $R^2$ statistics Table 1 given in Table 2.

According to the $R^2$ statistic the goodness of cumulative infiltration can be estimated by the RMK, MK, Swartzendruber, Kostiakov, Horton, Philip, SCS models, respectively.

Based on the results of ranking model given in Table 2 the CSC model obtained the lowest ranking between the all of the models and all of the criteria. Dashtaki et al. (2009) reported a better performance for Horton model than Kostiakov and Philip models. This finding is different with that obtained by Dashtaki et al. (2009) and Parhi et al. (2007) reported a better performance for RMK model than MK and Kostiakov models. These results are concordant with that obtained by Parhi et al. (2007). The mean RMSE indicated different pattern in term of model ranking. The mean RMSE reveals that the correspondence between measured and predicted infiltration is highest for MK model and lowest for SCS model. The results of present study indicated that the empirical models had best fit on the double ring data, because they are on the basis of data derived from field experiments without any pre-assumptions. However, Swartzendruber model showed better performance than Kostiakov, Horton and SCS empirical models. Based on the results of ranking model given in Table 2 the CSC model obtained the lowest ranking between the all of the models.

As a result of paired t-tests on the RMSE, we found that several pairs of models performed identically based on RMSE statistics. Philip-Swartzendruber, Philip-Horton and Swartzendruber-MK pairs performed identically based on RMSE statistics which showed that pairs of models made no great differences in predicted values at measurement points of cumulative infiltration in most of soils. This result indicated that models within each pair (Philip- Swartzendruber, Philip-Horton and Swartzendruber-MK) might have statistically identical performance even though the equations are different.

Effect of soil texture on performance of models: The $R^2$ percentiles are shown for various soil textures in Fig. 2. The $R^2$ percentiles were higher for the RMK, MK, Swartzendruber, Kostiakov models than other models for loam soils. The $R^2$ percentiles of RMK models were very close to 1.00 for majority of silty clay loam soils. In addition, for silt loam and clay loam soils RMK and MK models had a higher $R^2$. Using $R^2$ analysis, the RMK and MK were the best models for silt loam and clay loam soils and RMK, MK, Swartzendruber and Kostiakov were the best models for loam and silty clay loam soils. Other models were smaller number of soils better than RMK model.
Fig. 2: Box plot for $R^2$ percentiles as the goodness of fit of seven models for soil textural classes. 
PH = Philip, KO = Kostiakov, HO = Horton, SW = Swartzendruber, MK = Modified Kostiakov and RMK = Revised modified Kostiakov models

CONCLUSIONS

The results of this study indicated that all seven models account for $>90\%$ of the variance ($R^2$) in cumulative infiltration of majority of soils. Based on the mean RMSE and $R^2$ the MK model was the best model for prediction cumulative infiltration and SCS with two parameters was the worst model.

Using paired t-tests for RMSE, we found that Philip-Swartzendruber, Philip-Horton and Swartzendruber-MK model pairs could be considered to perform identically at the 95% significance level.

Texture of soil could affect the performance of cumulative infiltration models. Among four soil classes, the RMK model with four parameters showed better fits for loam, clay loam and silty clay loam soils and worse fit than MK model for silt loam soils.

REFERENCES


