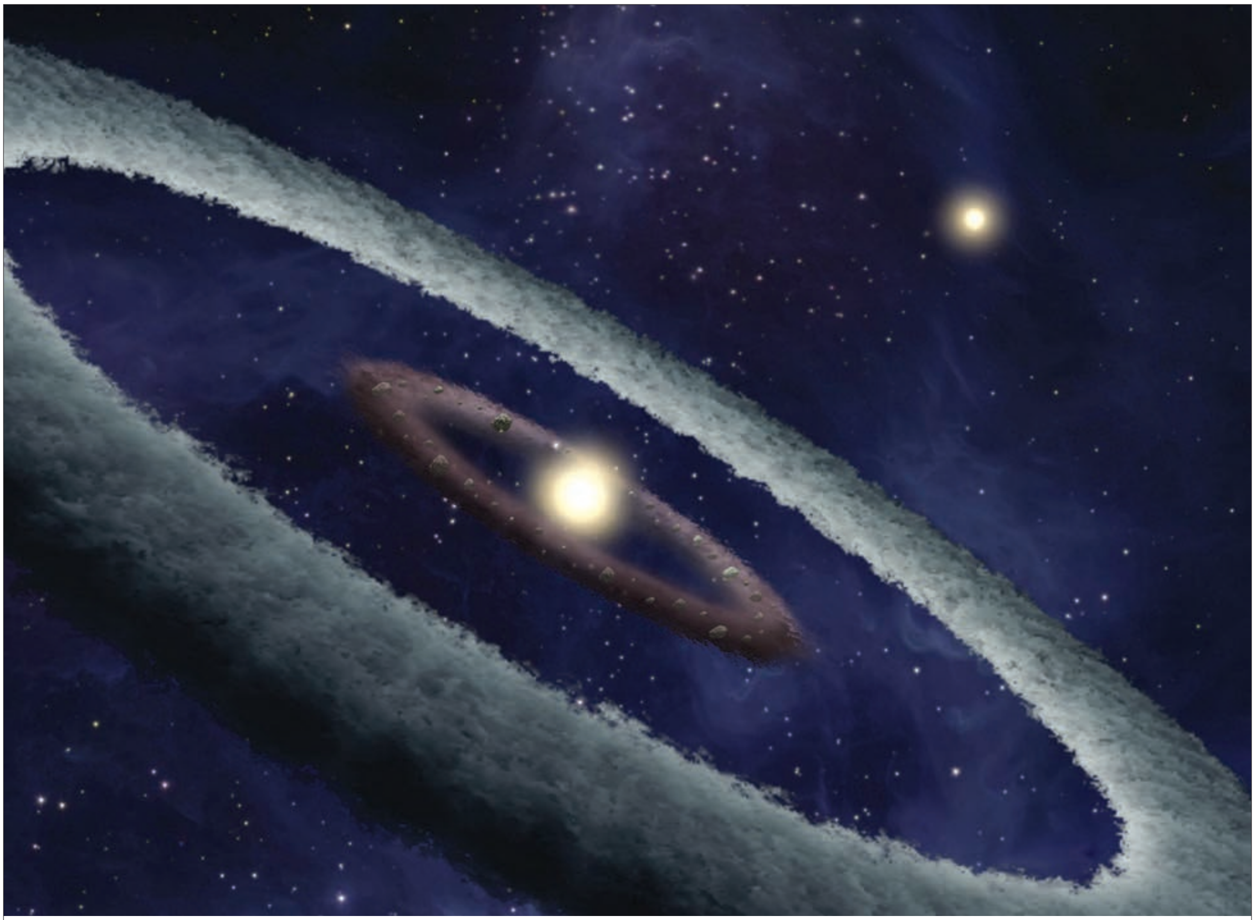

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On the Effectiveness of Variation in the Physical Variables on the Generalized Couette Flow with Heat Transfer in a Porous Medium

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Abstract: The influence of variation in physical variables on the steady generalized Couette flow with heat transfer in a porous medium is studied. The fluid is acted upon by a constant pressure gradient. The viscosity and the thermal conductivity are assumed to be temperature dependent. The two plates are kept at two constant but different temperatures and the viscous dissipation is considered in the energy equation. A numerical solution for the governing non-linear coupled equations of motion and the energy equation is obtained. The effect of porosity, the temperature dependent viscosity, thermal conductivity and electric conductivity on both the velocity and temperature distributions is examined.

