Seat Control Model Research on Railway Passenger Transport

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ABSTRACT
Based on the research on revenue management has been applied in our country's railway passenger transport market, the study applies method of transforming the continuous random variables into discrete random variables to improve the existing seat control model of multi-leg and multi-fare and simplifies the solution difficulty, by researching in the railway industry the existing seat optimum control model. Finally, an example shows the model's application feasibility.

Key words: Railway, passenger transport, seat control, revenue management

INTRODUCTION
With the rapid development of economy, the competition of highway, railway and civil aviation in price and serve becomes increasingly fierce. However, the subject status of railway passenger transportation by government pricing hasn't changed; information feedback delays; decisions are too slow which make the railway transport enterprise's competitiveness relatively weak in passenger market and causes railway's share reduced. Facing the challenges of severe competition, railway passenger transport department can change the status that the past can't accurately and rapidly respond according to the actual utilization condition of seats by using the price lever and executing price discrimination. Railway departments can determine the price difference according to busy or slack season and the geographical position based on the railway's wide distribution in our country and the consumption level widely different of in national regions (Xiaoping et al., 2000). But, in differential pricing, there is a problem: in general, the passengers who buy discounted train tickets have earlier trend to book seats than the high yield passengers and the demand of discounted ticket seats probably beyond available. If discounted ticket sales are overmuch, it will certainly reduce the railway transport income (Xiumin et al., 2003). When other factors remain unchanged, railway passenger transport departments should accurately master the relationship of seats stock changes and ticket fare adjustment, by researching the railroad revenue management and executing the optimum control on railway passenger transport seats.

Revenue Management (RM), is a tool widely used with the objective of maximization of the revenue of a firm, that helps to decide how much of inventory (seats in an airplane, rooms in a hotel or cars in a rental car fleet) to allocate to different types of market segments and at what prices/fares. Revenue in the railways can be understood as either yield per available seat mile or yield per revenue mile (Bharill and Rangaraj, 2008).
In recent years, many scholars have already studied actively the approach to maximize the railway passenger transport income and they realized the revenue management on railway passenger transport from price discrimination, flow control and community management three ways. Based on the single segment EMSR method and the network transport bid price method, Xiumin and Shudong (2005) raised and proved value decomposition—the theory of price transformation, converted complex network transport optimization problem into single segment multistage fare optimization problem. Then executed nested structure management mode for each section seat, combining the traditional EMSR dynamic control method. Zhenfei and Bo (2005) discussed the application problems of income management in the train's berth marketing. Binfeng (2007) gave a seat control stochastic programming model for multi-interval and various fares. Haichun and Jianfeng (2009) turned problems into linear programming under desired value of bid price method and applied method of random linear programming model to solve, etc. Armstrong and Meissner (2010) provided an overview of the published literature for both passenger and freight rail revenue management. Based on these, the paper, under the qualitative analysis, studied and improved the seat optimum control of revenue management on railway passenger transport through the mathematical model and the numerical analysis.

**PASSENGER TRANSPORT REVENUE MANAGEMENT SEATS CONTROL MODEL**

Railway passenger transportation seat control is very important for the railway revenue management; it is a process of allocating appropriate seats for different prices ticket and aiming at the total income maximization. i.e., a way to sell train tickets, based on passenger demand forecasting, using mathematical programming model to calculate seat’s sales upper limit for different fare levels to achieve the maximum profit.

**Technical terms in model:** Before discussing the concrete model, the paper first makes the following definition for some technical terms used in the model: Interval (leg) is the running interval of the train, its basic features include road length, train travel time and road capacity; O-D demand pair (segment) is a itinerary that contains one or more continuous intervals, its basic features include sections the journey through, travel route length and travel demand of this segment in different time sections; seats level, namely the fare level which means to combine existing seats level concept with price discount and corresponds with shipping space of aviation seat control.

**Railway passenger seat control basic model:** Define the fare of one seats level is $f_i$, assigned seat number is $S_i$. According to general demand hypothesis, each fare demand has been determined and meets the normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u)^2}{2\sigma^2}}, -\infty < x < +\infty$$

(1)

Because the passenger ticket sales amount is nonnegative number, the value of $x$ should become: 0$\leq x < +\infty$. Its purpose is to plus its cumulative probability when $x < 0$ to the cumulative probability when $x$ in the range [0, x]. Because the study we will talk about is that seats demand is greater than some numerical probability, not less than or equal to the number. Its probability distribution function is following:
If $D_i$ is the total demand probability of $f_i$, the probability of $S_i$ (seating) sold out is:

$$p(D_i > S_i) = 1 - \frac{1}{\sqrt{2\pi} \sigma} \int e^{-\frac{(y-\mu)^2}{2\sigma^2}} \, dy$$

