Structural Stability of Large-size Grating Tiling Device Based on Dynamic Stiffness

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ABSTRACT

A tiled grating is one of the most important components in a chirped-pulse amplification laser compressor, which is used in the inertial confinement fusion project. Dynamic stiffness is an important indicator of structural stability, which is important in a grating tiling device. This study focused on using a dynamic stiffness method to improve the stiffness and optimize the structure of a 2×1 grating tiling device with a tetrahedral truss support. Therefore, the dynamic stiffness of the device was analyzed to identify the weak components and determine their effects on the device. Then, the structural parameters of the grating tiling device were optimized based on the parameters of the weak components, which are represented by the first-order natural frequency curve. A modal analysis showed that the first-order natural frequency of the device increased from 74.17-134.68 Hz after the optimization. In addition, a stability experiment revealed that during the first 100 min, the dynamic response of the optimized device was less than that before the device was optimized. Thus, the structural stability was effectively enhanced.

Key words: Tiled grating, stability, dynamic stiffness, FEA, structural design, parameter optimization

INTRODUCTION

Chirped-Pulse Amplification (CPA) is an important method for achieving ultrashort laser energy amplification (Sharma and Kourakis, 2009). However, the damage threshold and aperture of a CPA compressor limits its output (Hornung et al., 2010; Kessler et al., 2004), implying that a multilayer dielectric (MLD) diffraction grating presently exhibits the best performance. However, current MLD diffraction gratings do not satisfy certain size requirements and several studies have been conducted to explore the use of grating tiling (Blanchot et al., 2010; Qian et al., 2011). In these studies, the stability of high-precision tiled gratings has been of particular interest (Zuo et al., 2006).

Studies on the stability of tiling systems have focused on structural and control algorithm adjustments. In Japanese FIREX-I device, capacitive sensors are used to monitor the change in displacement between the gratings and supports. The feedback control method and drivers are used to maintain stability of the device (Ezaki et al., 2008). The Laboratory for Laser Energetics used the near-field method to test the interference fringe pattern for controlling the gratings to achieve long-term stability (Kessler et al., 2004). Zhang et al. (2011) studied the environmental factors and used high-damping materials to improve the movement of the grating tiling device and a combined
surface to reduce the dynamic response and extend the stable time (Bai et al., 2013) used a flexure hinge to improve the stability of the device. In Chongqing University (Luo, 2012), a 2x2 tiled grating device based on a parallel board frame was developed by optical testing and the HHT method was used to analyze its vibrational characteristics. Studies on grating tiling have so far focused on the theory and control method, but there is still no perfect theory that can be used to optimize and improve the stability of grating tiling devices. In the present study, the dynamic stiffness theory was used to analyze the stability of a grating tiling device and improve its dynamic stiffness. The results of experiments confirmed the suitability of the proposed method for designing controlling the stability of a grating tiling device.

MATERIALS AND METHODS
Design of grating tiling device: A grating tiling device is used to splice two pieces of gratings together. This process is critical for achieving chip grating position accuracy (Wang et al., 2011) and with the exception of the displacement along the y-axis, the other five DOF position errors can be controlled in the range of nanometers. Based on the error-matching principle, it is only necessary to adjust the tiled grating piston (d), tip (θ), and tilt (θ) (Qiao et al., 2007). Moreover, the angle drift of the grating tiling device can be controlled to achieve a position accuracy in the range of microradians.

To realize three-DOF precision adjustment of the chip grating without introducing phase errors or affecting the time-space characteristics, the adjustment error of the design angle should be controlled to within 1 μrad. Figure 1 shows a model of the grating tiling device. The drivers of the adjustment components of the device are piezoelectric actuators, which have the advantages of simple structure, high reliability, high precision and the ability to generate large torques. The device contains two grating holders that are used for installing 210x420 mm gratings. The grating holder is designed with a high-stiffness tetrahedral truss structure for long-term stability. The

Fig. 1: Simplified model of the grating tiling device
adjustable grating is supported by a V-block and steel ball, to facilitate movement in all directions. The high stiffness of the holder (Guan and Liu, 2010) implies that it can be used in other grating tiling devices such as the OMEGA EP system (Qiao et al., 2007).

Overview of the method of analysis and improvement of tiled grating device: In this study, the dynamic stiffness method to analyze and improve the tiled grating device was adopted. First, the mathematical model of dynamic stiffness of the splicing mechanism is proposed. It forms the theoretical basis for the analysis and improvement of the tiled grating device. Second, this study presents the finite element analysis of the device and its weak components are determined. Using the response surface module of ANSYS to analyze the size parameters of the components, the sensitivity of each component to the natural frequency of the structure was determined. Finally, the solution to enhance the dynamic stiffness of the tiled grating device was presented.

