The Amelioration of Einstein’s Equation and Removing of Cosmological Difficulties

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ABSTRACT
In the framework of gravitational theory of general relativity, this article has systematically and radically solved the problem of galaxy formation and some significant cosmological puzzles. A flaw with Einstein’s equation of gravitational field is firstly corrected and the foundations of general relativity are perfected and developed and space-time is proved to be infinite, expansion and contraction of universe are proved to be in circles, the singular point of big bang is naturally eliminated, celestial bodies and galaxies are proved growing up with cosmic expansion, for example Earth’s mass and radius at present increase by 1.2 trillion tons and 0.45 mm, respectively in a year, in response to which geostationary satellites rise by 2.7 mm.

Key words: Background coordinates, standard coordinates, geodesic, negative pressure

INTRODUCTION
Though general relativity obtains considerable success, some significant fundamental problems such as the problem of singular point, the problem of horizon, the problem of distribution and existence of dark matter and dark energy, the problem of the formation of celestial bodies and galaxies, the mystery of solar neutrino, as well as the problem of asymmetry of particle and antiparticle, always are not solved naturally and satisfactorily. These problems long remain implies strongly that the fundamentals of general relativity have flaw and needs further perfection. For the purpose, this paper begins with determining the vacuum solution of Einstein’s field equation in the background coordinate system, then by correcting rationally Einstein’s field equation from an all new perspective these get problems removed.

THE STATIC METRIC OF SPHERICAL SYMMETRY IN BACKGROUND COORDINATE SYSTEM
In this study light’s speed c = 1. According to general relativity, for the static and spherically symmetric case, in the standard coordinate system (Weinberg, 1972; Huanwu and Xieng, 1998), the correct form of invariant line element outside gravitational source is given by:

\[ ds^2 = -dt^2 + \left(1 - \frac{2GM}{r}\right)dr^2 + \left(1 - \frac{2GM}{r}\right)^{-1}d\theta^2 + \sin^2\theta d\phi^2 \] (1)
Equation 1 is a solution of vacuum field equation $R_{\mu\nu}$ in the coordinate system with $t, l, \theta, \phi$ as independent variables. Here, $\tau$ is proper time, $M$ is the total mass of gravitational source; $l$ is usually called standard radial coordinate which doesn’t have clear physical significance and only in the far field is approximately viewed as true radius. In order to describe clearly dynamic behavior and definite position of a particle in gravitational field and enable general relativity to have common language with other theories including Newton’s gravitational theory and compare results with one another, it is necessary to transform (1) into the form expressed in background coordinates. Hence we take $l = l(r)$. In this study, $r$ is defined as background coordinate (Zhou, 1983; Fock, 1964) and refers to true radius, that is to say, its meaning is the same as that used usually in quantum mechanics or electrodynamics. $t, \theta, \phi$ are standard coordinates and can also be viewed as background coordinates which represent true time and angle. In the following we try to determine $l = l(r)$ by the introduction of an additional transformation equation and such operation is allowed is because metric tensor satisfies Bianchi identity and if a metric is a solution of field equation in one coordinate system it is also a solution under arbitrary coordinate transformation and the meaning of using coordinate transformation is to guarantee the new metrics meet field equation.

According to general relativity the dynamical equation of particle outside source is geodesic:

$$\frac{d^2 x^\mu}{dt^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{dt} \frac{dx^\lambda}{dt} - \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{dt} \frac{dx^\lambda}{dt} = 0$$  (2)

where, $x_0 = t$ and indexes $\lambda, \nu, \mu, \sigma, \alpha, \beta = 1, 2, 3$. Equation 2 can exist in any coordinate system and is a basic equation to solve acceleration of moving particle.

When a particle of mass $m$ is moving along radius in the static gravitational field of spherical symmetry, giving consideration to the effect of its speed, in the background coordinate system, in the far field (weak field) the radial component of Eq. 2 should reduce to the following relativistic dynamic Eq. 3 rather than others:

$$\frac{d}{dt} \left( \frac{dr}{dt} \right) m = \frac{mGM}{r^2}$$  (3)

where, $m$ refers to relativistic dynamic mass, namely $m = \frac{m_0}{\sqrt{1-v^2}}$. Why the radial component should reduce to (3) is that (3) stands for the equality of gravitational mass and inertial mass and also stands for the speed of light is the limit one. In order to enable it to reduce to (3) we may introduce a transformation equation as follows:

$$\frac{dt}{dr} = \sqrt{\frac{2GM}{r} \exp(-\frac{GM}{r})}$$  (4)

The correctness of Eq. 4 will be seen later, it determines a coordinate transformation of $1-r$. By means of separating variables, the solution of Eq. 4 is easily given by:
Here, constant \(C_1\) is determined according to the continuity of function \(l = l(r)\) on the boundary of source and the back Eq. 23 can give out the boundary value \(l(r_s)\), \(r_s\) denotes source's radius (celestial body radius). Note that \(\Theta\) makes sure \(l=r\) for \(r \to \infty\), prove as follows.

Form Eq. 4 we see \(l \to \infty\) for \(r \to \infty\) and considering of:

\[
\lim_{x \to \infty} \ln x = 0
\]

it holds that for \(l \to \infty\) the left-hand side of \(\Theta\) is:

\[
\theta \left( \sqrt{1+\frac{2GM}{l}} - \frac{GM}{l} \ln \frac{1+\sqrt{1+\frac{2GM}{l}}}{l} \right) \approx l
\]

and for \(r \to \infty\), the right-hand side of \(\Theta\) is:

\[
r \left( \frac{C_1 + 1 - \frac{GM}{r} \ln (1 + \frac{GM}{r}) G^2 M^2}{2r^2 + G^2 M^2 12r^2 + \cdots} \right) \approx r
\]

Under transformation of Eq. 4, Eq. 1 becomes the following Eq. 5 which is an exact solution of vacuum field equation \(R_{\mu \nu} = 0\) in background coordinate system \(x^\mu = (t, r, \theta, \phi)\):

\[
ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{2GM}{r}dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right)
\]  

(5)

Note that now \(l = l(r)\) is already a concrete function of \(r\) which is determined by \(\Theta\) and can not be written explicitly. And here \(t, r, \theta, \phi\) are independent coordinate variables.

In the far field, the line element 5 provides:

\[
\begin{align*}
g_{00} &= -1 + \frac{2GM}{l} \approx -1 + \frac{2GM}{r}, \quad g_{01} = \exp(-\frac{2GM}{r}) \approx 1 - \frac{2GM}{r}, \quad g_{22} = l^2(r) \approx r^2, \\
g_{01} &= l^2(r) \sin^2 \theta = r^2 \sin^2 \theta, \quad \Gamma^0_{10} = \frac{GM}{r^2}, \quad \Gamma^0_{11} = \frac{GM}{r^2}, \quad \Gamma^0_{01} = \frac{GM}{r^2}, \quad \Gamma^0_{10} = \frac{GM}{r^2}, \quad \Gamma^0_{01} = 0, \quad \Gamma^0_{01} = 0,
\end{align*}
\]

and introducing them into Eq. 2 and putting:

\[
\mu = 1, \quad \dot{\theta} = 0, \quad \dot{\phi} = 0, \quad v = \frac{dr}{dt}
\]

we obtain:
\[ \frac{d^2r}{dt^2} + (1-v^2) \frac{GM}{r^3} = 0 \] (6)

Which is equivalent to Eq. 3.

**Proof:** Assume:

\[ d\theta = d\phi = 0, \quad m = \frac{m_0}{\sqrt{1-v^2}} \]

from Eq. 3 we have:

\[ 0 = \frac{d}{dt} \left[ \left( \frac{dr}{dt} \right) m \right] + \frac{mGM}{r} = m\left( \frac{d}{dt} \left( 1-v^2 \right)^{\frac{1}{2}} \right) \frac{dv}{dt} + \frac{mGM}{r} \frac{dr}{dt} m \frac{r}{v_0} + m \frac{GM}{r} = m \left[ (1-v^2) \frac{d^2r}{dt^2} + \frac{GM}{r} \right] \]

which immediately yields Eq. 6.

By far, we may say that Eq. 5 is just the appropriate line element expressed in background coordinate system \((t, r, \theta, \phi)\) which we look for and satisfies vacuum field equation and entire requirements on physics.

