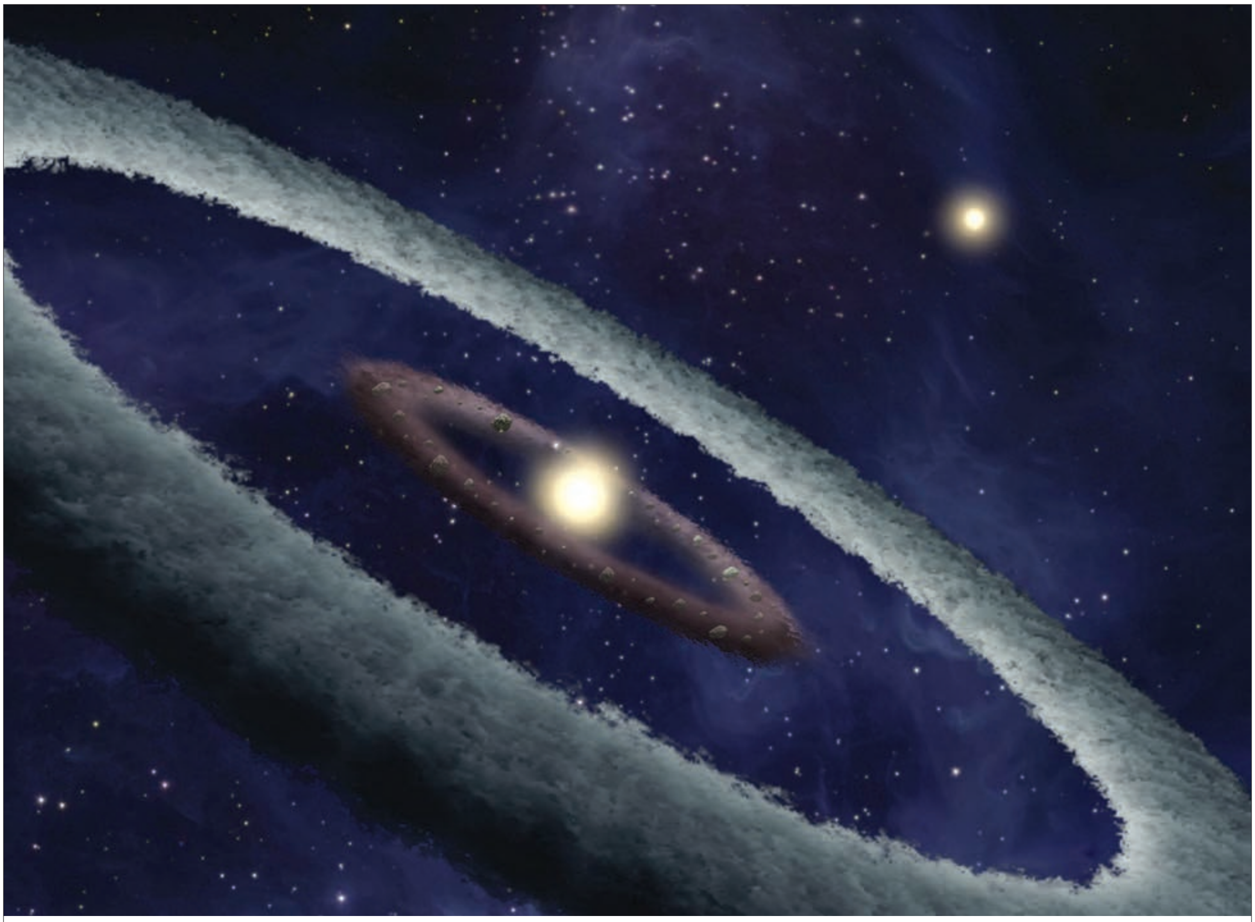

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Quantum Maxwell-Bloch Equation for Laser Model of Quasar

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Abstract: Quantum Maxwell-Bloch Equation (QMBE) for the description of the nature of laser action in astrophysical jets of quasars and AGNs have been developed. The theory self consistently incorporates population dynamics of the active volume and accounts for the three-dimensional nature of the gain medium as is expected would be the case in astrophysical jets. Most importantly, the model self consistently incorporates atomic parameters, which are fundamental for quantitative comparison with experimental results.