When $D_i > S_i$, the actual booked seat count is $S_i$; when $D_i < S_i$, the actual booked seat count is $D_i$, so the real earnings correspond with fare ($f_i$) is: $R_i = f_i \times \min\{S_i, D_i\}$. R’s expected revenue is:

$$E(R_i) = f_i \left[ \int_0^{S_i} f(x_i) \, dx_i + S_i \int_0^\infty f(x_i) \, dx_i \right]$$

for $D_i$‘s distribution is known where $f(x_i)$ is $D_i$‘s distribution density function. To a train whose seat count is $N$ in its single running interval and there are $i = 1, 2, ..., I$ fares level, so the railway passenger transport’s seats control basic model of single-leg and multi-fare structure is following:

$$\max \quad Z = E\left( \sum_{i=1}^I f_i \times \min\{D_i, S_i\} \right)$$

s.t. $\sum_{i=1}^I S_i \leq N, S_i \geq 0$ and it is an integer.

The above model can well express the probabilistic of demand and it is a stochastic programming model we can use stochastic linear programming model (SLP) method to solve.

**Seat control basic model for multi-interval various fares structure**

**Model parameter and variable’s description:** $k = 1, 2, ..., K$ is interval; $i = 1, 2, ..., I$ is seats level; $j = 1, 2, ..., J$ is an O-D demand pair, i.e., passenger flow ODF (origin-destination flow); $f_i$ is a per seat fare of passenger flow FODF (fare ODF) that meets $j$ O-D demand and $i$ seats level, $x_i$ is seats count assigned to one FODF, $X$ is a $D$ column vector comprised of all $x_i$, $d_i$ is a demand that meets one FODF where $t = 1, 2, ..., T$ and $T = I \times J$; $A$ is $K \times T$ incidence matrix which represents the relationship between $t$ FODF and $k$ interval, $s_{kt} = 1$ represents the $t$-th FODF uses interval $k$, or $s_{kt} = 0$, where $t = 1, 2, ..., T$ is the number of FODF, $k = 1, 2, ..., K$ is the number of interval; $c_k$ is the ability of train to provide seats in the $k$-th interval and $C = (c_1, c_2, ..., c_K)^T$.

**Model description:** In order to facilitate the problem solving, the paper will transform a continuous random variable into a discrete random variable. Suppose $d_i$ has $n$ kinds of values and $\{d_{i1}, d_{i2}, ..., d_{in}\}$, the size of $n$ can be determined by actual condition and hopes to achieve. Here make $d_{i0} = 0$ and divide $x$ into $n$ parts: $\{x_{i1}, x_{i2}, ..., x_{in}\}$, corresponding to $[d_{ik}, d_{ik+1})$ $(k = 1, 2, 3, 4)$ and $\sum_{i} x_{ik} = x_i$. The target of model is to maximize the train’s operation benefit, the concrete model is following:

$$142$$
\[
\begin{align*}
\max & \quad Z = \frac{1}{t} \sum_{i=1}^{t} \left( \frac{1}{p} \sum_{k=1}^{p} x_{ik} \right) - \frac{2}{t} \sum_{i=1}^{t} \left( p(d_i < d_{i,k-1}) \times x_{ik} \right) \\
\text{s.t.} & \quad A \times X \leq C \\
& \quad x_{ik} \leq d_{i,k} - d_{i,k-1} \quad k = 2, 3, ..., n \\
& \quad x_{i,k} \geq 0 \text{ and it is an integer}
\end{align*}
\]

The first constraint is the shipping capacity balance constraint of train in each interval, i.e. the actual distributed seat number should be smaller than the ability of train to provide seats in this interval (carrability). The second and third is passenger flow balance constraint, represents the reserved seats of corresponding range \([d_{i,k-1}, d_{i,k}]\) should be smaller than the demand in the range. If \(d_i < d_{i,k-1}\), these reserved seats will become vacant seats, the subtrahend in the objective function is used to correct uncertainty, i.e., the loss resulting from the train is not full.

EXAMPLES ANALYSIS

Suppose there is a train run from A to C, containing a inter station B, all the seats are sleeping berth, the total number is 150, ticketing use the full-price train ticket and train ticket which is sold at a 20% discount, i.e., \(I = 2\). As shown in Fig. 1 for the passenger train route diagram, so the running interval of train is AB and BC, i.e. \(K = 2\), there are three O-D demand pairs: AB segment, AC segment, BC segment, i.e. \(T = 6\). Ticket price, no. and demand are shown in Table 1. As in the previous hypothesis we can get the form of matrix \(A\) as follows:

\[
A = \begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix} \quad (3-1)
\]

To a train, each interval capacity (the maximum capacity of the train) should remain the same, so we can get the following inequality from constraint condition 1:

\[
\sum_{i=1}^{t} x_{ik} \leq 150 \quad (3-2)
\]

\[
\sum_{k=2}^{t} x_{ik} \leq 150 \quad (3-3)
\]