Obviously it is, however, neglected usually, necessary to identify which of the solutions that satisfy field equation in the same coordinate system is correct or more correct. As an example worthy of mentioning, we point out that applying directly \(l = r\) in (1), namely \(l\) is directly explained as background coordinate, gives the following another exact solution expressed in the same coordinate system \(x^\mu = (x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)\):

\[ ds^2 = \left( 1 - \frac{2GM}{r} \right) dt^2 + \left( 1 - \frac{2GM}{r} \right)^{-1} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \] (7)

However, in accordance with Eq. 7 the corresponding geodesic can’t reduce to Eq. 3 in weak field, instead it reduces to:

\[ \frac{d^2r}{dt^2} + (1-3v^2) \frac{GM}{r^3} = 0 \] (8)

**Proof:** Equation 7 provides:

\[ g_{00} = 1 + \frac{2GM}{r}, \quad g_{11} = \left( 1 - \frac{2GM}{r} \right)^{-1}, \quad g_{22} = r^2, \quad g_{33} = r^4 \sin^2 \theta, \quad g_{0\mu} = 0 (\mu \neq \nu), \]

\[ \Gamma^0_{11} = \frac{1}{2} \frac{g^{00}}{g^{ii}} \left[ \frac{\partial g_{ii}}{\partial x^0} + \frac{\partial g_{0i}}{\partial x^i} - \frac{\partial g_{00}}{\partial x^i} \right] = -\frac{GM}{(1-2GM/r)^3}, \quad \Gamma^0_0 = \frac{GM}{(1-2GM/r)^3}, \quad \Gamma^0_i = \frac{(1-2GM/r)GM}{r^3}, \quad \Gamma^0_j = 0 \]

substituting them into Eq. 2 and taking \( \mu = 1 \) and \( d\phi = d\theta = 0 \) yield immediately:
\[
\frac{dr}{dt} = -\Gamma^\alpha_{\beta\gamma} v^\alpha v^\beta + 2v^\alpha \Gamma^\alpha_{\beta\gamma} = -(1 - \frac{2GM}{r}) \frac{GM}{r^2} + \frac{3GM}{(1 - 2GM/r)^2} v^2 \quad \text{and for} \quad \frac{2GM}{r} \ll 1
\]

this equation distinctly reduces to Eq. 8 which isn't Eq. 3. It is easily found that Eq. 8 not only goes against the elementary principle of equality of gravitational mass and inertial mass but also leads to incorrect conclusion that gravitational field becomes repulsive one for a particle whose speed exceeds 0.58c. Hence, Eq. 8 must be wrong and implies Eq. 7 can't describe high speed and has a certain shortcoming compared with Eq. 5.

Note that the angle of orbital precession of Mercury described by Eq. 5 is still the same as that described by line element Eq. 7 (Huanwu and Xieng, 1998), the angle of orbital precession doesn't change under the transformation of radial coordinates. In a word Eq. 5 is the correct line element expressed in background coordinate system and again, though general relativity is fully covariant and can use all sorts of coordinates, we must use background coordinates when we take geodesic equation to compare with Newtonian gravitational law expressed in background coordinates, otherwise they don't have the common language and the meaning of each term in geodesic equation is unclear and the comparison is distinctly ruled out. This shows that the special advantage of using the coordinates with clear physical meaning. And using background coordinates general relativity becomes naturally flat spacetime's gravitational theory and common language with other theory of physics. Of course, on earth using which sort of coordinates is in accordance with specific conditions and questions to demand to solve and sometimes we have to use the sort of coordinates whose physical meaning is not too clear in order to simplify mathematical calculation but this certainly misses out or covers up some important information and even can not link theory with observations.

Must point out, though Schwarzschild radial coordinate doesn't be explained as background coordinate (namely true radius) in standard textbooks, one treats it as true radius involuntarily in practice, this makes a certain confusion on logic and concept. For example, while computing deflected angle of light on Sun's surface, one takes the value of radial coordinate on the surface for Sun's true radius, serious question doesn't happen thanks to the difference between 1 and r slight (see the calculated result in section V). In this study, in order to hint the difference Schwarzschild radial coordinate is denoted l and true radius is denoted r, therefore, this study is actually to perfect and refining the fundamentals of general relativity. As a result of careful calculation step by step, we find Einstein's field equation may change and by applying the revised field equation we see that many difficult problems of cosmology can all be readily solved and maybe new physics will be brought out.

**THE AMELIORATION OF EINSTEIN'S GRAVITATIONAL FIELD EQUATION**

It is seen from the above discussions that in the case of weak field approximation:

\[
g_{00} = -1 + \frac{2GM}{r}
\]

and:

\[
g_{tt} = 1 - \frac{2GM}{r}
\]
instead of the previous:
\[ g_{tt} = 1 + \frac{2GM}{r} \]

which are just the requirement of that Eq. 6 can hold and hint us to alter the coupling constant \( \gamma \) in gravitational field equation:
\[ R_{\mu\nu} = \gamma (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) \]

Note that the coupling constant \( \gamma \) relates to the form of weak field approximation metrics \( g_{\mu\nu} \) and is confirmed in the course of solving weak field approximation metrics and the change of the metrics means that the coupling constant \( \gamma \) need change too. So, the content of the section III is really to renew solving under a certain condition Einstein's field equation including solving constant \( \gamma \) and here pressure \( p \) isn't assumed as zero in advance and it is also to be solved.

And now we set out to reconfirm the coefficient \( \gamma \) by solving weak field approximation metrics \( g_{\mu\nu} \). Here:
\[ T_{\mu\nu} = (\rho + p) U_{\mu} U_{\nu} + pg_{\mu\nu} \text{ and } U^{\mu} = \frac{dx^{\mu}}{dr}, U_{\mu} = g_{\mu\nu} U^{\nu} \]

And from \( ds^2 = -dt^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \), we have \( U_{\mu} U^{\mu} = 1 \), hence it follows that:
\[ T = g^{\mu\nu} (\rho + p) U_{\mu} U^{\nu} + pg^{\mu\nu} g_{\mu\nu} = (\rho + p) U_{\mu} U^{\mu} + 4p = 3p - p \]

Here, pressure \( p \) isn't assumed as zero in advance and it is also to be solved. Similar to previous calculation used in standard textbooks, the following discussions are still carried out in the background right-angled coordinate system \( x^0 = (x^0, x^1, x^2, x^3) = (t, x, y, z) \) And for weak field we have \( g_{\mu\nu} = h_{\mu\nu}, \eta_{\rho\sigma} \) and \( |h_{\mu\nu}| << 1 \). Here, Minkowskian metrics \( \eta_{00} = -1, \eta_{11} = \eta_{22} = \eta_{33} = 1 \) and \( \eta_{\mu\nu} = 0 \). (\( \mu + \nu \)). Omitting the terms that are less than \( o(h^2) \) we have (Weinberg, 1972):
\[ \Gamma_{\varphi} = -\frac{1}{2} \eta^{\mu\nu} \left( \frac{\partial g_{\mu\nu}}{\partial x^\varphi} + \frac{\partial g_{\nu\mu}}{\partial x^\varphi} - \frac{\partial g_{\mu\nu}}{\partial x^\varphi} \right), h_{\mu}^\varphi = \eta^{\rho\varphi} h_{\rho\mu}, \text{and } h = h_\mu^\mu = \eta^{\mu\nu} h_{\mu\nu} \]

Correspondingly, Ricci tensor:
\[ R_{\mu\nu} = \Gamma_\lambda^\mu \Gamma_\lambda^\nu - \Gamma_\mu^\lambda \Gamma_\nu^\lambda = \frac{1}{2} \eta^{\mu\rho} h_{\rho\lambda} h_{\lambda\nu} + \frac{1}{2} \eta^{\rho\lambda} h_{\rho\mu} h_{\lambda\nu} - h_{\mu\nu} \]

where, the semicolons denote covariant derivative and the commas denote common derivative.
May as well use harmonic condition:

\[ h_{\mu\nu} = \frac{1}{2} h_{\mu\nu} \]  

(9)

Differentiating Eq. 9 with respect to \( x^\mu \) yields

\[ h_{\mu\nu,\gamma} = \frac{1}{2} h_{\mu\nu,\gamma} \]

Similarly:

\[ h_{\sigma\nu,\gamma} = \frac{1}{2} h_{\nu,\nu,\gamma} \]

Using \( h_{\mu,\nu} = h_{\mu,\nu} \) and adding up the above two equations yield:

\[ h_{\mu\nu,\gamma} - h_{\mu,\nu,\gamma} - h_{\sigma,\nu,\gamma} = 0 \]