Key words: QMBE, laser, active volume, astrophysical jets, quasars, AGNs

INTRODUCTION

Most of the observed phenomena in extragalactic radio sources have been suggested to have their origin in the predominance electromagnetic radio impinging on the particles: electrons, protons, neutrons etc in these already excited states. The analysis of the emission lines on the assumption of redshift in quasar stellar objects (QSOs) reaffirms that the strength of these emissions has their origin in laser action (Varshni, 1974a and b; Varshni and Lam, 1974). The active medium, a veritable tool needed to initiate this laser action is the collection of atoms, molecules or ions that form the accretion disk which normally emits at the optical region of the electromagnetic spectrum.

To understand and appreciate the phenomenon of laser action in jets of astrophysical sites: quasars, AGNs, we will look at it from three stage perspective. First, in thermal equilibrium, most of the atoms in the active laser medium (accretion disk) are in the ground state; pumping normally induced by electromagnetic radiation interaction with the atoms leads to particle ionization with number density n(E) in the excited state E given by the Boltzmann relation:

$$n(E) = n_o \exp\left(-\frac{E}{\kappa T}\right) \tag{1}$$

where n_o is the equilibrium density state, T is the excitation temperature and κ is the Boltzmann constant. This further causes a non-radiative transition to the metastable state, which in AGN jets corresponds to the edge of the gap, which is a state of higher energy level with relatively long relaxation time. A consequence of this long relaxation time, accumulation of particles is expected in this state.

In considering some of the morphological aspects of astrophysical jets, the role of coherent mechanism needs to be understood as applied in astrophysical context. This is important sequel to the fact that coherent emission processes provides relatively natural and plausible explanation of the luminosity of gaps that are both features of galactic and extragalactic jets (Baker *et al*., 1988). Also, it reaffirms that a QSO is a star with a rapidly expanding atmosphere in which population inversion is predominant (Ekpe, 2004).

The suggestion of the application of a coherent model of quasar is fundamentally based on the idea of plasma processes in planetary magnetosphere (Matsumoto *et al*., 1996; Kudoh and Shibata,

1995) and laboratory experiments (laboratory plasma devices) (Baker *et al*., 1988) where coherent emission is a single particle emission process with energy high enough to cause collective emission that subdues completely the incoherent processes. It also predicts an astrophysical jet with higher energy rate extraction per particle for electron jet population that can as well be applied in incoherent synchrotron emission models.

The purpose of this research is to apply the quantum Maxwell-Bloch equation in an astrophysical jet of quasars and AGNs to study the phenomenon of laser action. The formalism is derived based on the study of the interaction between radiation and matter incorporating the field-field correlations and field-dipole correlations. This approach is totally different from the previous study (Ekpe, 2004) where the correlation parameters that introduces damping factor in the field were not considered.

In this study, an attempt will be made within the context of coherent emission model to show how the QMaxwell-Bloch equations (QMBEs) can be used in determining the physical parameters needed for the study of possible laser action in quasars. However, no attempt will be made to explain the energetic aspect of the laser operation developed using the laser rate equation approach (Narducci and Abraham, 1988).

THEORETICAL FRAMEWORK

In this study, we present the theoretical approach to studying laser action in astrophysical jets of quasars and AGNs. We first lay clam to the fundamental theories and ideas that supports possible laser action in astrophysical jets (Vashni, 1974a) and on that basis, develop a general theoretical framework for discussing laser systems. The second part will focus on the basis for the development of the general formalism and we will finally develop from the basic model the required model needed for the description of laser actions in astrophysical jets of quasars and AGNs. Most importantly, the model to be developed based on the QMBE will self consistently incorporate atomic interaction parameters such as coherent effects (Gammaitoni *et al*., 1998; Hodges *et al*., 1997) single photon processes (Abraham, 1989), dynamical stark shifts (Meystre and Sargent, 1991), quantum noise (Hodges *et al*., 1993; Harvey *et al*., 1994) and homogeneous broadening (Abraham *et al*., 1988). These parameters are needed for quantitative comparison with experimental findings.