Use the initial conditions Table 1 given to modeling according to Eq. 5 we know per \(d_i (t = 1, ..., 6)\) has 4 ways to sure value, so \(x_{ik} (t = 1, ..., 6)\) that corresponds with demand range \([d_{i,k-1}, d_{i,k}]\) (\(k = 1, 2, 3, 4\)) also has 4 ways to sure value. And for meeting the constraint condition of model, there will be the following conditions: When \(t = 0\): \(0 \leq x_{i,1} \leq 40, 0 \leq x_{i,2} \leq 10, 0 \leq x_{i,3} \leq 20, 0 \leq x_{i,4} \leq 10; \) When \(t = 2\): \(0 \leq x_{2,1} \leq 65, 0 \leq x_{2,2} \leq 25, 0 \leq x_{2,3} \leq 10, 0 \leq x_{2,4} \leq 20; \) When \(t = 3\): \(0 \leq x_{3,1} \leq 25, 0 \leq x_{3,2} \leq 10, 0 \leq x_{3,3} \leq 25, 0 \leq x_{3,4} \leq 20; \) When \(t = 4\): \(0 \leq x_{4,1} \leq 50, 0 \leq x_{4,2} \leq 10, 0 \leq x_{4,3} \leq 15, 0 \leq x_{4,4} \leq 25; \) When \(t = 5\): \(0 \leq x_{5,1} \leq 40, 0 \leq x_{5,2} \leq 10, 0 \leq x_{5,3} \leq 10, 0 \leq x_{5,4} \leq 20; \) When \(t = 6\): \(0 \leq x_{6,1} \leq 65, 0 \leq x_{6,2} \leq 10, 0 \leq x_{6,3} \leq 20, 0 \leq x_{6,4} \leq 25. \)
Fig. 1: Train route diagram

<table>
<thead>
<tr>
<th>O-D pair</th>
<th>Fare No.</th>
<th>Fare</th>
<th>The probability of demand range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>t = 1</td>
<td>1600</td>
<td>[0.40)–95% [40,59)–90% [50,70)–80% [70,80)–65%</td>
</tr>
<tr>
<td></td>
<td>t = 2</td>
<td>1280</td>
<td>[0.65)–95% [65,90)–90% [90,100)–80% [100,120)–65%</td>
</tr>
<tr>
<td>A-C</td>
<td>t = 3</td>
<td>2000</td>
<td>[0.25)–95% [25,35)–90% [35,50)–80% [50,80)–65%</td>
</tr>
<tr>
<td></td>
<td>t = 4</td>
<td>1600</td>
<td>[0.50)–95% [50,80)–90% [80,100)–80% [100,120)–65%</td>
</tr>
<tr>
<td>B-C</td>
<td>t = 5</td>
<td>800</td>
<td>[0.40)–95% [40,50)–90% [50,80)–80% [80,80)–65%</td>
</tr>
<tr>
<td></td>
<td>t = 6</td>
<td>640</td>
<td>[0.65)–95% [65,75)–90% [75,100)–80% [100,120)–65%</td>
</tr>
</tbody>
</table>

Values in parenthesis represent the probability of demand for within seats with respective CI.

Table 2: Computational results

<table>
<thead>
<tr>
<th>Station</th>
<th>Fare 1</th>
<th>Fare 2</th>
<th>Fare 1</th>
<th>Fare 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40</td>
<td>50</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>Value range</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cumulative value</td>
<td>60</td>
<td>50</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
<td>-</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Value range</td>
<td>-</td>
<td>-</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>10</td>
<td>0</td>
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<td></td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cumulative value</td>
<td>-</td>
<td>-</td>
<td>60</td>
<td>50</td>
</tr>
</tbody>
</table>

\[
\sum_{i=1}^{6} x_{i,k} = x_i (t=1, \ldots, 6).
\]

There are 24 variables, by solving the model, get the results as shown in Table 2.

The data are not accurate in Table 1, in practical applications, the railway departments can use statistical package in analyzing complex survey data to analyze passenger flow volume. There are three statistical packages: SAS, SPSS and STATA, can be used (Oyeyemi et al., 2010). However, the experimental results in table 2 verify the rationality of the seat control model built up in the study.

CONCLUSION

This study played a positive role for the railway departments to gain better economic benefit, based on analyzing and studying the railway passenger seat inventory control optimization, improved the seat control model for multi-interval various fares, made the model better considering
the loss the train produced because of uncertain filling situation and introduced this uncertainty into the model to modify and offset it. In practical applications, the railway departments should base on the detailed investigation and statistical analysis about passenger transport market to get accurate forecasted data, formulate rational fare policy flexibly and use the results calculated by the seat control model built up in the study, there will be guiding significance for railway passenger's practical work.

ACKNOWLEDGMENTS
The study described in this research was supported by National High-Tech Research and Development Program of China (863 Program) under Grant No. 2009AA062703 and the State Key Laboratory of Rail Traffic Control and Safety of Beijing Jiaotong University under Contract No. RCS20080R006.

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