Key words: Fluid mechanics, heat transfer, porous medium, variable properties

INTRODUCTION

The flow between parallel plates is a classical problem that has important applications in magnetohydrodynamic power generators and pumps etc. Hartmann and Lazarus (1937) studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary and insulating plates. Then the problem was extended in numerous ways. Closed form solutions for the velocity fields were obtained (Tao, 1960; Alpher, 1961; Sutton *et al*., 1965; Cramer and Pai, 1973) under different physical effects. Some exact and numerical solutions for the heat transfer problem are found in (Nigam and Singh, 1960; Attia *et al*., 1996). In the above mentioned cases the Hall term was ignored in applying Ohm's law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable (Cramer and Pai, 1973). Under these conditions, the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force. Abo-El-Dahab (1993) studied the effect of Hall currents on the steady Hartmann flow subjected to a uniform suction and injection at the bounding plates. Tani (1962) studied the Hall effect on the steady motion of electrically conducting and viscous fluids in channels. Soundalgekar *et al*. (1979) and Soundalgekar and Uplekor (1986) studied the effect of Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed either to be constant (Soundalgekar *et al*., 1979) or varying linearly along the plates in the direction of the flow (Soundalgekar and Upleker, 1986). Attia (1998) studied the Hall current effects on the velocity and temperature fields of an unsteady Hartmann flow with uniform suction and injection.

Most of these studies are based on constant physical properties. It is known that some physical properties are functions of temperature (Herwig and Wicken, 1986) and assuming constant properties is a good approximation as long as small differences in temperature are involved. More accurate prediction for the flow and heat transfer can be achieved by considering the variation of the physical

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properties with temperature. Klemp *et al*. (1990) studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb (1996) studied the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity. Later Attia (1999) extended the problem to the transient state.

In the present study, the problem considered is the steady flow of a viscous incompressible fluid with heat transfer in a porous medium. The flow in the porous media deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium (Joseph *et al*., 1982; Ingham and Pop, 2002; Khaled and Vafai, 2003). The upper plate is moving with a uniform velocity while the lower plate is kept stationary and the fluid is acted upon by a exponential decaying pressure gradient. The viscosity and the thermal conductivity are assumed to vary with temperature. The two plates are kept at two constant but different temperatures. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. Thus, the coupled set of the equations of motion and the energy equation including the viscous dissipation term becomes non-linear and is solved numerically using the finite difference approximations to obtain the velocity and temperature distributions.

FORMULATION OF THE PROBLEM

The fluid is assumed to be flowing between two infinite horizontal plates located at the $y=\pm h$ planes. The upper plate is moving with a uniform velocity U_o while the lower plate is kept stationary. The flow is through a porous medium where the Darcy model is assumed (Khaled and Nafai, 2003). The two plates are kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2>T_1$. A constant pressure gradient is applied in the x-direction. The viscosity of the fluid is assumed to vary exponentially with temperature while its thermal conductivity are assumed to depend linearly on temperature. The viscous dissipation is taken into consideration. Since the plates are infinite in the x and z-directions, the physical quantities do not change in these directions and the problem is essentially one-dimensional.

The flow of the fluid is governed by the Navier-Stokes equation

$$\rho(\vec{v}.\vec{\nabla}v) = -\vec{\nabla}P + \vec{\nabla}.(\mu\vec{\nabla}v) + \vec{f}_b \tag{1}$$

where $\vec{v}$ is the velocity vector, P is the pressure, μ is the viscosity of the fluid and $\vec{f}_b$ is the body force per unit volume. Using Eq. 1 and 2, the two components of the Navier-Stokes equation are

$$-\frac{dP}{dx} + \frac{d}{dy}\left(\mu\frac{du}{dy}\right) - \frac{\mu}{K}u = 0 \tag{2}$$

where K is the Darcy permeability (Khaled and Vafai, 2003). The no-slip condition at the plates implies that

$$y = -h : u = 0 \tag{3a}$$

$$y = -h : u = U_o \tag{3b}$$

The energy equation describing the temperature distribution for the fluid is given by White (1991):

$$\frac{1}{\rho c_p}\left(\frac{d}{dy}\left(k\frac{dT}{dy}\right)+\mu\left(\frac{du}{dy}\right)^2\right)=0 \tag{4}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid and k is thermal conductivity of the fluid. The last term in the left-hand side of Eq. 4 represents the viscous dissipation, respectively.