BASIC FORMULATION

Considering a homogeneously broadened two-photon laser system (Fader, 1985) governed by the Schrödinger equation (Abraham, 1989). Let us further assume a ring cavity model with a Gaussian transverse profile that supports only TEM_{00} mode and a unidirectional cavity field that resembles a plane wave with constant intensity profile along the transverse direction (Narducci and Abraham, 1988; Yariv, 1987; Abraham, 1989). The equation describing the coherent interaction between the electromagnetic radiation (incident photon) and a homogeneously broadened two-photon system are:

$$\frac{\partial P(z,t)}{\partial t}+\left[\frac{1}{T_2}+i(\omega_c-\omega_a)\right]P(z,t)=i\mu^2\frac{\Delta N}{\hbar}E(z,t) \tag{2}$$

$$\frac{\partial E(z,t)}{\partial t}=i\frac{\omega_a}{2\varepsilon_0 cn}P(z,t) \tag{3}$$

$$\frac{\partial \Delta N}{\partial t}=\frac{\mu}{2\hbar}\left[P(z,t)E^*(z,t)+P^*(z,t)E(z,t)\right] \tag{4}$$

where, E (z,t) and P(z,t) are the complex electric field and resonant atomic polarization envelopes propagating along the z-axis, t is the local time, ω_a and ω_c are the atomic line and cavity resonance frequencies, respectively, μ is the modulus of the atomic transition dipole moment and all other symbols retain their usual meanings.

Assuming a low-loss cavity and an incident pulse $E_{in}(t)$ in a regime $<<T_2$, the change in the pulse shape on passing through the active volume can be obtained approximately as:

$$\Delta E(t) \sim \exp\left(-\frac{t}{T_2}\right)\exp i\delta_{ac}t \tag{5}$$

where, $\delta_{ac} = \omega_a - \omega_c$ is the detuning parameter between the centre of the atomic line and the cavity resonance selected as reference.

As a consequence of the ring cavity model adopted for this discussion, let the two mirrors 1 and 2 which corresponds to the boundary of the gap acting like the hot and cold parallel plates in the Rayleigh-Bourdon convection be of the same amplitude with reflectivity coefficient $R^{1/2}$ of an active volume of length ℓ and ring resonator of length Λ. The boundary conditions for the Maxwell's fields is:

$$E(z,t) = RE\left(\ell, t - \frac{\Lambda - \ell}{c}\right) \tag{6}$$

which as a consequence of time delay and scale factor, R on the right hand side, do not depict the usual periodicity as demanded by nature (Abraham, 1989).

Following Narducci and Abraham (1988), after define a new set of dimensiouless variables:

$$F(z,t) = \frac{\mu E(o,t)}{\hbar(\gamma_\perp \gamma_c)^{\frac{1}{2}}} \tag{7a}$$

$$P(z,t) = \left(\frac{\gamma_\perp}{\gamma_c}\right)^{\frac{1}{2}} P(o,t) \tag{7b}$$

$$\Delta N(z,t) = \Delta N(o,t) \tag{7c}$$

where, $\gamma_c = \frac{1}{T_1}$ and $\gamma_\perp = \frac{1}{T_2}$ are the population and polarization relaxation rates, respectively, we develop a full set of Maxwell-Bloch equations:

$$\frac{\partial A(z,t)}{\partial z} + \frac{1}{c}\frac{\partial A(z,t)}{\partial z} = -\frac{N\mu^2\omega_a}{2\hbar\varepsilon_o cn\gamma_\perp} P(z,t) \tag{8}$$

$$\frac{\partial P(z,t)}{\partial t} - \gamma_\perp\left[P(z,t) + A(z,t)\Delta N(z,t)\right] - \delta_{ac}P(z,t) \tag{9}$$

$$\frac{\partial\Delta(z,t)}{\partial t}=\frac{1}{2}\gamma_c\left[P^*(z,t)A(z,t)+P(z,t)A^*(z,t)\right]-\gamma_c\left[\Delta N(z,t)-1\right] \tag{10}$$