Mathematical model of dynamic stiffness of splicing mechanism: Dynamic stiffness is a major indicator of the vibration characteristic of a mechanical structure subjected to excitation forces and it is defined as the excitation force produced by the mechanical structure per unit amplitude. Static stiffness is a measure of the ability to resist deformation under static loading, whereas dynamic stiffness is a measure of the ability to resist deformation under dynamic loading. The former is determined from the deformation of the structure under a static load and the latter from the natural frequency. The dynamic stiffness is considered in this study with the aim of decreasing the amplitude of the mechanical structure, excluding the resonance frequency as much as possible under excitation by ambient vibration and decreasing the response amplitude excited by random vibration. It can therefore be used to develop a reliable theoretical basis and engineering approach for designing the structure of the machine, eliminating a weak link and improving the stability based on the structural formation and dynamic stiffness.

A tiled grating system is mathematically a linear structure and equivalent to the superposition of several harmonic vibrations when excited by the ambient environment. Its dynamic characteristics can therefore be described by mechanical impedances—the displacement impedance (dynamic stiffness), velocity impedance and acceleration impedance (effective mass) (Zuo, 1987). The critical aspect of precision tiled grating is the changes that occur over time and the dynamic stiffness is thus used to evaluate the structural stability.

A locked grating tiling device is equivalent to a mass-spring-damping system with one DOF and its response under vibration can be simulated by the stable random vibration response model of a linear system with one DOF (Zuo, 1987). The vibration model is shown in Fig. 2.

According to Newton’s second law, the differential equation of the vibration is:

\[ m\ddot{y}(t) + c\dot{y}(t) + ky(t) = -m\ddot{x}(t) \]  \hspace{1cm} (1)

where, \( y(t) = z(t) - x(t) \) is the displacement of the structure relative to the foundation, \( \dot{y}(t) \) is the velocity of the structure, \( \ddot{y}(t) \) is the acceleration of the structure, \( \ddot{x}(t) \) is the vibration acceleration of the foundation, \( m \) is the mass of the device, \( c \) is the structural damping of the device and \( k \) is the static stiffness matrix of the device.
Fig. 2: Vibration model of a one-DOF system

The natural frequency and damping ratio are respectively given by:

$$\omega_n = \sqrt{\frac{k}{m}}$$

and:

$$\zeta = \frac{c}{2m\omega_n}$$

and:

$$H(\omega) = \frac{1}{\omega_n^2 - \omega^2 + i2\zeta\omega\omega_n} \quad (2)$$

As per definition, the dynamic stiffness can be expressed as:

$$K_d = \frac{F(t)}{|y(t)|} = \frac{m\ddot{x}(t)}{|H(\omega)\times(t)|} = K_s \sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + (2\zeta \frac{\omega}{\omega_n})^2} \quad (3)$$

where, $K_d$ and $K_s$ are respectively the dynamic stiffness and static stiffness of the structure of the device, $\omega$ is the frequency of the external excitation and $\omega_n$ is the natural frequency of the device.
If the frequency ratio \( \tilde{\omega} = \omega / \omega_n \) is introduced into Eq. 3, the equation can be simplified as:

\[
K_i = K_i \sqrt{(1 - \tilde{\omega}^2)^2 + (2\zeta \tilde{\omega})^2}
\] (4)

In a series system, the relationship between the system stiffness and the stiffness of the links is as follows:

\[
K = (K_i^1 + K_i^2 + \cdots + K_i^n)^2
\] (5)

According to Eq. 4, when the excitation frequency \( \omega \) is much lower than the resonance frequency \( \omega_n \), i.e., when the ratio of the excitation frequency \( \omega \) to the natural frequency \( \omega_n \) is close to 0, the dynamic stiffness is approximately equal to the static stiffness. If \( \omega \) is almost equal to \( \omega_n \), then the dynamic stiffness will be significantly affected by the static stiffness and damping ratio \( \zeta \). This will put the structure under great risk owing to resonance. If \( \omega \) is higher than \( \omega_n \), then the dynamic stiffness would be affected by the static stiffness as well as by the increase in the damping ratio with an exponential increase in the speed. This indicates that it is better for \( \omega \) to be as small as possible to avoid the risk of resonance and eliminate the amplitude excited by the ambient environment. Therefore, it is necessary to increase the dynamic stiffness. Thus, by increasing the resonance frequency (i.e., by increasing the value of the denominator), \( \omega \) can be made very low to increase the dynamic stiffness.

