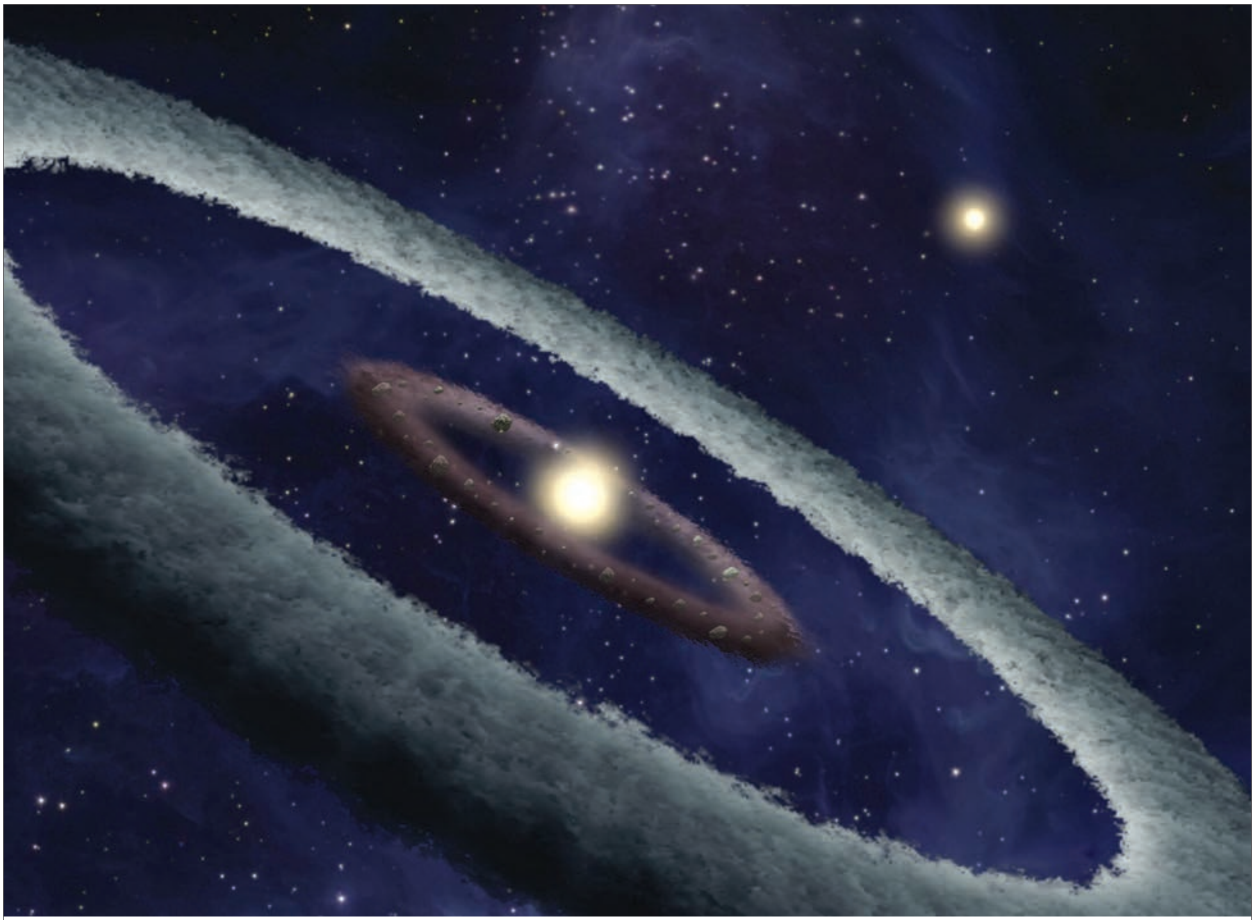

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Effect of Nonthermal Electron on Dust-acoustic Shock Waves in Dusty Plasma

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ABSTRACT

A dusty plasma system containing non-thermal electron distributions, Boltzmann distributed ions and mobile charge fluctuating positive dust has been considered. The nonlinear propagation of the dust-acoustic (DA) waves in such dusty plasma has been investigated by employing the reductive perturbation method. The effect of non-thermal electrons on the height and thickness of DA shock waves are also studied. It has been found that the thickness of the Dust acoustic shock composition decreases as the non-thermal parameter increases, while the amplitude of the shock composition thickness varies with the charge fluctuating dust.