Hence, we obtain:

\[ \nabla^\mu h_{\mu} - \frac{\partial h_{\mu}}{\partial t} = 2\gamma(T_{\mu\nu} - \frac{1}{2} T_{\mu\nu}) = 2\gamma[(\rho + p)U_{\mu}U_{\nu} + \frac{\rho - p}{2} \eta_{\mu\nu}] \]

which have retarded solutions:

\[ h_{\mu\nu} = \frac{\gamma}{4\pi} \int \frac{2(\rho + p)U_{\mu} + (\rho - p)\eta_{\mu\nu}}{\xi} \, dx' \, dy' \, dz' \]

Here:

\[ \xi = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \]

the terms in the integral sign take the values of \( t' = t - \xi \). Note that the above retarded solutions can be used in arbitrary cases of motion of source. Hence, in order to get the external metrics

\[ g_{00} = -1 + \frac{2GM}{r} \quad \text{and} \quad g_{ii} = 1 - \frac{2GM}{r} \]

in the case of static spherical symmetry \((U_0 = \eta_{ij}U^i = -1, \ U_j = 0)\), it must be required that the constant coefficient \( \gamma = 4\pi G \) and simultaneously pressure \( p \) satisfies:

\[ \int_{\xi}^0 \frac{p}{\xi} \, dx' \, dy' \, dz' = -\int_{\xi}^0 \frac{p}{\xi} \, dx' \, dy' \, dz' = -\frac{M}{r} \quad \text{for} \quad r = \sqrt{x^2 + y^2 + z^2 + \xi} \]
which means:

\[ \int p\,dx\,dy\,dz = -\int p\,dx\,dy\,dz = -M \]  

(10)

In view of Eq. 3 it must hold that \( h_{ij} = 0 \) in the static case. Next we solve the other three \( h_{ij} \).

Inserting \( h_{ij} = \eta^{ij} h_{ij} \) and \( h = \eta^{ij} h_{ij} = h_{ij} + 3 h_{ij} \) into Eq. 9 and noticing \( h_{11}, h_{22}, h_{33}, h_{ij} = h_{ij} = h_{ij} = h_{ij} \), \( h_{00} = 0, h_{0i} = h_{i0} = 0 \), we obtain three equations as follows:

\[
\begin{align*}
    h_{33} + h_{j1,1} &= \frac{1}{2} (h_{11} - h_{00})_{1j} \\
    h_{12,j} + h_{j1,j} &= \frac{1}{2} (h_{11} - h_{00})_{j1} \\
    h_{13,j} + h_{j1,j} &= \frac{1}{2} (h_{11} - h_{00})_{j1}
\end{align*}
\]

After a certain calculation we arrive at:

\[
\begin{align*}
    h_{0j,j} &= \frac{1}{4} \left[ (h_{11} - h_{00})_{0j} + (h_{0j} - h_{00})_{1j} - (h_{11} - h_{00})_{j1} \right]
\end{align*}
\]

Here, \( i \neq j \), \( j \neq k \), and \( i, j, k = 1, 2, 3 \). With the condition \( h \to 0 \) for \( r \to \infty \), \( h_{ij} \) are solved by:

\[
\begin{align*}
    h_{ij} &= \frac{1}{4} \int_{\infty}^{r} \int_{0}^{\theta} \left( \frac{\partial^2}{\partial (r')^2} + \frac{\partial^2}{\partial (\theta')^2} - \frac{\partial^2}{\partial (\phi')^2} \right) \left( h_{11} - h_{00} \right) \, dx' \, dx'
\end{align*}
\]

Note that \( x^1 = x, \, x^2 = y, \, x^3 = z \). On the other hand, for the weak field case Bianchi identity can give the ordinary conservation law \( T^{\mu}_{\nu,\mu} = 0 \).

**Proof**: Because:

\[ R_{ij}^{\nu} = R_{j}^{\nu} + \Gamma^{\nu}_{k} R_{j}^{k} - \Gamma^{\nu}_{j} R^{k}_{k} = R_{ij}^{\nu} + o(h^2) = R_{ij}^{\nu} \]

then:

\[ 0 = (R_{r}^{r} - \frac{1}{2} R \delta_{r}^{r})_{\nu} = R_{r}^{\nu} - \frac{1}{2} R_{r} = R_{r}^{\nu} - \frac{1}{2} R_{\nu} \]

and moreover field equation gives \( R = -\gamma T \) and:

\[ R_{i}^{\nu} = \gamma (T_{i}^{\nu} - \frac{1}{2} T \delta_{i}^{\nu}) = \gamma (T_{r}^{\nu} - \frac{1}{2} T_{\nu}) = \gamma T_{\nu} + \frac{1}{2} R_{\nu}, \]

hence, \( T_{\nu} = 0 \).
And for the static case, using $\nabla^2 h = (\rho + p)U$, $(p\rho) = 0$ yields $\frac{\partial p}{\partial x^2} = 0$. Considering $\Delta^2 (h_{\nu\nu} - h_{\mu\mu}) = 16\pi G\rho$, it is immediately verified that:

$$\nabla^2 h_{\mu\nu} = \frac{1}{4} \int_s^t \int_\sigma [\nabla^2 (\frac{\partial U}{\partial x}) + \frac{\partial U}{\partial x}] \nabla^2 (h_{\nu\nu} - h_{\mu\mu}) \ dx^4 \ dx^4 = 0$$

That is to say, $h_{\nu\mu}$ worked out here is indeed reasonable approximate solution of field equation with $\gamma = 4\pi G$.

And again, as a special case of spherical symmetry, if the source's density is a constant, namely $\frac{\partial \rho}{\partial x^2} = 0$, since $\frac{\partial \rho}{\partial x^2} = 0$, we can infer from Eq. 10 a very useful and significant result:

$$p = \rho$$

which can be regarded as the form of pressure in weak field in the case of that density $\rho$ is even. It is obviously too subjective to take gravitational source's pressure for zero in advance, in fact, by intense calculation we see that the pressure takes negative value where matter exists and the places where matter exists turn out to be so-called pseudo-vacuum (Gondolo and Fresse, 2003; Guth, 1981). Obviously the pressure as gravitational source isn't so-called thermodynamic pressure.

To sum up, we can conclude that in any coordinate system gravitational field equation is revised as:

$$R_{\mu\nu} - \frac{\lambda}{2} g_{\mu\nu} = 4\pi G T_{\mu\nu}$$

where, $\lambda$ replaces previous $-8$, obviously Eq. 11 preserves general covariance.

Of course Eq. 5 satisfies Eq. 11 because both $p$ and $\rho$ vanish outside gravitational source and Eq. 11 becomes $R_{\mu\nu} = 0$ outside source.

APPLICATIONS AND TESTS OF EQ. (11) IN COSMOLOGY

It is decided by practice in the final analysis whether a theory is right or not. The application of Eq. 11 in cosmology proves strongly that the revision is quite successful.

With $l$ as radial coordinate, in the co-moving coordinates Friedmann-Robertson-Walker metric is given by Weinberg (1972) and Sawangwit and Shank (2010):

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{1}{1 - kl^2} dl^2 + l^2 d\theta^2 + l^2 \sin^2 \theta d\phi^2 \right]$$

$a(t)$ is universe expansion factor and metric:

$$g_{00} = -1, g_{11} = \frac{a^2(t)}{1 - kl^2}, g_{22} = a^2(t), g_{33} = a^2(t) \sin^2 \theta, g_{44} = 0 (\mu \neq \nu)$$

and substituting they into (11) yields the following equation like Friedmann's:
\[
\left(\frac{da(t)}{dt}\right)^2 + k = -\frac{4\pi G}{3} \rho a^3(t)
\]  
(12)

Consequently, \( k \) must be negative, and the cosmological constant is so far proved infinite or open. And again, in virtue of \( T^\mu{}\nu = (\rho U^\mu) U_\nu = 2U_\mu (U^\nu) \), \( 2U_\mu (U^\nu) \rho = 2U^\mu (U_\nu) = 0 \), it follows that \( d (\rho a^3) + p d a^2 = 0 \) and:

\[
p d \left( \frac{1}{n} \right) + d \left( \frac{p}{n^2} \right) = 0
\]  
(13)

Here, \( n \) represents the density of particle (galaxy) number. Since \( \rho \) is assumed homogeneous, we may use the weak field condition \( p = -\rho \) and substituting it into Eq. 13 yields \( d\rho = 0 \), that is to say, \( \ddot{\rho} = 0 \) or:

\[
p = -\rho = \text{const} = -\rho_0
\]  
(14)

which is the most appropriate expression of energy conservation in infinite spacetime and indicates the singular point of big bang did not exist. In addition (13) implies the mass of galaxy changing with cosmic expansion since \( \rho/n \) stands for per particle mass. And further, the solution of Eq. 12, namely expanding factor, is given by:

\[
a(t) = A \sin \left( \sqrt{\frac{4\pi G \rho_0}{3}} t \right)
\]  
(15)

Here, \( A \) is a positive constant. So far, cosmic expansion and contraction are proved to be in circles like a harmonic oscillator. (15) means that the expansion of universe is decelerating and its contraction is accelerating, this fact is compatible with the newest data observed, Fig. 1 (Schwarz et al., 1993; Dai et al., 2004; Wang et al., 2009). We realize that the conclusion universe’s expansion is accelerating is wrong at all. In fact a decelerating expansion is more acceptable for philosophy. We should be sobering that the accelerating universe is not from direct measured data and instead it depends quite on cosmic model and if something is wrong with the model the conclusion certainly fails.