DYNAMICS OF THE ACTIVE MEDIUM CARRIER

The electron-photon interactions invariably, causes dissipation. Many models have been proposed to deal with this many-particle interactions (Chow *et al*., 1994). Let us assume that the electron density fluctuation be as a consequence of electron-photon interactions and define the ambipolar carrier density N(r) as:

$$N(r)=\frac{1}{8\pi^3}\int f^e(r,k)d^3k \tag{11}$$

with light carrier interaction which shows how the field-dipole correlations converts electrons into photons as:

$$\frac{\partial}{\partial t}N(r)\Big|_{cL}=ig_o\frac{\sqrt{v_o}}{8\pi^3}\int d^3k\sum_i\left[C_{ii}(r;r,k)-C^*_{ii}(r;r,k)\right] \tag{12}$$

where, the index i denotes dipole density of the linear polarization direction of i = x,y,z. The total cavity carrier density is thus calculated to be:

$$\frac{\partial}{\partial t}N(r)=D_{amb}\Delta N(r)+j(r)-\gamma N(r)+ig_o\frac{\sqrt{v_o}}{8\pi^3}\int d^3k\sum_i\left[C_{ii}(r;r,k)-C^*_{ii}(r;r,k)\right] \tag{13}$$

where, g_0 is the coupling frequency given as:

$$g_o=\left(\frac{\omega_o}{2\hbar\varepsilon v_o}\right)^{\frac{1}{2}}\left|d_{cv}\right| \tag{14}$$

and d_{cv} is the dipole element matrix

DYNAMICS OF THE RADIATION AND ATOMIC SYSTEM

To appreciate the dynamics of the whole system, let us again consider a two-photon level approximation of a four level lasing medium. The spatiotemporal dynamics of a lasing system can be simulated successfully by adopting the semiclassical Maxwell-Bloch equations and ignoring the light field effects in the quantum field. A consequence of this assumption, its noise terms are completely compatible with the conditions as demanded by quantum mechanics (Holger and Ortwin, 1998). After small modifications to suit the four-level atoms (Siegmann, 1986), we obtain in three dimensional from Eq. 8-10 the following set of Maxwell-Bloch equations:

$$\frac{\partial P(r,t)}{\partial t}=-\left[\gamma_\perp+i\delta_{ac}\right]P(r,t)-\frac{i\mu^2(r)}{\hbar}\Delta N(r,t)A(r,t)+\mu n(E)\eta(r,t) \tag{15}$$

$$\frac{\partial \Delta N(r,t)}{\partial t} = \gamma_c\left[R(r,t) - \Delta N(r,t)\right] + \frac{i}{4\hbar}\left[A(r,t)P^*(r,t) - A^*(r,t)P(r,t)\right] + n(E)\varphi(r,t) \tag{16}$$

$$\left[c^2\nabla^2 - \varepsilon(r)\frac{\partial^2}{\partial t^2} - \kappa(r)\frac{\partial}{\partial t}\right]E(z,t)e^{-i\omega t} = 4\pi\left[\frac{\varepsilon(r)+2}{3}\right]\frac{\partial^2}{\partial t^2}P(r,t)e^{-i\omega t} \tag{17}$$

where, A(r,t) is the field local to the atoms, R(r,t) is the pump rate, n(E) is the number density of lasing centres in the pumped region, η(r,t) and φ(r,t) depicts the Langevin forces for intrinsic noise due to quantum fluctuations of the polarization and population inversion, respectively and all other symbols retain their usual meanings.