Key words: Dust-acoustic, shock waves, non-thermal electrons, perturbation

INTRODUCTION

Dust and plasmas exist together in the universe and they make dusty plasmas. Dusty plasmas are found in cometary tails, asteroid zones, planetary ring, interstellar media, lower part of Earth's ionosphere and magnetosphere, etc. (Goertz, 1989; Mende's and Rosenberg, 1994; Shukla, 2001; Horanyi and Mendis, 1986; Verheest, 2001; Horanyi, 1996). Noticeable applications of dusty plasmas are also found in laboratory devices (Barkan *et al.*, 1995; Merlino *et al.*, 1998; Homann *et al.*, 1997). There has been a rapidly growing interest in the field of dusty plasmas because of its great variety of new phenomena associated with wave and instabilities (Verheest, 1992; Pieper and Goree, 1996; Bliokh and Yaroshenko, 1985). The existing plasma wave spectra are not only modified by the presence of charged dust grains in a plasma (De-Angels *et al.*, 1988; Shukla and Stenflo, 1992), it also brings about new novel eigen modes such as Dust Acoustic (DA) wave (Rao *et al.*, 1990; Barkan *et al.*, 1996), Dust Ion Acoustic (DIA) waves (Shukla and Silin, 1992; Barkan *et al.*, 1996) etc.

From the first theoretical study on the ultra low frequency DA waves in dusty plasma by Rao *et al.* (1990) and motivated by the experimental observations of these waves (Barkan *et al.*, 1996; Pieper and Goree, 1996), numerous investigations have been carried out to study the different aspects of the physics of dusty plasmas during the past few years. However, most of the investigations were mainly on dusty plasmas with negatively charged dust grains (Amin *et al.*, 1998; Popel and Yu, 1995; Ma and Liu, 1997), in this regard, nonlinear solutions and double layers in dusty plasmas have been investigated by several authors (Bharuthram and Shukla, 1992; Mamun *et al.*, 1996). However, in the space plasmas environments, some plasma systems are found with positively charged dust grains (Mendis and Horanyi, 1991; Chew *et al.*, 1993; Haunes *et al.*, 1996; Horanyi *et al.*, 1993). Such dust grains with net positive charge are due to processes such as irradiation by Ultraviolet (UV) light; thermionic emission produced by radiative heating as well as secondary emission of electrons from the surface of the dust grains (Verheest, 1992; Shukla and Mamun, 2002).

Recently, Paul *et al.* (2009), investigated the nonlinear propagation of DA waves accounting for the charge fluctuating positive dust and Boltzmann-distributed electrons and ions. For this purpose, they derived the Burgers equation, by employing the reductive perturbation method (Washimi and Taniuti, 1996). They showed that, the dust charge fluctuation is a source of dissipation and is responsible for the formation of collisionless DA shock waves in such dusty plasma. Since in a real dusty plasmas; the electron behaviour can be powerfully modified by the nonlinear potential of the localized DA composition by generating a population of fast vigorous electrons, the present paper is mainly to determine how the electron non-thermality effect can be expected to modify the result of Paul *et al.* (2009). This simplification involves a little increase in algebraic intricacy of the pertinent formulas. This notwithstanding, the basic principles do not change.

BASIC EQUATION

We consider unmagnetized collisionless dusty plasma consisting of non-thermal electrons, Boltzmann-distributed ions and charge fluctuating positively charged mobile dust. We assume for simplicity that all the grains have the same charge, equal to $q_d = +z_d e$, with z_d representing the charge state of the dust component. Hence, charge neutrality at equilibrium is given by $n_{e0} = n_{i0}+z_{d0} n_{d0}$, where $n_{e0}(n_{i0})$ is the equilibrium electron (ion) number density, n_{d0} is the dust density at equilibrium, z_{d0} represent equilibrium charge state of the dust component. All the dust grain is assumed to be spheres of radius r_d. The basic equations for one-dimensional DA waves for such a dusty plasma is given as:

$$\frac{\partial n_d}{\partial t} + u_d \frac{\partial}{\partial x}(n_d u_d) = 0 \tag{1}$$

$$\frac{\partial n_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = -\frac{z_d e \partial \phi}{m_d \partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_e - n_i - z_d n_d) \tag{3}$$

where, ϕ is the electrostatic potential, n_d, n_e, n_i are respectively, the number density for the plasma species for dust, electrons and ions, u_d is the dust fluid speed. The non-thermal electron distribution is given as (Carins *et al.*, 1995):

$$n_e = n_{e0}\left(1 - \beta_n \frac{e\phi}{k_B T_e} + \beta_n \frac{e^2\phi^2}{k_B^2 T_e^2}\right) e^{\frac{e\phi}{k_B T_e}} \tag{4}$$

Where:

$$\beta_n = \frac{4\alpha_1}{1+3\alpha_1} \tag{5}$$

and the Boltzmann distributed ion as:

$$n_i = n_{io} e - \frac{e\phi}{k_B T_e} \tag{6}$$

where, α_1 is a parameter determining the number of non-thermal electrons present in our plasma model, k_B is the Boltzmann constant and T_e (T_i) is the electron (ion) temperature. Neglecting all other charging processes, we assume that the dust is charged by photoemission current (I^+_p), thermionic emission current (I^+_t) and electron absorption current (I^-_e) only. The charge state z_d of the dust component is not constant but varies according to the following equation (Paul *et al.*, 2009; Shukla and Mamun, 2002):

$$\frac{\partial z_d}{\partial t} + u_d \frac{\partial z_d}{\partial x} = \frac{1}{e}(I_p^+ + I_t^+ + I_e^-) \tag{7}$$