The temperature of the fluid must satisfy the boundary conditions,

$$T = T_1,\ y = -h \tag{5a}$$

$$T = T_2,\ y = h \tag{5b}$$

The viscosity of the fluid is assumed to vary with temperature and is defined as, $\mu = \mu_o f_1(T)$ and μ_o is the viscosity of the fluid at $T = T_1$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form (Attia and Kotb, 1996), $f_1(T) = \exp(-a_1(T-T_1))$. In some cases the parameter a_1 may be negative, i.e., the coefficient of viscosity increases with temperature (Attia and Kotb, 1996; Attia, 1999).

Also the thermal conductivity of the fluid is varying with temperature as $k = k_o f_2(T)$ and k_o is the thermal conductivity of the fluid at $T = T_1$. We assume linear dependence for the thermal conductivity upon the temperature in the form $k = k_o[1+b_1(T-T_1)]$ (White, 1991), where the parameter b_1 may be positive or negative (White, 1991).

The problem is simplified by writing the equations in the non-dimensional form. To achieve this define the following non-dimensional quantities,

$$\hat{y}=\frac{y}{h},\hat{x}=\frac{x}{h},\hat{G}=\frac{hG}{\rho U_o^2},\hat{u}=\frac{u}{U_o},\hat{T}=\frac{T-T_1}{T_2-T_1},$$

$\hat{f}_1(T)=e^{-a_1(T_2-T_1)T}=e^{-aT}$, a is the Viscosity parameter,
$\hat{f}_2(T)=1+b_1(T_2-T_1)T=1+bT$, b is the thermal conductivity parameter,
$Re = \rho U_o h/\mu_o$, the Reynolds number,
$M = h\mu_o/(\rho U_o K)$,parameter,
$Pr = \mu_o c_p/k_o$, number,
$Ec = U_o^2/c_p(T_2-T_1)$, the Eckert number,
$\tau_L = (\partial \hat{u}/\partial \hat{y})$ $\hat{y} = -1$ is the axial skin friction coefficient at the lower plate,
$\tau_U = (\partial \hat{u}/\partial \hat{y})$ $\hat{y} = 1$ is the axial skin friction coefficient at the upper plate,
$Nu_L = (\partial T/\partial \hat{y})$ $\hat{y} = -1$ is the Nusselt number at the lower plate,
$Nu_U = (\partial T/\partial \hat{y})$ $\hat{y} = 1$ is the Nusselt number at the upper plate,
In terms of the above non-dimensional quantities the velocity and energy Eq. 3 to 6 read (the hats are dropped for convenience)

$$G+f_1(T)\frac{d^2u}{dy^2}+\frac{df_1(T)}{dy}\frac{du}{dy}-f_1(T)Mu=0 \tag{6}$$

$$y=-1, u=0 \tag{7a}$$

$$y=1, u=1 \tag{7b}$$

$$\frac{1}{Pr}f_2(T)\frac{d^2T}{dy^2}+\frac{1}{Pr}\frac{df_2(T)}{dy}\frac{dT}{dy}+Ecf_1(T)\left(\frac{du}{dy}\right)^2=0 \tag{8}$$

$$T = 0,\ y = -1 \tag{9a}$$

$$T = 1,\ y = 1 \tag{9b}$$

Equation 6 and 8 represent coupled system of non-linear ordinary differential equations which are solved numerically under the boundary conditions (7) and (9) using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method is used and an iterative scheme is used to solve the linearized system of difference equations. The resulting block tri-diagonal system is solved using the generalized Thomas-algorithm (Ames, 1977). Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the *y*-direction. The computational domain is divided into meshes each of dimension Δy. We define the variables v = du/dy and H = dT/dy to reduce the second order differential Eq. 6 and 8 to first order differential equations. The finite difference representations for the resulting first order differential take the form

$$\begin{gathered}G+\left(\frac{\bar{f}_1(T)_{i+1}+\bar{f}_1(T)_i}{2}\right)\left(\frac{v_{i+1}-v_i}{\Delta y}\right)+\left(\frac{\bar{f}_1(T)_{i+1}-\bar{f}_1(T)_i}{\Delta y}\right)\left(\frac{u_{i+1}-u_i}{2}\right)- \\ M\left(\frac{\bar{f}_1(T)_{i+1}+\bar{f}_1(T)_i}{2}\right)\left(\frac{u_{i+1}-u_i}{2}\right)=0\end{gathered} \tag{10}$$

$$\begin{gathered}\frac{1}{Pr}\left(\frac{\bar{f}_2(T)_{i+1}+\bar{f}_2(T)_i}{2}\right)\left(\frac{H_{i+1}-H_i}{\Delta y}\right)+\frac{1}{Pr}\left(\frac{\bar{f}_2(T)_{i+1}-\bar{f}_2(T)_i}{\Delta y}\right)\left(\frac{H_{i+1}+H_i}{2}\right)+ \\ Ec\left(\frac{\bar{f}_1(T)_{i+1}+\bar{f}_1(T)_i}{2}\right)\left(\frac{\bar{v}_{i+1}+\bar{v}_i}{2}\right)\left(\frac{v_{i+1}+v_i}{2}\right)=0\end{gathered} \tag{11}$$