Now we compute the relation between distance and red-shift. May as well put \( a(t) = 1 \), the light from a galaxy to us satisfies (Weinberg, 1972):

\[
1 + z = \frac{1}{a(t)} \quad \text{and} \quad dz = -\frac{da}{a^2(t)}
\]

Here, \( z \) denotes red-shift. And writing:

\[
\frac{4\pi G \rho_0}{3H_0^2} = q_0
\]

\( H(t_0) = H_0 \), we infer from Eq. 12:
The curved line is of Eq. 16 with $q_0 = 0.14$ and $H_0 = 70 \text{ km/s Mpc}^{-1}$.

Fig. 1: The recent hubble diagram of 69 GRBs and 192 SNe Ia

\[
H = a^{-1} \frac{da}{dt} = H_0 \sqrt{(1 + q_0)(1 + z)^2 - q_0} \text{ and } k = -H_0^2 (1 + q_0)
\]

Note that the subscript "0" refers to the present-day values. For the propagation of light line $ds^2 = 0$, then:

\[
\frac{dt}{a(t)} = \frac{dz}{H} = \frac{dl}{\sqrt{(1 - kl^2)}} = \int_0^z \frac{dl'}{\sqrt{1 - kl'^2}}
\]

$l_*$ denotes the galaxy's invariant coordinate. In view of luminosity-distance:

\[
d_* = (1 + z) \int_0^z \frac{dl}{\sqrt{1 - kl'^2}}
\]

we work out a new relation between distance and re-shift:

\[
H_0 d_* = \frac{z + 1}{\sqrt{q_0} + 1} \ln \frac{(z + 1) \sqrt{q_0 + 1} - \sqrt{(q_0 + 1)(z + 1)^2 - q_0}}{1 + \sqrt{q_0 + 1}}
\]  

(16)

As $z \rightarrow 0$, expanding the right hand side of Eq. 16 into power series with respect of $z$ (16) becomes:

\[
H_0 d_* = z + \frac{1 - q_0}{2} z^2 + \frac{3q_0 - 2q_0 - 1}{6} z^3 + ...
\]

which is the same result as that obtained via pure kinematics. The curved line in Fig. 1 (Dai et al., 2004; Wang et al., 2009) is the image of Eq. 16 with $q_0 = 0.14$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

The situation described by the curved line agrees well with the recent data of observations. Note that recent observations show that:

\[
q_0 = \frac{4\pi G \rho}{3H_0^2} = \frac{\Omega_m}{2} = 0.1 \pm 0.05
\]

(Linder, 2003; Hamuy, 2003; Cunha et al., 2004).
Note that Distance-Modulus is equal to 51 g d+25 and the unit of d is Mpc.

Next we calculate “our” cosmic age, namely the time from last a (t) = 0 (at the moment, t may as well take 0) to today. Writing H (t0) = H0, from:

\[ H = \frac{\dot{a}}{a} = 2 \left( \frac{\pi G \rho}{3} \right)^{\frac{1}{2}} \left( 1 \right) \left( \frac{\pi G \rho}{3} \right)^{\frac{1}{2}} \]

in the case that q0 takes 0.14 “our” cosmic age is calculated as:

\[ t_0 = \frac{t_0}{H_0 \sqrt{\Omega_0}} = 1.37 \times 10^9 a \]  \hspace{1cm} (17)

which agrees with observations. Besides, we can also compute how a galaxy’s mass changes with time. Writing a galaxy mass m (t), taking account of \( \rho = \text{const} = \frac{N m(t)}{a^3(t)} \), where N is equivalent to a proportional coefficient, immediately it is concluded that:

\[ \frac{m(t)}{a^3(t)} = \frac{m(t_0)}{a^3(t_0)} \]  \hspace{1cm} (18)

which implies that galaxies can grow up without mergers and consists with recent observations (Genzel et al., 2006). The formula (18) defines how a galaxy mass changes with evolution of universe.

And again, because any point can be thought the centre of universe’s expansion, Eq. 18 can be looked as the rule of mass’s change of any celestial body or galaxy. And applying (18) to the earth of today, we find that the increase of the earth’s mass in a year is:

\[ \Delta m = \left[ \frac{a^3(t_0)}{a^3(t)} \right] - 1 \Delta m(t_0) \approx 3 H_0 m_0 = 12.46 \times 10^4 \text{kg} \]  \hspace{1cm} (19)

And also deduce that the expanding speed of the radius of the earth is today v_0 = H_0 \rho_0 = 0.45 mm/s.

By the way, from:

\[ a(t_0) = A \sin \left( t_0 \sqrt{\frac{4 \pi G \rho_0}{3}} \right) = 1 \]

it can be decided that constant:

\[ A = \frac{1}{\sin \left( t_0 \sqrt{\frac{4 \pi G \rho_0}{3}} \right)} \]

and further we have the following relation of reshift Z and universe time t:
\[ 1 + z = \frac{1}{a(t)} = \sin \left( t \sqrt{\frac{4\pi G \rho_0}{3}} \right) \left/ \sin \left( t \sqrt{\frac{4\pi G \rho_0}{3}} \right) \right. \]

Here, \( t \) is the time at which photons was given out from the celestial body. The relation can be used to evaluate low limit of celestial body age.

We can also decide the density of galaxy number of any time \( t \). Take \( n_g \) for number density of galaxy of today \( t_0 \) and use proper speed \( v_p = H d_p \), where, \( d_c \) denotes proper distance of galaxy, then \( dd_p = H d_p \, dt \), further:

\[ \frac{d}{d_{d_0}} = \exp \int_{t_0}^{t} H \, dt \]

and since galaxy number conserves, namely \( n d_{d_0}^3 = n_0 d_{d_0}^3 \), number density of galaxy of any time \( t \) reads therefore:

\[ n = n_0 \exp \int_{t_0}^{t} H \, dt = n_0 \left( \sin \left( t \sqrt{\frac{4\pi G \rho_0}{3}} \right) \left/ \sin \left( t_0 \sqrt{\frac{4\pi G \rho_0}{3}} \right) \right. \right)^{\frac{2\pi}{N_1(0)}} \]

**EXACT INTERIOR SOLUTION OF EQ. 11 AND MECHANISM OF CELESTIAL BODY'S EXPANSION**

In the case of static spherical symmetry, inside a celestial body (gravitational source), with \( l \) as standard radial coordinate the exact interior solution of Eq. 11 is given by:

\[ d \sigma^2 = -\exp \left[ C_2 + \int_{r}^{l} f(\theta) \left( 1 + \frac{\omega(\theta)}{l} \right)^{-1} \, dl + \left( 1 + \frac{\omega(\theta)}{l} \right)^{-1} \, dl + f(\theta)^2 + \sin^2 \theta \, d\phi^2 \right] \]

in which:

\[ \omega(\theta) = 4\pi \int_{0}^{\rho(\theta)} l^2 \, dl, f(\theta) = \frac{Q}{r_0} \left[ 4\pi l^2 \rho(\theta) + \omega(\theta) \right], l_0 = \frac{1}{l} \]

Constant:

\[ C_2 = \ln \left( 1 - \frac{2GM}{l_0} \right) \]

it makes sure \( g_{\infty} \) is continual on the boundary of the celestial body. Note that as scalar \( \rho = \rho(\theta) = \rho(\tau) \), \( p = p(\theta) = p(\tau) \) and outside gravitational source both \( p \) and \( \rho \) vanish, namely \( \rho(0) = \rho(\tau) = \rho(\tau) = p(0) = p(\tau) = 0 \) for \( r > r_0 \).