**Modal analysis of tiling device and identification of weak components:** Equation 5 indicates that the most effective way to improve the overall system stiffness is to improve the stiffness of the weakest parts, namely, the weak links. By a modal analysis, the modal shapes of all the resonance points can be determined. The weak link at a resonance point produces a resonance component or fringe and the weakest link of the device produces a resonance component that determines the dynamic stiffness of the entire device (Yu et al., 2013).

A modal analysis of the tiling device was carried out. SOLID95 units were used for the meshing of the finite element model of the device shown in Fig. 3 and the material parameters are listed in Table 1.

The first four modes of the device are listed in Table 2 and the vibration mode is shown in Fig. 4. The results show that the natural frequency of the fourth-order mode of the device was 274.06 Hz and that its vibration frequency was above that of the laboratory environment.

### Table 1: Material parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Material</th>
<th>Density (kg m(^{-3}))</th>
<th>Elastic modulus (Pa)</th>
<th>Poisson’s ratio</th>
</tr>
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<tbody>
<tr>
<td>Support plate</td>
<td>Aluminium</td>
<td>2770</td>
<td>71×1011</td>
<td>0.33</td>
</tr>
<tr>
<td>Orating</td>
<td>K9 glass 45</td>
<td>2658</td>
<td>0.68×1011</td>
<td>0.23</td>
</tr>
<tr>
<td>Other parts</td>
<td>45 steel</td>
<td>7850</td>
<td>2.00×1011</td>
<td>0.30</td>
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</table>

### Table 2: Frequency of modality

<table>
<thead>
<tr>
<th>Order</th>
<th>Natural frequency (Hz)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>74.17</td>
</tr>
<tr>
<td>2</td>
<td>94.87</td>
</tr>
<tr>
<td>3</td>
<td>160.97</td>
</tr>
<tr>
<td>4</td>
<td>274.06</td>
</tr>
</tbody>
</table>
In Fig. 4, the first-order vibration mode shows the pitch motion of grating of the device; the second-order vibration mode shows the warping of the grating at the pitch angle; the third-order vibration mode shows the swinging of the grating surface around the grating bus and the fourth-order vibration mode shows the grating with almost no vibration owing to the vibration of the tiling device. It is clear that the low-order modes significantly affect the direction of the

Fig. 3: Finite element model of precision grating tiling device

Fig. 4(a-d): Typical modes of grating tiling device, (a) First-order mode, (b) Second-order mode, (c) Third-order mode and (d) Fourth-order mode
beams. The vibration modes reveal that the support plate and the rods supported by the tetrahedral truss experienced large deformations, implying that they are the weakest components.

**Parameter optimization of tiling device based on dynamic stiffness:** The deflection of the grating can have a significant effect on the direction of the beam. The effect is predominant for rotation about the x and y-axes (Ma et al., 2005). Hence, in the adjustment of the design for structural stability, the vibration mode and natural frequency of the rotation in the x-y plane in order to satisfy the stability design requirements was considered.

The bending deformations of the inclined and erect rods occurred along the x-axis and the support plate was subjected to a combination of bending and twisting. These behaviors are characteristic of weak components. The structural stability of the device can thus be improved by increasing the section modulus of the inclined and erect rods along the x-axis and increasing the thickness (size along the y-axis) of the support plate. Because the size of the inclined rod along the z-axis is limited by space, only an increase in the thickness along the x-axis can be used to increase the section modulus. An increase in the size of the erect rod along the y-axis can thus be used to achieve the best static stiffness.

In the function \( g = g(x_1, x_2, ..., x_n) \), the sensitivity of the variable \( x_i \) can be expressed as:

\[
S_i = \frac{\partial g}{\partial x_i}
\]

where, \( g \) may be a performance parameter such as the natural frequency of the structure and \( x_i \) an optimal parameter such as the thickness. The sensitivity value determines the extent to which the performance parameter is influenced by the variation in the optimal parameter. When optimizing the structural dynamics, the determined natural frequency sensitivity would be different for different values of thickness and the thickness may be varied by calculation to improve the natural frequency. Using the response surface module of ANSYS to analyze the size parameters of the components, the effect of all the parameters on the first-order natural frequency (namely, the sensitivity of each component to the natural frequency of the structure) can be determined. In the analysis, the parameters of the weak components, whose parameters needed to be adjusted to improve the stability, were used as input parameters. The relationships between the thickness of the support plate and the first-order natural frequencies of the support plate, the erect rod and the inclined rod are shown in Fig. 5.

**Statistical analysis:** FEA software is used to analyze the dynamic stiffness problem for the tiled grating device. The first-order natural frequencies of these components were obtained as the output of the analysis.