The variables with bars are given initial guesses and an iterative scheme is used to solve the linearized system of difference equations. Computations have been made for α =5, Pr = 1 and Ec = 0.2. Grid-independence studies show that the computational domain $-1<y<1$ can be divided into intervals with step size Δy = 0.005. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns u, v, T and H for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1<y<1$. Less than 7 approximations are required to satisfy this convergence criteria for all ranges of the parameters studied here.

RESULTS AND DISCUSSION

Figure 1 presents the velocity distribution as functions of *y* for various values of the parameters a and M and for b = 0. It is clear that increasing the parameter M decreases the velocity u for all values of a as a result of increasing the damping force on u. On the other hand, increasing the parameter a increases u for all M due to the decrease in viscosity. It is also concluded that the influence of the parameter a on u is more pronounced for smaller porosity parameter.

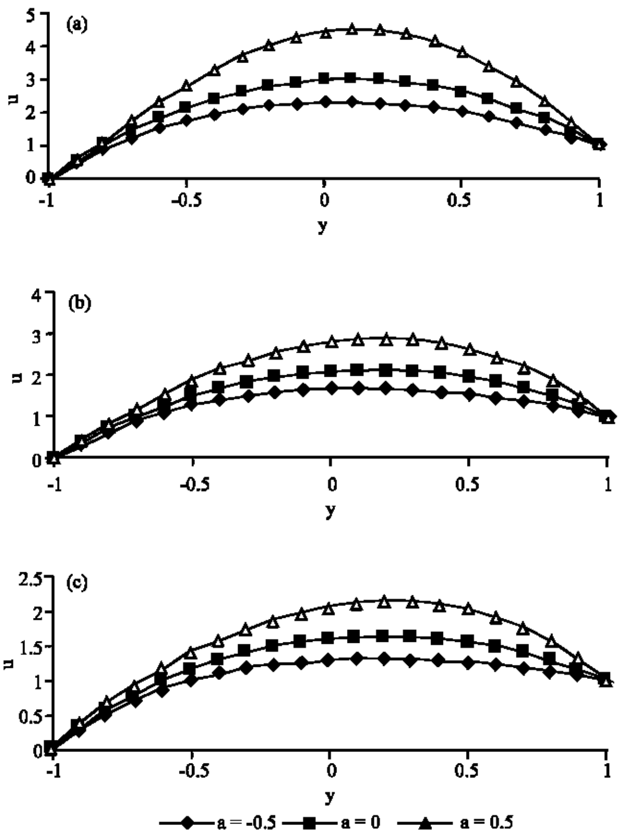

Fig. 1(a): Variation of u with y for various values of a (b = 0, M = 0), (b) Variation of u with y for various values of a (b = 0, M = 1) and © Variation of u with y for various values of a (b = 0, M = 2)

Figure 2 presents the temperature distribution as functions of y for various values of the parameters a and M and for b = 0. Increasing the parameter M decreases T as a result of decreasing *u* and, in turn, decreases the viscous dissipation. On the other hand, increasing the parameter *a* increases T for all values of M due to its effect in increasing u and consequently increasing the viscous dissipation. It is clear also that the effect of a on T is more apparent for smaller M.

Figure 3 presents the temperature distribution as functions of y for various values of the parameters b and M and for a = 0. The figure indicates that the effect of b on T depends on t and increasing b increases T at small times, but decreases T when t is large. This occurs because, at low times, the centre of the channel acquires heat by conduction from the hot plate, but after large time, when u is large, the viscous dissipation is large at the centre and centre looses heat by conduction. It is clear also that the effect of b on T is more apparent for smaller M. It is noticed that the parameter b has no significant effect on u in spite of the coupling between the momentum and energy equations.