A cursory look at Eq. 15-17 show that for one to solve them or use it for any meaningful discussion, the relation between the local field A(r,t) and the macroscopic field E(r,t) must be established. With recourse to the Lorenz-Lorentz approximation (Jackson, 1975) to good order, we obtain this relation as:

$$A(r,t) \approx \left[\frac{\varepsilon(r)+2}{3}\right]E(r,t) \tag{18}$$

Using Eq. 10 in 15-17, we obtain a set of Maxwell-Bloch equations with linear local field corrections adequate for describing the relation of the atomic and optical dynamics for laser action in an astrophysical jet as:

$$\frac{\partial P(r,t)}{\partial t} = -\left[\gamma_\perp + i\delta_{ac}\right]P(r,t) - \frac{i\mu^2(r)}{\hbar}\Delta N(r,t)\left[\frac{\varepsilon(r)+2}{3}\right]E(r,t) + \mu n(E)\eta(r,t) \tag{19}$$

$$\frac{\partial \Delta N(r,t)}{\partial t} = \gamma_c\left[R(r,t) - \Delta N(r,t)\right] + \frac{i}{4\hbar}\left[\left[\frac{\varepsilon(r)+2}{3}\right]E(r,t)P^*(r,t) - \left[\frac{\varepsilon(r)+2}{3}\right]E^*(r,t)P(r,t)\right] + n(E)\varphi(r,t) \tag{20}$$

$$\left[c^2\nabla^2 - \varepsilon(r)\frac{\partial^2}{\partial t^2} - \kappa(r)\frac{\partial}{\partial t}\right]E(z,t)e^{-i\omega t} = 4\pi\left[\frac{\varepsilon(r)+2}{3}\right]\frac{\partial^2}{\partial t^2}P(r,t)e^{-i\omega t} \tag{21}$$

A perusal at Eq. 19-21 show that the dynamical parameters (variables) are now the macroscopic electric field E(r,t), the scaled dipole moment density (polarization field) P(r,t) and the population inversion density (loosely population inversion) ΔN(r,t). They are the basis parameters for full discussion of noise influenced dynamics of coupled matter-field systems which are consistent with the requirements of quantum mechanics and are in natural standing to the requirements that are needed for physical realizability of laser action in astrophysical jets of quasars and AGNs.

QUANTUM MAXWELL-BLOCH EQUATIONS (QMBE) FOR LASER ACTION IN JETS

The full description of the cavity (or local) field needs consideration of the complete three dimensional wave equation, a consequence of the finite transverse nature of the resonator cross section (Narducci and Abraham, 1988; Lamb, 1964; Abraham *et al*., 1988). This is also important considering the presumed complex nature of the astrophysical jets that are sites of laser actions in quasars and

AGNs, which cannot in principle be described classically. With recourse to the previous sections, we follow (Holger and Ortwin, 1998) to establish the quantum mechanical Maxwell-Bloch equation (QMBE) for bulk gain medium in three dimensions as:

$$\frac{\partial}{\partial t}N(r)=D_{amb}\Delta N(r)+j(r)-\gamma N(r)+ig_o\frac{\sqrt{v_o}}{8\pi^3}\int d^3k\sum_i\left[C_{ii}(r;r,k)-C^*_{ii}(r;r,k)\right] \tag{22}$$

$$\begin{aligned}\frac{\partial}{\partial t}C_{ij}(r;r',k)=&-\left[\Gamma(k)+i\Omega(k)\right]C_{ij}(r;r',k)-\sum_k\frac{\partial}{\partial r_i}\varepsilon_r^{-1}\frac{\partial}{\partial r_k}C_{kj}(r;r',k)+k_o^2C_{ij}(r;r',k)\\&+ig_o\frac{2\sqrt{v_o}}{3}\left[2f^e_{eq}(k;N(r)-1)\right]I_{ij}(r;r')+ig_o\frac{2\sqrt{v_o}}{3}\delta_{ij}f^{e2}_{eq}(k;N(r))\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\partial}{\partial t}I_{ij}(r;r',k)=&-\frac{i\omega_o}{2k_o^2}\sum_k\left[\frac{\partial}{\partial r_k}\varepsilon_r^{-1}\frac{\partial}{\partial r_k}-\frac{\partial}{\partial r'_k}\varepsilon_r^{-1}\frac{\partial}{\partial r'_k}\right]I_{ij}(r;r')+\frac{i\omega_o}{2k_o^2}\sum_k\left[\frac{\partial}{\partial r_i}\varepsilon_r^{-1}\frac{\partial}{\partial r_k}I_{kj}(r;r')-\frac{\partial}{\partial r'_j}\varepsilon_r^{-1}\frac{\partial}{\partial r'_k}I_{ik}(r;r')\right]\\&-ig_o\frac{\sqrt{v_o}}{8\pi^3}\int d^3k\sum_i\left[C_{ij}(r;r',k)-C^*_{jk}(r;r',k)\right]\end{aligned} \tag{24}$$