**RESULTS**

**Relative contributions of sensitivities of weak members:** Figure 5 shows that the first-order natural frequency of the device can be increased from 74.17 to 130-150 Hz by increasing the thickness of the support plate, the vertical rod and the inclined rod. However, a continuous increase in the thickness of the three elements eventually results in a state wherein the first-order natural frequency of the device can no longer be significantly changed. This is because the change in the size parameters of the components is inevitably accompanied by a change in mass. The equation \( \omega = \sqrt{K/m} \) indicates that the effect of the static stiffness \( k \) on the natural frequency of the device is the same as that of the mass. If the increase in the static stiffness is more than that in the mass for a given
Fig. 5: First-order natural frequency curve of the weak members

Fig. 6: Relative contributions of sensitivities of weak members

Table 3: Original and optimized stability parameters of weak components

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Support plate (a×b) (mm)</th>
<th>Vertical rod (a×b) (mm)</th>
<th>Inclined rod (a×b) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>6×200</td>
<td>6×5</td>
<td>10×6</td>
</tr>
<tr>
<td>Optimized</td>
<td>8×200</td>
<td>8×5</td>
<td>10×8</td>
</tr>
</tbody>
</table>

a, b: Parameters of the component sections shown in Fig. 3

increase in the size of the components, the natural frequency of the device would increase. Conversely, if the increase in static stiffness is less than that in the mass, the natural frequency of the device would decrease.

An analysis revealed that the sensitivities of the thickness of the support plate, vertical rod and the inclined rod respectively account for 54, 40.9 and 5.1% of the overall sensitivity (Fig. 6). The first-order natural frequency of the device is significantly affected by the support plate, with the vertical and inclined rods having little effect. Accordingly, the relevant parameters of the weakness components were adjusted (Table 3).
Structural stability validation of the grating tiling device: After adjusting the relevant parameters of the weak components, the natural frequency of the device was significantly improved. Table 4 lists the first four orders of the natural frequency after the adjustment.

DISCUSSION

A comparison between the data shown in Table 2 and 3 reveals that the vibration mode of the device after the adjustment was the same as that before the adjustment, although the first-order natural frequency was increased from 74.17-134.68 Hz, an improvement of 81.6%. Compared with the first natural frequency of the early tiled grating device, the first natural frequency is 128.04 Hz (Luo, 2012), which shows that the new one is better.

To investigate the dynamic response of the device in the working state, an experiment on the stability of the grating tiling device was performed. Figure 7 shows the vibration power spectrum of the experimental environment.

The equation of the dynamic stiffness, \( K_d = F(t)/|y(t)| \), indicates that an improvement in the dynamic stiffness for a certain force was accompanied by a reduction in the displacement \( y(t) \). Figure 8 shows the experimentally determined beam pointing of the grating tiling device during the adjustment of the parameters of the weak components. Figure 8 shows that the angular displacement of the device was always lower after the adjustment, with the exception of the individual measuring points. In addition, the angle stability could be controlled between \( \pm 1 \) and \( \pm 1 \text{ grad} \), which satisfies the requirements for tiling grating and the angular displacement was less than the index proposed in the literature (Zuo et al., 2007). The adjusted structural stability thus met the requirements for stability design. It was observed that the dynamic

<table>
<thead>
<tr>
<th>Order</th>
<th>Natural frequency (Hz)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>134.68</td>
</tr>
<tr>
<td>2</td>
<td>292.87</td>
</tr>
<tr>
<td>3</td>
<td>411.36</td>
</tr>
<tr>
<td>4</td>
<td>502.25</td>
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</table>

Fig. 7: Acceleration PSD of experimental environment
CONCLUSION

In this study, a 2×1 grating tiling device with a tetrahedral truss support was designed. A modal analysis of the device was conducted to identify its weak components and the relative effects of the weak components on the sensitivity of the dynamic stiffness was determined using a dynamic stiffness index. In addition, the relevant parameters of the system were optimized to eliminate the element of subjectivity in the design. The analytical results showed that the first-order natural frequency of the grating tiling device was increased from 74.17-134.68 Hz by adjustment of the parameters and it was also experimentally determined that the angular displacement of the grating tiling device was reduced by the adjustment, with the exception of the individual measuring points. The dynamic stiffness of the device was thus confirmed to have been improved. The analytical and experimental results validated the effectiveness of the proposed design method and its suitability in obtaining certain reference values for the design of similar devices. It is different from the traditional optical method to device a optic device and FEA software is used to analyze the dynamic stiffness problem for the tiled grating device. Therefore, the design process, in addition to being cost effective and less time consuming, verified the correctness of the design from another perspective.

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