In order to determine the interior form of Eq. 20 in background coordinates, Eq. 4 is naturally extended as inside source:
\[
\frac{dl}{dr} = \sqrt{1 + \frac{G\rho(r)}{l}} \exp\left(-G\int_0^r \rho \, dx \, dy' \, dz'\right)
\]

(21)

Obvious under the transformation of Eq. 21, line element (20) turns into:

\[
d^2 = -\exp]\left[C_2 + \int_0^r f(r)\left(1 + \frac{\rho(r)}{l}\right) \, dl\right] \, dr^2 + \exp\left[-2G\int_0^r \rho \, dx \, dy' \, dz'\right] \, dr^2 + f^2(\phi^2 + \sin^2 \phi \, d\phi')
\]

(22)

Here, \( l = l(r) \) is a specific function of \( r \) which is determined by Eq. 21. Line element (22) is just the exact solution looked for and expressed in background coordinate system \((x^0, x^1, x^2, x^3) = (t, r, \theta, \phi))\). Note that the solution of Eq. 21 satisfy the initial condition \( l(0) = 0 \). In fact, because there is no acceleration tendency for every direction at the centre gravitational source, \( \frac{dg_{00}}{dr} \) must be zero and from Eq. 22 we have:

\[
0 = \frac{dg_{00}}{dr} = \frac{dl}{dr} \frac{dg_{00}}{dl} = \frac{dl}{dr} f(r)\left(1 + \frac{\rho(r)}{l}\right) \exp\left[C_2 + \int_0^r f(r)\left(1 + \frac{\rho(r)}{l}\right) \, dl\right]
\]

which indicates \( f(l) = 0 \) at the centre and so that \( l = l(0) = 0 \) at the centre. And if:

\[
\rho = \text{const} = \frac{3M}{4\pi r_c^3}
\]

then:

\[
\int_0^r \frac{\rho \, dx \, dy' \, dz'}{\xi} = \frac{3M}{2r_c} - \frac{M}{2r_c} r_c, \quad \omega(l) = 4\pi \int_0^l \rho(l) r^2 dl = \frac{M}{r_c} l^2
\]

the solution of Eq. 21 is easily given by:

\[
\sqrt{\frac{r_c}{GM}} \ln\left(\sqrt{\frac{GM}{r_c^3} + \sqrt{1 + \frac{GM}{r_c^3} l^2}}\right) = \left[r + \frac{GM}{6\pi r_c^3} l^2 + \frac{1}{40} \left(\frac{GM}{r_c^3}\right)^2 l^4 + \cdots \right] \exp\left(-\frac{3GM}{2r_c}\right)
\]

(23)

Though energy density \( \rho \), generally speaking, isn’t a constant, we may take its average value or piecewise integrate on \( r \) in practice for the convenience of calculation. As an important example, on the surface of the Sun \( r = r_s = 6.96 \times 10^6 \text{ m}-1720 \text{ m}, M = 1.99 \times 10^{30} \text{ kg}, \) using (23), that is taking average value of \( \rho \), we can work out the surface’s \( l = l (r_s) = 6.96 \times 10^6 \text{ m}-1720 \text{ m} \) which is highly equal to the Sun’s radius. And likewise, we can work out \( l = 6371.000038 \text{ km} \) on the Earth’s surface and this almost equals the Earth’s radius 6371 km.

So far, using the continuity of \( l = l (r) \) not only we can determine the constant \( C_1 \) but also can calculate the deflected angle of light line on the surface of Sun. For photon’s propagation outside Sun from (5) we have:
\[ 0 = \text{ds}^2 = -\left(1 - \frac{2GM}{l}\right)dt^2 + \exp\left(-\frac{2GM}{r}\right)dr^2 + r^2 \left(dt^2 + \sin^2 \theta d\phi^2\right) = -\left(1 - \frac{2GM}{l}\right)dt^2 + \left(1 - \frac{2GM}{l}\right)^{-1} dr^2 + (d\theta^2 + \sin^2 \theta d\phi^2) l^2 \]

Similar to former calculation, the deflected angle is given by

\[ \alpha = \frac{4GM}{l} = \frac{4GM}{1(r_s)} = 1.78^* \]

which is more consistent with observational result (1.89*) compared with former theoretical value:

\[ \alpha = \frac{4GM}{r} = \frac{4GM}{r_s} = 1.75^* \]

On the other hand, the conserved law gives:

\[ \frac{dp}{df} = G(p + p') \left(2\pi l p + \frac{\alpha}{2} \right) \left(r + K\omega(l)\right)^{-1} \quad (24) \]

On the boundary the gravity acceleration should be continual, according to Eq. 2, using Eq. 4, 5, 21 and 22 we have:

\[ (\Gamma_{\partial r})_{\partial r} = (\Gamma_{\omega})_{\partial r}, \text{ that is, } (\frac{d\phi_{\partial r}}{dr}_{\partial r})_{\partial r} = (\frac{d\phi_{\partial r}}{dr})_{\partial r} \]

it follows that:

\[ \left[ \frac{dl}{dr} \frac{d}{dl} \left(1 - \frac{2GM}{l}\right)_{\partial r} \right]_{\partial r} = \left[ \frac{dl}{dr} \frac{d}{dl} \exp \left[ C \gamma + \frac{1}{l^2} f(l) \left(1 + \frac{\omega(l)}{l}\right)^{-1} \right] \right]_{\partial r} \]

And after simplifying further, it becomes:

\[ [4\pi l^2 p + \omega(l)] \sqrt{l - 2GM} = -2M \sqrt{l + K\omega(l)} \quad (25) \]

which is the boundary condition \( p \) must satisfy and the condition determines \( p \) negative within celestial body.

For general cases, inside source, gravitational field is still weak which means \( l = l(r) = r, \frac{2GM}{r} \ll 1 \) and from (25) the boundary pressure:

\[ p \approx -\frac{3M}{4\pi l^2} = -\bar{p} \]

which is consistent with (10). Here, \( \bar{p} \) denotes average.
As an emphasis, we must point out that when (1) or (5) is applied to a mass point of the surface of the static source, it exists that:

\[ 0 \geq ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 \]

which indicates that

\[ 1 - \frac{2GM}{r} \]

of static source is nonnegative.

Next let us consider a small volume \( V_i \) of mass \( m_i \) inside source, \( dV_i \) denotes \( V_i \)'s change caused from the expansion of space-time, in view of Eq. 12 we have \( dm_i = \rho_0 dV_i \), hence:

\[ dp_i = \frac{d(m_i)}{V_i} = \left(\rho_i + p_i\right)\frac{dV_i}{V_i} = \left(\rho_i + p_i\right)\frac{da^2(t)}{a^2(t)} \]

which means that for arbitrary point it holds that:

\[ \frac{dp_i}{dt} = -\rho + p_i \frac{da^2(t)}{a^2(t)} \]

Equation 26 determines how matter density changes locally. It is seen from Eq. 26 that when celestial bodies expand with cosmic expansion its density may be unchanging in the case of \( \rho + p = 0 \). So far, we deduce that bursts of celestial bodies and formation of earthquakes originate both from unceasing accumulation of inside matter and change of distribution and it is the negative pressure that gets matter in celestial body continuously produce (Nashed, 2011).

**CRACKING OF THE PUZZLE OF DARK MATTER**

The negative pressure as important gravitational source is invisible and it is the negative pressure that appears as the form of dark matter and leads to the phenomenon of missing mass or say that so-called dark matter is just the negative pressure, this fact are showed as follows.