Table 1 shows the dependence of the temperature at the centre of the channel on a and b for M = 1. In Table 1, T increases with increasing a for all values of b. On the other hand, for smaller values of a, increasing b increases T, while for higher values of a increasing b decreases it. This

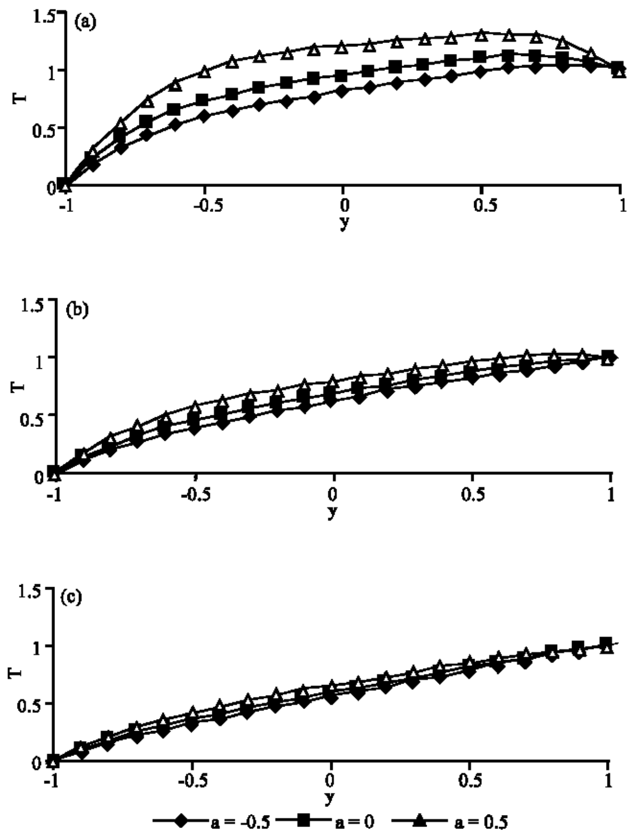

Fig. 2a: Variation of T with y for various values of a (b = 0, M = 0), (b) Variation of T with y for various values of a (b = 0, M = 1) and © Variation of T with y for various values of a (b = 0, M = 2)

Table 1: Variation of the steady state temperature T at y = 0 for various values of a and b (M = 1)

T	a = -0.5	a = -0.1	a = 0.0	a = 0.1	a = 0.5
b = -0.5	0.5898	0.6569	0.6785	0.7033	0.8543
b = -0.1	0.6186	0.6699	0.6859	0.7037	0.8034
b = 0.0	0.6243	0.6727	0.6876	0.7042	0.7959
b = 0.1	0.6292	0.6749	0.6889	0.7044	0.7893
b = 0.5	0.6419	0.6789	0.6901	0.7025	0.7677

is because decreasing a decreases the velocity and its gradient which decreases dissipation and makes the centre gain heat by conduction. Higher values of a increases dissipation and the centre looses heat by conduction which result in a decrease in T when increasing b.

Table 2a and b present the effect of the parameters a and b on the skin friction coefficients at both walls τ_L and τ_U, respectively, for M = 1. Increasing a increases τ_L and the magnitude of τ_U for all b. The effect of b on τ_L and τ_U depends on a. For small a, increasing b increases τ_L but decreases the magnitude of τ_U. On the other hand, for higher values of a, increasing b decreases τ_L but increases the magnitude of τ_U.

Tables 3a and b present the effect of the parameters a and b on the Nusselt numbers at both walls Nu_L and Nu_U, respectively, for M = 1. Increasing α increases Nu_L but decreases Nu_U. The effect of the parameter b on Nu_L depends on the value of b. Increasing b decreases Nu_L but increasing b more increases Nu_L. On the other hand, increasing b decreases the magnitude of Nu_U.