$$\frac{\partial}{\partial t}P_i(r;k)=-\left[\Gamma(k)+i\Omega(k)\right]P_i(r;k)+ig_o\frac{2\sqrt{v_o}}{3}\left[2f^e_{eq}(k;N(r)-1)\right]E_i(r) \tag{25}$$

$$\frac{\partial}{\partial t}E_i(r)=-\frac{i\omega_o}{2k_o^2}\sum_k\left[\frac{\partial}{\partial r_k}\varepsilon_r^{-1}\frac{\partial}{\partial r_k}E_i(r)-\frac{\partial}{\partial r_i}\varepsilon_r^{-1}\frac{\partial}{\partial r_i}E_k(r)+k_o^2E_i(r)\right]-ig_o\frac{\sqrt{v_o}}{8\pi^3}\int P_i(r,k)d^3k \tag{26}$$

with the loss due to cavity damping of the astrophysical gap given by

$$\kappa=\frac{c}{2\ell\sqrt{\varepsilon_r}}\mathrm{In}(R_1R_2) \tag{27}$$

where, $I(r;r',k)$ and $C(r;r',k)$ are the field-field correlation and field-dipole correlation respectively, $\Gamma(k)$ is the total momentum dependent scattering rate in the carrier system, $\Omega(k)$ is the momentum dependent frequency and all other symbols, retain their usual meanings.

Taking a perusal at the developed QMBE, it will be observed that they depict average spatial coherence of the light field which for any physical system need not only explain the behaviour of the light field in the active volume. This is however not a problem as it can be achieved simply by letting all material properties to vanish outside the finite active volume and the filed propagation then calcnlated outside the volume by allowing ε_r vary in vacuum.

ANALYTIC SOLUTION OF THE QUANTUM MAXWELL BLOCH EQUATIONS

The analytic solution of the QMBE is evidently to help in the understanding of one of the most important phenomenon that reduces the possibility of stimnlated emission. The spontaneous emission process and amplified spontaneous emission a consequence of the field-field correlations $I(r;r',k)$ and the field-dipole correlations $C(r;r',k)$ uttermoslty, reduces the amplification needed to sustain stimulated emission mechanism.

The dynamics of the optical field and the dipole density from the developed QMBE on careful observation, is found to be linear hence, the equation of motion can be integrated to obtain the Green function for the dynamics of the field consistent with our assumption of bulk gain medium as:

$$\frac{\partial}{\partial t}E(r,t)\Big|_{cL} = g_o^2\frac{\nu_o}{12\pi^2}\int d^3k\int_0^{\infty} d\tau e^{-[\Gamma(k)+i\Omega(k)\tau]}\times\left[2f^e(k)-1\right]E(r,t-\tau) \tag{28}$$

Solving the integral over the entire space ($d\tau$), we obtain that the amplification rate $G(\omega)$ for the optical field of frequency ω in the bulk medium is:

$$G_{bulk}(\omega) = g_o^2\frac{\nu_o}{12\pi^2}\int d^3k\frac{\Gamma(k)}{\Gamma^2(k)+\left[\Omega(k)-\omega\right]^2}\left[2f^e(k)-1\right] \tag{29}$$