Speaking generally, within a galaxy the metric field is weak field and when a galaxy is treated as a celestial body of spherical symmetry, according to the discussion in section III, within the galaxy (\( 0 < r < r_{g} \)) pressure \( p = \text{const} < 0 \). From (10) we infer

\[ p = \text{const} = -\frac{3M}{4\pi r_{g}^3} \]

and further we have:

\[ h_0 = -G\int_{0}^{3\pi} \int_{\xi} dx' dy' dz' = -4\pi G \left\{ \int_0^{r^*} \int_0^{r^*} \rho(\rho + p) d\rho d\rho' - \int_0^{r^*} \rho d\rho \right\} - 6G\pi \rho \gamma^3 + 2G\gamma \pi \rho \gamma^2 \]
According to Eq. 2 the gravity acceleration (or gravitational field strength) within the galaxy is given by:

\[ g = -r^2 = \frac{1}{2} \frac{d\theta}{dr} = 2\pi G \frac{m(r)}{r^2} + 2\pi G \int_0^r r^2 \, dr = 2\pi G r^2 \frac{M(r)}{2r^2} \]

where, \( m(r) = 4\pi \int_0^r r^2 \, dr \) and \( g \) may be positive or negative since pressure is negative and the negative \( g \) indicates the direction of acceleration is towards centre. And according to Eq. 2 the corresponding round orbital speed \( v_r \) is given by:

\[ v_r^2 = -2\pi G r^2 \frac{M(r)}{2r} \quad (27) \]

From Eq. 27 it is seen that when \( m(r) \) looks even on the verge of zero near the centre of the galaxy the speed \( v \) can become high too and this explains the phenomenon of so-called missing mass. Again, from Eq. 27 we get \( 2rv_r^2 = 4\pi G r^2 \cdot M(r) \) and if \( v \) is a constant between \( r_1 \) and \( r_2 \), differentiating this equation and using:

\[ v_r^2(r_1 < r < r_2) = -2\pi G r^2 \frac{M(r)}{2r} = \frac{3M}{4\pi r^3} - \frac{Gm(r)}{2r} \]

yield:

\[ \rho(r_1 < r < r_2) = -3p - \frac{v_r^2}{2\pi cr^2} = \frac{9M}{4\pi r^3} \frac{3M}{4\pi r^3} r^2 \frac{M(r)}{4\pi r^3} \quad (28) \]

which is the condition a typical spiral galaxy with a halo satisfies. May as well set \( r_1 = nr_2 (0 < n < 1) \), then:

\[ m(r_2) = m(r_1) + 4\pi \int_0^{r_2} r^2 \, dr = \frac{3M}{4\pi r^3} (1 - n^3) r^3 + \frac{m(r_2)}{n} \]

and in consideration of \( 0 \leq m(r_2) \leq M \) we concluded that:

\[ 0 \leq r^3 \leq \frac{nM - m(r_2)}{3nM (1 - n^3)} \quad (29) \]

which indicates it is impossible for \( r_2 \) to arrive at the galaxy's edge \( r_\ast \) in the case of \( n < \sqrt{2/3} \). Obviously, if \( \rho \) begins to decrease from \( r_2 \) to \( r_\ast \), both \( v_r \) and \( \mid g \mid \) begin to increase. Of course, it isn't easy to observe the speed of the particles between \( r_2 \) and \( r_\ast \) because near the edge \( r_\ast \) matter becomes virtually very thin. The curve in Fig. 2 describes the situation predicted by Eq. 27 and 29 and it is in conformity with recent observational results (Cayrel et al., 2001).
Fig. 2: The velocity distribution diagram

So far, we conclude that so-called dark matter is just the effect of the negative pressure or say that the negative pressure is just so-called dark matter and the dark matter (Genzel et al., 2003; Li et al., 2008) puzzle has naturally been cracked. Of course, so-called dark energy problem is also removed since cosmological constant is reconfirmed as zero again and the concept of dark energy becomes unnecessary in the new amendme.

MOTION IN CENTRE FIELD AND FORMATION OF GALAXIES AND BACKGROUND PHOTONS

Equation 14 indicates that not only space is expanding but also celestial bodies or galaxies themselves, that is, like a expanding balloon, the ink prints on it also expand on the same proportion. This is just the elementary mechanism of galaxy formation. In order to illuminate galaxy formation clearer we look into the motion in centre field. Let M denote mass of centre body. Generally speaking, its gravitational field is weak, geodesic reduces to Newton's law, for a object moving around the centre body we have:

\[
\frac{4\pi r^2}{T^2} = \frac{GM}{r}
\]

(30)

where, \(r\) is the radius of round orbit, \(T\) is revolution period. Noticing M to be variable now and to satisfy (18) and using (30) we infer \(\Delta r = r(1 + \Delta T/T)^{\frac{3}{2}}a(t + \Delta t)/a(t) - r\) from \(t\) to \(t + \Delta t\). And putting \(\Delta t = 0\) we have:

\[
v = \frac{dr}{dt} = rH + \frac{2r}{3T} \frac{dT}{dt}
\]

(31)

where, the final term is explained as perturbation and gravitational radiation. For instance, apply Eq. 31 to the motion round today's Earth, for geostationary satellite, neglecting perturbation and gravitational radiation, namely taking \(dT_0 = 0\) we find that its orbit radius will increase by \(\Delta r_0 = 2.7\) mm in a year. And for the motion of Moon, observations show that its orbit radius increases by 0.38 cm in a year today, then using Eq. 31 we conclude that the orbit period \(T_0\) of Moon will slow by 0.0001 s in a year today.

When Eq. 31 is used to the edge of a spiral galaxy, it is concluded that the terminuses of spiral arms gradually stretch outward. Of course, other points near the terminuses continuously follow and form involutes (Fig. 3).

Equation 31 means that separating speed from centre lies on \(v = rH\) neglecting perturbation and radiation damp.
Earlier                                           Early                                         Today

Expanding centre                                  Gradually stretching out arm

Fig. 3: Sketch map of formation and evolution of spiral arms

Fig. 4: The global picture of galaxy evolution and distribution in different stages

It is important to realize that the spin of a system is the composition of orbit motion of many particles, spin and orbit motion do not have essential difference. For celestial body's expansion, lying on $v = rH$ means its spin period not to change.

Note that the existence of Eq. 31 doesn't mean the destruction of conservation of angular momentum on whole because mass $M$ is connected with the factor $a(t)$ which embodies the interaction among galaxies, the nonconservation of angular momentum of individual galaxy is admitted.

Again, the fact that space, celestial bodies and galaxies simultaneously expand proportionally links the homogeneity of today's universe in a large range with that of early universe in a small range, because the large range is just the amplification of early the small range. Background radiation has proven early universe to be homogeneous in quite small range. Therefore, our conclusion is in accordance with observations.

The following Fig. 4 is the global picture of galaxy evolution and distribution under $\rho = \text{const}$ in different time stages, the earlier, the smaller and the denser. Figure 5 is the picture of galaxies seen by today's telescope and the farther, the earlier and the evener.

Note that the horizon at moment $t>0$ is now according to Eq. 15:

$$d_L(t) = a(t) \int_0^t \frac{1}{a(t')} dt = \sin \left( \frac{\sqrt{\frac{4\pi G\rho_c}{3}}}{3} \right) \int_0^t \frac{dt}{\sin \left( \frac{\sqrt{\frac{4\pi G\rho_c}{3}}}{3} \right)}$$

So-called horizon puzzle or homogeneity puzzle does not exist in the present theory framework at all.

Naturally, the microwave background radiation measured today is the compositive effect of various photons emitted by unnumberable galaxies remote, whose distances to us are
unidentifiable, which comprised infinitely deep thin gas and could absorb any frequency photon and therefore possess black body feature.

Note that the state that horizon vanishes is unobservable though $d_{\lambda}(t) = 0$ for $t = 0$, because any observation carried out needs a time lag $\Delta t$.

Now, referring to Fig. 5 we try to solve galaxy number $dN_\gamma$ between $z$-$z+dz$ which is an observational quantity for our telescope today. From the discussion above we know proper distance of galaxies of reshift $z$ is given by:

$$d_\gamma = \int_0^z \frac{dz}{H} = \frac{1}{H_0 \sqrt{q_0} + 1} \ln \frac{(z+1) \sqrt{q_0} + 1 + \sqrt{(q_0 + 1)(z+1)^2 - q_0}}{1 + \sqrt{q_0} + 1}$$

where, $H = H_0 \sqrt{(1 + q_0)(1 + z)^2 - q_0}$ and number density of galaxies near proper distance $d_\gamma$ reads:

$$n = n_0 \exp \int_0^z 3Hdt = n_0 \exp \int_0^z \frac{3}{1+z} dz = n_0 (1+z)^3$$

where, $z$ is reshift of galaxies near proper distance $d_\gamma$, we easily obtain the following result:

$$dN_\gamma = 4\pi n_0 \frac{d}{dz} \frac{d_\gamma}{dz} = 4\pi n_0 \frac{d}{dz} \frac{dz}{d_\gamma} = 4\pi n_0 \frac{(1+z)^3}{(q_0 + 1)H_0^2 \sqrt{(1 + q_0)(1 + z)^2 - q_0}} \ln \frac{(z+1) \sqrt{q_0} + 1 + \sqrt{(q_0 + 1)(z+1)^2 - q_0}}{1 + \sqrt{q_0} + 1}$$

where, $n_0$ may take the value of number density of the galaxies around us.

Finally, we prove that the dark body spectrum of background photons keep on in the course of the propagation. Assume that background photons arrive at $B$ from $A$ between $t = t_A$ and $t = t_B$. See Fig. 6.