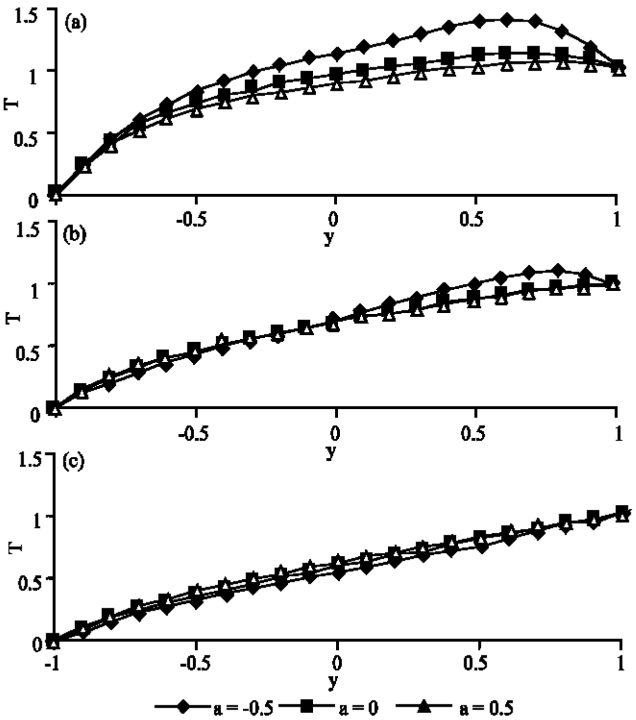

Fig. 3a: Variation of T with y for various values of b (a = 0, M = 0), (b) Variation of T with y for various values of b (a = 0, M = 1) and © Variation of T with y for various values of b (a = 0, M = 2)

Table 2a: Variation of the steady state skin friction coefficient at the lower plate τ_L for various values of a and b (M = 1)

τ_L	a = -0.5	a = -0.1	a = 0.0	a = 0.1	a = 0.5
b = -0.5	3.7462	4.0166	4.0838	4.1507	4.4173
b = -0.1	3.7602	4.0211	4.0837	4.1455	4.3848
b = 0.0	3.7626	4.0219	4.0837	4.1446	4.3791
b = 0.1	3.7649	4.0228	4.0837	4.1436	4.3741
b = 0.5	3.7724	4.0251	4.0837	4.1409	4.3587

Table 2b: Variation of the steady state skin friction coefficient at the lower plate τ_U for various values of a and b (M = 1)

τ_U	a = -0.5	a = -0.1	a = 0.0	a = 0.1	a = 0.5
b = -0.5	-1.6888	-2.4984	-2.7737	-3.0841	-4.7516
b = -0.1	-1.6499	-2.4927	-2.7738	-3.0888	-4.7675
b = 0.0	-1.6434	-2.4917	-2.7738	-3.0894	-4.7691
b = 0.1	-1.6377	-2.4907	-2.7738	-3.0903	-4.7702
b = 0.5	-1.6211	-2.4883	-2.7738	-3.0922	-4.7711

Table 3a: Variation of the steady state Nusselt number at the lower plate Nu_U for various values of a and b (M = 1)

Nu_L	a = -0.5	a = -0.1	a = 0.0	a = 0.1	a = 0.5
b = -0.5	1.2619	1.4180	1.4677	1.5235	1.8355
b = -0.1	1.2303	1.3949	1.4457	1.5018	1.8035
b = 0.0	1.2313	1.3975	1.4484	1.5046	1.8043
b = 0.1	1.2342	1.4017	1.4527	1.5089	1.8068
b = 0.5	1.2531	1.4242	1.4757	1.5319	1.8239

Table 3b: Variation of the steady state Nusselt number at the upper plate Nu_U for various values of a and b (M = 1)

Nu_U	a = -0.5	a = -0.1	a = 0.0	a = 0.1	a = 0.5
b = -0.5	0.2554	- 0.0644	-0.1652	-0.2782	-0.9157
b = -0.1	0.1886	0.0037	-0.0536	-0.1173	-0.4661
b = 0.0	0.1711	0.0037	-0.0481	-0.1054	-0.4174
b = 0.1	0.1546	0.0017	-0.0454	-0.0976	-0.3793
b = 0.5	0.0991	- 0.0144	-0.0491	-0.0872	-0.2888

CONCLUSIONS

The steady flow between two parallel plates in a porous medium is studied. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. The effects of the porosity parameter M, the viscosity parameter a and the thermal conductivity parameter b on the velocity and temperature fields are discussed. It is found that the parameters a and b have a more pronounced effect on the velocity and temperature distributions for smaller values of M. On the other hand the parameter b has no significant effect on the velocity u, however, it has a marked effect on the temperature and its effect depends greatly on the parameters M and a. It is of interest to find that the variation of the Nusselt number at the lower plate with the thermal conductivity parameter b depends on the values of b.

NOMENCLATURE

a: Viscosity parameter
b: Thermal conductivity parameter
c_p: Specific heat at constant pressure
Ec: Eckert number
k: Thermal conductivity
K: Darcy permeability
P: Pressure gradient
Pr: Prandtl number
M: Porosity parameter
T: Temperature of the fluid
T_1: Temperature of the lower plate
T_2: Temperature of the upper plate
U_o: Velocity of the upper plat
u: Velocity component if the x-direction
v: Velocity of the fluid
x: Axial direction
y: Distance in the vertical direction
μ: Viscosity of the fluid
ñ: Density of the fluid

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