The effect of the field-field correlations $I(r;r',k)$ and the field-dipole correlations $C(r;r',k)$ introduces spontaneous emission and amplified spontaneous emission. To establish the source of these emissions which acts as parasite in the active volume, the QMBE for the field-field correlations is solved to obtain the ratio of the stimulated emission processes to spontaneous emission mechanism, η_r as

$$\eta_r = \frac{\frac{\partial}{\partial t}C_{ij}(r;r',k)\Big|_{stimulated}}{\frac{\partial}{\partial t}C_{ij}(r;r',k)\Big|_{sponteneous}} = \frac{I_{ij}(r;r')\left(2f^e(k)-1\right)}{\delta(r-r')\delta_{ij}f^{e2}(k)} \tag{30}$$

where, $\delta(r-r')$ is the photon density per mode. Assuming that $2f^e(k) - 1 \approx f^{e2}(k)$ we obtain from Eq. 29 to good order the density of the spontaneous emission rate $S_{bulk}(\omega)$ for bulk medium as:

$$S_{bulk}(\omega) = n(r)_{light}\,g_o^2\frac{\nu_o}{6\pi^3}\int d^3k\frac{\Gamma(k)}{\Gamma^2(k)+\left[\Omega(k)-\omega\right]^2}f^{e2}(k) \tag{31}$$

where, factor 2 has been introduced to account for the intensity used instead of field consideration and $n(r)_{light}$ is the density of the light field modes per unit volume of the cavity of the edge gap frequency ω_0 of the jet given as

$$n(r)_{light} = \frac{\omega_o^2}{\pi^2c^3}\varepsilon_r^{\frac{3}{2}} = \frac{k_o^2}{\pi^2c}\varepsilon_r^{\frac{3}{2}} \tag{32}$$

In order to establish the total rate of spontaneous emission per unit cavity volume, we integrate Eq. 31 over all frequencies to obtain

$$\int S_{bulk}(\omega)d\omega = n(r)_{light}\,g_o^2\frac{\nu_o}{6\pi^2}\int f^{e2}(k)d^3k \tag{33}$$

But the integral $\int^{e2}(k)d^3k = 4\pi^3 N$, hence, we obtain from (33) that $\int S_{bulk}(\omega)d\omega = \eta_{Ts}$ is

$$\begin{aligned}\eta_{Ts} &= n(r)_{light}\,g_o^2\frac{\nu_o}{6\pi^2}\times 4\pi^3N\\ &= n(r)_{light}\frac{2\pi}{3}\frac{\nu_o}{6\pi^2}g_o^2N\end{aligned} \tag{34}$$

Substituting the values of g_0^2 and $n(r)_{light}$ from Eq. 14 and 32, respectively, we obtain η_{Ts} in terms of the dipole transition matrix element as

$$\eta_{Ts} = \frac{\omega_o^3}{2\hbar\varepsilon_o\varepsilon_r^{-\frac{1}{2}}\pi^2\nu_o c^3}\left|d_{cv}\right|^2 \tag{35}$$

From Eq. 34, let us define for generality (any cavity design), the rate of spontaneous emission $\frac{1}{\tau_s}$ as

$$\frac{1}{\tau_s} = \frac{2\pi}{3} n(r)_{light}\, g_o^2 N \tag{36}$$

Hence, we obtain the total density of spontaneous emission process as

$$S_{total} = \frac{N}{\tau_s} \tag{37}$$

Equation 37 is particularly important in the cavity design as it gives the total density of the parasite sucking the stimulated emission mechanism in the astrophysical jets of quasars and AGN.

CONCLUSIONS

A laser model that self consistently includes the dynamics of population inversion, local field correction, coherent mode coupling as well as quantum noise in the context of the two-photon laser system have been formalized. The quantum Maxwell-Bloch equation (QMBE) suitable for discussing the spatially inhomogeneous nature of laser action in astrophysical jets that takes into account the quantum mechanical nature of the field: local and light field have also been developed.

The model though describes the familiar results for simple cavities where the gain volume occupies the active volume; it further describes the coupled cavity system envisaged of a typical astrophysical jet undergoing laser action. However, this approach though theoretical, it is predicted to conform with experimental requirements as the QMBE applied here are fully developed from quantum mechanical approach based on the quantum description of the interaction of radiation (light field) with the quantum states of the active volume which are purely natural.