![Diagram of galaxies and telescope](image)

**Fig. 5:** The actual picture of galaxies seen by today's telescope

![Diagram of dark body spectrum](image)

**Fig. 6:** The traveling background photons toward us
Take \( \sigma (v, t) dv \) for photon's number density between \( v - v + dv \) at the time \( t \). Assume that photon number is conserved in the course of propagation, then we have:

\[
\sigma(v_a(t_a)) dv_a a'(t_a) = \sigma(v_b(t_b)) dv_b a'(t_b)
\]

Take \( T_A \) for the temperature of background photons at the time \( t = t_A \), in consideration of that average kinetic energy of photons is equal to \( 3/2 k_B T \), where, \( k_B \) is Boltzmann constant, we have:

\[
T_A = \frac{2 \int \sigma(v_a(t_a)) dv_a R^2(t_a)}{3 k_B \int \sigma(v_a(t_a)) dv_a R^2(t_a)}
\]

and based on the same reason:

\[
T_B = \frac{2 \int \sigma(v_b(t_b)) dv_b R^2(t_b)}{3 k_B \int \sigma(v_b(t_b)) dv_b R^2(t_b)}
\]

And using reshift relation \( V_A a(t_A) = V_B a(t_B) \), where \( V_A \) and \( V_B \) are the frequency of the same photon at the time \( t = t_A \) and \( t = t_B \), it is proven that:

\[
a(t_B) T_B = a(t_A) T_A = \frac{2 \int \sigma(v_a(t_a)) dv_a a'(t_a)}{3 k_B \int \sigma(v_a(t_a)) dv_a a'(t_a)} = \frac{2 a(t_b) \int \sigma(v_b(t_b)) dv_b a'(t_b)}{3 k_B \int \sigma(v_b(t_b)) dv_b a'(t_b)} = T_a a(t_b)
\]

where, \( T_B \) denotes the temperature of background photons at the time \( t = t_B \). If background photons at position \( A \) satisfy black body spectrum, that is to say:

\[
\sigma(v, t) dv = \frac{8 \pi v^3 dv}{\exp \frac{2 \pi hv}{k T_A} - 1}
\]

then using \( v_A a(t_A) = v_B a(t_B) \), namely \( a(t_A) dv_A = a(t_B) dv_B \), we obtain:

\[
\sigma(v_b(t_b)) dv_b = \frac{\sigma(v_a(t_a)) dv_a a'(t_b)}{a'(t_b)} = \frac{8 \pi v_A^3 dv_A}{k T_A} a'(t_b) = \frac{8 \pi v_B^3 dv_B}{k T_B} a'(t_b)
\]

So far our conclusion has been proven. Note that if \( t_A = 0 \), \( a(0) = 0 \), then any \( v_A - v_B = 0 \) which implies that the background photons to come to us were given off in different time and the photons whose reshift are bigger were given off in earlier time and from farther source. As a result, we get the conclusion that the lower frequency of background photons measured today is, the smaller their density fluctuation or anisotropy is. The WMAP is only a telescope which see farther than other telescopes, the range seen by WMAP is still quite small compared with whole infinite universe and the existence of cosmic background radiation has nothing to do with whether universe occurred big bang or not.
Though the temperature of background photons become lower and lower due to their reshift, the average temperature of cosmic matter should be unchanged all along, the singular point of big bang should not exist. In the new theory cosmic temperature keeps unchanged since cosmic energy density is proven unchanged, all observations can parallel and even better be explained. The temperature of background photons observed today never represents that of universe today, it is worthy of laying stress that the temperature of universe means the average of all matter’s temperature but not only background photons’ temperature and talking about temperature leaving matter has no meaning. In principle, cosmic temperature can be measured directly, we may suppose that the temperative of the Milky Way represents that of universe because it is a moderate galaxy, that is to say, the temperative of universe is far higher than that of background photons measured today.

QUANTUM PROCESS OF CONTINUOUS CREATION OF MATTER IN CELESTIAL BODIES

\[ P = \rho \] tells us that the negative pressure in celestial bodies is actually a negative energy field and there \( P \) and \( \rho \) excite with each other and generate simultaneously. Connecting with particle physics it is naturally deduced that in celestial bodies many particle-antiparticle pairs (including neutron and antineutron, proton and antiproton, electron and positron and so on) can ceaselessly produce and annihilate, the antiparticles lie in negative energy level-----can’t be observed, the particles lie in positive energy level and the absolute value of energy of particle and antiparticle is equal. Let \( \Delta t \) denote the lifetime of a kind of particle-antiparticle pairs, namely the average time from production to annihilation, according to uncertain principle the range \( \Delta E \) of energy satisfies

\[ \Delta E \geq \frac{h}{2\Delta t} \]  

which shows that instantaneous energy of new particle may be very high. Note that not all of the particles annihilate as soon as they come into being, only those which don’t not have opportunity in the time \( \Delta t \) to react with the surrounding particles or to collide and change their energy can annihilate, once the reaction with other particles or the collision occur the annihilation no longer carry out and in this case the negative energy field detains a negative energy antiparticle while the particle becomes constituents of matter. Therefore, the negative energy field is too a quantum field to consist of various negative energy antiparticles. Of course, an antiparticle of energy \( \epsilon \) can be excited to energy \( \epsilon \) by a meson of energy \( 2\epsilon \) and becomes the antiparticle that can be observed. For no other reason than that many antiparticles lie in negative energy level and can’t be observed, we perceive that particles and antiparticles aren’t symmetrical. As a result of general relativity, Eq. 14 exposes already that matter and antimatter are symmetrical.

Obviously the negative pressure field, not only thermal nuclear reactions, provides energy source of star radiation, therefore, the mystery of solar neutrino doesn’t exist in the new theory framework.

Considering of tunneling effect in quantum theory, many nuclear reactions are able to complete slowly in celestial bodies even if the temperature (average kinetic energy of particles) is low which implies that in the case of low temperature elements can also compose. As for what kind of nuclear reaction is in evidence, this depends on temperature of celestial bodies. And as a result, the abundance of elements in a celestial body is the effect of various nuclear reaction for long time.
For a celestial body of temperature $T$, we may as well treat all atoms in it as a open thermodynamic system, whose giant distribution function according to quantum statistics is given by:

$$\rho = \exp(-\Psi - \sum_{i=1}^{k} \alpha_i N_i - \beta E)$$

where, $N_i$ denotes the number of atoms of $i$-th kind element. And let $m_i$ denote its mass, the total energy:

$$E = \sum_{i=1}^{k} N_i m_i$$

then the average value of atom number of element of $j$-th kind reads:

$$\bar{N}_j = \frac{\sum_{\alpha=1}^{\infty} \sum_{\beta=1}^{\infty} \sum_{\lambda=1}^{\infty} \exp N_j \left[-\psi - \sum_{\alpha=1}^{\infty} N_j (\alpha_i + \beta m_i)\right]}{\sum_{\alpha=1}^{\infty} \sum_{\beta=1}^{\infty} \sum_{\lambda=1}^{\infty} \exp \left[-\psi - \sum_{\alpha=1}^{\infty} N_j (\alpha_i + \beta m_i)\right]} = \frac{1}{m_j} \frac{\partial}{\partial \beta} \ln \left(\sum_{\alpha=1}^{\infty} \exp(-\alpha_i - \beta m_j)N_j\right) = \frac{1}{m_j} \frac{\partial}{\partial \beta} \ln(1 - e^{-\alpha_i - \beta m_j}) = \frac{1}{\exp(\alpha_j + \beta m_j) - 1} = \frac{1}{\exp\left(\frac{m_j - \mu_j}{kT}\right) - 1}$$

Here, $\mu_j$ amounts to the chemical potential of the group, $T$ is the temperature of the celestial body, namely average kinetic energy of all atoms, $k$ is Boltzmann constant. From above relation we have for arbitrary two elements $A$ and $B$:

$$\frac{N_A}{N_B} = \frac{\exp\left(\frac{m_A c^2 - \mu_A}{kT}\right)}{\exp\left(\frac{m_B c^2 - \mu_B}{kT}\right)}$$

which decides the abundance of elements in a celestial body. Observations of astronomy show that element abundance is different in different celestial bodies which is consistent with (33). Observations of astronomy show that the abundance of elements is in accordance seen from large scope which implies both temperature and chemical potential are uniform seen from large scope. Observations of astronomy show that all elements in other celestial bodies can also be found out on the earth which implies that the origin of various elements is in the same way, namely they originate all production and annihilation of particle-antiparticle pairs (Attia, 2007; Chad-Umoren, 2010; Pal et al., 2007).