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REFERENCES

Abraham, N.B., P. Mandel and L.M. Narducci, 1988. Dynamical Instabilities and Pulsations in Lasers. In: Process in Optics XXV. Wolf, E. (Ed.), Elsevier, Amsterdam, pp: 25-28.

Abraham, N.B., 1989. An Overview of Optical Instabilities and Chaos and an Introduction to Some Models and Current Areas of Research in Laser Instabilities. In: Dynamics of Non-Linear Optical Systems. Pesquera, L. and F.J. Bermejo (Eds.), World Scientific Publ. Co., Pte: Singapore, pp: 3-29.

Baker, D.N., E.B. Joseph, B. Gregory and A.E. Jean, 1988. The collective emission of electromagnetic waves from astrophysical jets: Luminosity, gaps, BL lactertae objects and efficient energy transport. Astrophys. J., 326: 110-124.

Chow, W.W., S.W. Koch and M. Sargent, 1994. Semiconductor Laser Physics. Springer, Berlin.

Ekpe, O.E., 2004. Coulomb Scattering in Laser-Jet Model of Quasars. An Unpublished Ph.D Thesis, EBSU, Abakaliki, Nigeria, pp: 3-20.

Fader, W.J., 1985. Theory of two-coupled laser. IEEE J. Quantum Electr., QE-21: 1838-1844.

Gammaitoni, L., P. Hänggi, P. Jung and F. Marchesoni, 1998. Stochastic resonance. Rev. Mod. Phys., 70: 223-287.

Harvey, J.D., J.M. Dudley, P.F. Curley, C. Spielmann and F. Krausz, 1994. Coherent effects in a self-mode-locked Ti: Sapphire laser. Opt. Lett., 19: 972.

Hodges, S.E., M. Munroe, J. Cooper and M.G. Raymer, 1993. Compound cavity laser modes for arbitrary interface reflectivity. Opt. Lett., 18: 1481-1483.

Hodges, S.E., M. Munroe, J. Cooper and M.G. Raymer, 1997. Multimode laser model with coupled cavities and quantum noise. J. Opt. Soc. Am., 180: 191-193.

Holger, F.H. and H. Ortwin, 1998. Quantum maxwell-bloch equations for spatially inhomogeneous semiconductor lasers. arXiv: Physics/9807011, v2: 1-20.

Jackson, J.D., 1975. Classical Electrodynamics. Wiley, NY: USA., pp: 282-283.

Kudoh, T. and K. Shibata, 1995. Mass flux and terminal velocities of magnetically driven jets from accretion disks. Astrophys. J., 452: L41-L44.

Lamb, W.E., 1964. Theory of an optical maser. Phys. Rev., 134: A1429-A1450.

Matusumoto, R., Y. Uchida, S. Hirose, K. Shibata, M.R. Hayashi and A. Ferrari, 1996. Radio jets and the formation of active galaxies: Accretion avalanches on the torus by effect of a large scale magnetic field. Astrophys. J., 461: 115-126.

Narducci, L.M. and N.B. Abraham, 1988. Laser Physics and Laser Instabilities. World Scientific Publishing Co. Pte Ltd., Singapore, pp: 15-75.

Sargent, M., 1991. Elements of Quantum Optics. Springer-Verlag, NY, USA.

Siegmann, A.E., 1986. Lasers. University Science, Chap. 3, Mill Valley Calif, pp: 19-21.

Varshni, Y.P., 1974a. The redshift hypothesis and the plasma laser star model for quasi-stellar objects. Bull. Am. Astron. Soc., 6: 308.

Varshni, Y.P., 1974b. Proposed absorption redshift systems for 4c 05.34 are due to chance coincidences. Bull. Am. Astron. Soc., 6: 449.

Varshni, Y.P. and C.S. Lam, 1974. Emission line 4686 in the quasi-stellar objects. J. Roy. Astron. Soc., Canada, 68: 264.

Yari, A., 1987. Introduction to Optical Electronics. Holt-Rinehart, NY: USA., pp: 318-319.