**CONCLUSIONS**

Density and pressure of universe do not change all along (Bonamente et al., 2001), the singularity of big bang didn’t exist (Mei, 2011) and matter in universe is produced continuously and slowly. With cosmic expansion celestial bodies and galaxies expand too which is just the fundamental mechanism of celestial body or galaxy formation. The dark matter to appear as
negative pressure is just the antimatter that lies in negative energy level and thus cannot be observed which can not exist alone and must be accompanied by ordinary matter.

APPENDICES

A: the deduction of Eq. 20 and 24

According to description of general relativity, in the case of static spherical symmetry, in standard coordinate system the form of invariant line element is written as:

$$\text{d}s^2 = -\text{d}t^2 + A(t)\text{d}l^2 + \text{d}\text{r}^2 + \text{d}\theta^2 + \sin^2 \theta \text{d}\phi^2$$

where, \( l \) is called standard radial coordinate, space-time coordinate \( x^\mu = (x^0, x^1, x^2, x^3) = (t, l, \theta, \phi) \).

\( g_{00} = g_0 = B(l), g_{11} = g_l = A(l), g_{22} = g_{\theta\theta} = l^2, g_{\phi\phi} = g_{\phi\phi} = \sin^2 \theta \) the other components are equal to zero.

From the definition of inverse Matrix we work out \( g_{\mu\nu} = \frac{1}{B} \frac{\partial B}{\partial t}, g_{l\mu} = \frac{1}{A}, g_{\theta\theta} = \frac{1}{l^2}, g_{\phi\phi} = \frac{1}{\sin^2 \theta} \) the others are equal to zero. Form \( \Gamma^\nu_{\mu\nu} = \frac{1}{2} g^{\nu\rho} \left( \frac{\partial g_{\mu\rho}}{\partial x^\mu} + \frac{\partial g_{\nu\rho}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right) \), we work out \( \Gamma^l_{tt} = \frac{A'}{2A}, \Gamma^l_{tz} = \frac{B'}{2B}, \Gamma^l_{\phi\phi} = -\frac{\sin \theta \cos \theta}{l} \) the others are zero, where

\( A' = \frac{dA}{dl}, B' = dB \). Form \( R_{\mu\rho} = \Gamma^\sigma_{\rho\nu} \Gamma^\nu_{\mu\sigma} + \Gamma^\sigma_{\mu\nu} \Gamma^\nu_{\rho\sigma} - \Gamma^\sigma_{\rho\mu} \Gamma^\nu_{\sigma\nu} \), we work out \( R_{tt} = \frac{B^2 - B'A' - B'' - B'^2}{2A} \) the others are zero. On the other hand \( T_{\mu\nu} = (\rho + p) u^\mu u^\nu + p g_{\mu\nu}, g_{\mu\nu} u^\mu u^\nu = -1, T = g^{\mu\nu} T_{\mu\nu} = 3p - \rho \) and for the case of static spherical symmetry \( p = \rho(0), \rho = \rho(0) \), \( u^r = -\sqrt{B}, U_\phi = 0 \) then we work out:

\[
T_{tt} - T \frac{T_{tt}}{2} = \frac{B(3p + \rho)}{2}, T_{zl} - T \frac{T_{zl}}{2} = l^2 \sin^2 \theta \frac{(\rho - p)}{2}, T_{zl} - T \frac{T_{zl}}{2} = l^2 (\rho - p) \frac{1}{2}, T_{ll} - T \frac{T_{ll}}{2} = A(\rho - p)
\]

the other corresponding components are zero. Field Eq. 11 is equivalent to \( R_{\mu\nu} = 4\pi G(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}T) \), we get the following three independent equations:

\[
R_{tt} = \frac{B'}{2A} + \frac{B'}{4A} \left( \frac{A'}{A} + \frac{B'}{B} \right) - \frac{B''}{4B} = 2\pi G(p + 3p)B
\]

\[
R_{zl} = \frac{B'}{2A} + \frac{B'}{4A} \left( \frac{A'}{A} + \frac{B'}{B} \right) - \frac{A'}{A} = 2\pi G(p - p)A
\]

\[
R_{ll} = \frac{1}{2A} \left( \frac{A'}{A} + \frac{B'}{B} \right) - 1 = 2\pi G(p - p)\frac{l^2}{4}
\]

And the other corresponding equations are identities. Then we have

\[
R_{tt} + R_{zl} + R_{ll} = -\frac{1}{l^2} + \frac{1}{l^2} - \frac{A'}{Al} = 4\pi G\rho \text{ namely } \left\{ \frac{1}{A} \right\} = 1 + 4\pi G\rho l^2 \text{ and since } A(0) \text{ is limited, we infer}
\]

\[
A(l) = \left(1 + \frac{Go(l)}{l}ight)^{-1}, \text{ where, } Go(l) = 4\pi \int_0^l \rho(l^2) dl.
\]

On the other hand, the conservation law \( T_{\mu\nu} = 0 \) gives \( \frac{B'}{B} = \frac{2\rho'}{\rho + p} \) then from:

\[
R_{tt} = \frac{1}{2} \left(1 + \frac{Go(l)}{l} \right) \left[ \frac{G}{F} (\omega(1 + Go)) \left( \frac{-2p'}{\rho + p} \right) + (1 + Go) \right] - 2\pi G(p - p)\frac{l^2}{4}
\]

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after being simplified:

$$\frac{dp}{dl} = G(p+p) \left( 2\pi^2p + \frac{\omega}{2} \right) \left( 1 + lG\varepsilon \right)^{-1}$$

And again, from:

$$\frac{B - 2p}{B + p} = -2G \left( 2\pi^2p + \frac{\omega}{2} \right) \left( 1 + lG\varepsilon \right)^{-1}$$

we obtain:

$$B(l) = \exp \left[ C_1 + \int^l f(l) \left( 1 + \frac{\phi(l)}{l} \right)^{-1} dl \right]$$

where:

$$f(l) = \frac{G}{l^2} \left[ 4\pi l^2p(l) + \phi(l) \right]$$

and constant:

$$C_1 = \ln \left( 1 - \frac{2GM}{l} \right)$$

it makes sure B(l) is continuous on the bound r_s (surface of source). Note that the value of l(r_s) on the bound is determined by Eq. 23.

B: The deduction of luminosity distance:

$$d_L = (1 + Z) \int_0^{r_s} \frac{dl}{\sqrt{1 - kl^2}}$$

At the moment t proper distance of a galaxy is defined as:

$$d_p = a(t) \int_0^{r_s} \frac{dl}{\sqrt{1 - kl^2}}$$

Let a telescope of area A faces the galaxy. Within time \( \delta t_0 \) the galaxy emitted n photons of total energy nhv_s and within time \( \delta t_0 \) they arrive at the telescope. Spectrum radiate power of galaxy is defined as \( L = \frac{nhv_s}{\delta t_0} \). Power received by telescope is:

$$p = \frac{nhv_s}{\delta t_0} \frac{A}{4\pi d_p^2(t_0)}$$

Using \( v_s = \frac{v_s(t_a)}{a(t_a)} \) and \( \frac{1}{\delta t_0} = \frac{a(t_a)}{\delta t_a a(t_a)} \) we have:
\[ p = \frac{\text{nhv}}{\delta_0} \frac{A}{4\pi d_i^2(t_i)} \frac{\text{nhv} a^2(t_i)}{4\pi a_i^2(t_i)} = \frac{\text{La} a^2(t_i)}{4\pi a_i^2(t_i) d_i^2(t_i)} \]

Vision luminosity received by telescope is defined as:

\[ I = \frac{p}{A} = \frac{\text{La}^2(t_i)}{4\pi a_i^2(t_i) d_i^2(t_i)} \]

We know that vision luminosity of light source in Euclidean space is:

\[ I = \frac{L}{4\pi d^2} \]

and generalizing the definition to curve space, luminosity distance:

\[ d_L = \frac{I}{\sqrt{4\pi}} = \frac{a(t_i)}{a(t_i)} d_i(t_i) \]

Using:

\[ \frac{a(t_f)}{a(t_i)} = 1 + Z \]

and putting \( a(t_o) = 1 \), finally we obtain:

\[ d_L = (1+Z)d_i(t_i) = (1+Z) \int_{t_i}^{t_f} \frac{dt}{\sqrt{1-k t^4}} \]

REFERENCES