Where:

$$I_p^+ = \pi r_d^2 eJY \exp\left(-\frac{z_d e^2}{k_B r_d T_{ph}}\right) \tag{8}$$

$$I_p^+ = \pi r_d^2 e\left(-\frac{2\pi m_e k_B T_p}{h^2}\right)^{\frac{3}{2}} \left(\frac{8k_B T_p}{\pi m_e}\right)^{\frac{1}{2}} \times \left(1+\frac{z_d e^2}{r_d k_B T_p}\right) \exp\left(-\frac{z_d e^2}{r_d k_B T_p} - \frac{w_e}{k_B T_p}\right) \tag{9}$$

$$I_e^- = \pi r_d^2 e n_{e0} \exp\left(\frac{e\phi}{k_B T_p}\right)\left(\frac{8k_B T_p}{\pi m_e}\right)^{\frac{1}{2}} \times \left(1+\frac{z_d e^2}{r_d k_B T_p}\right) \tag{10}$$

where, h is the Planck's constant, T_{ph} is the photon temperature, w_e is the work function, J is the UV photon flux, Y is the yield of photons. The typical values of w_e, J and Y are given respectively as 8.2 eV, 5.0×10^4 photons /cm^2/s and 0.1. For convenience, we express the set of Eq. 1 to 7 in normalized form by introducing the following normalized variables: $N_d = n_d/n_{do}$, u_d-u_d/C_d, $\phi = e\phi/k_B T_e$, $Z_d = z_d/z_{do}$, $X = x/\lambda_{Dd}$, $T = tw_{pd}$, $\lambda_{Dd} = (k_B T_e/4\pi z^2_{do})^{1/2}$, $C_d \equiv (z_{do} k_B T_e/m_d)^{1/2}$ and $w_{pd} = (4\pi z^2_{do} n_{do} e^2/m_d)^{1/2}$ to obtain the following equations:

$$\frac{\partial N_d}{\partial T} + \frac{\partial}{\partial X}(N_d U_d) = 0 \tag{11}$$

$$\frac{\partial U_d}{\partial T} + Ud\frac{\partial U_d}{\partial X} = -Z_d \frac{\partial \phi}{\partial X} \tag{12}$$

$$\frac{\partial^2 \phi}{\partial X^2} = (1+\mu)(1-\beta_n \phi^2)e^{\phi} - \mu_i e^{-\sigma\phi} - Z_d N_d \tag{13}$$

$$\frac{\partial Z_d}{\partial T}+U_d\frac{\partial Z_d}{\partial X} = \mu\left[Pe^{-\alpha Zd}+Q(1+\beta Z_d)e^{-\beta Zd}+Re^{\phi}(1+\beta Z_d)\right] \tag{14}$$

where, σ is T_e/T_i; μ_i is $n_{io}/z_{do}n_{do}$; μ_i is $n_{io}/z_{do}n_{do}$; μ is $\pi\, r^2{}_d/z_{do}w_{pd}$; P is JY; Q is $2e^{-we/kBTe}(2\pi m_e k_B T_e)/h^{2\,3/2}(8k_BT_e/\pi m_e)^{1/2}$, R is $n_{e0}(8k_BT_e/\pi m_e)^{1/2}$, α is $z_{do}e^2/r_dk_BT_{ph}$; β is $z_{do}e^2/r_dk_BT_e$.

NONLINEAR DUST ACOUSTIC SHOCK WAVES

To study the dynamics of nonlinear dust acoustic shock waves in the presence of non-thermal electrons, Boltzmann distributed ions and charge fluctuating positive dust grains; we employ the reductive perturbation technique (Washimi and Taniuti, 1996). We introduce the stretched coordinates (Das *et al.*, 1997) $\xi = \epsilon(X\text{-}V_0T)$ and $\tau = \epsilon^2T$, where ϵ is a small parameter and V_0 is the DA shock waves velocity normalized by C_d. The variables N_d, U_d, Z_d and ϕ are expanded as:

$$\begin{gathered} N_d = 1+\epsilon N_{d1}+\epsilon^2 N_{d2}+.... \\ U_d = \epsilon U_{d1}+\epsilon^2 U_{d2}+....... \\ \\ Z_d = 1+\epsilon Z_{d1}+\epsilon^2 Z_{d2}+.... \\ \phi = \epsilon\phi_1+\epsilon^2\phi_2+.... \end{gathered} \tag{15}$$

Now, substituting these expansions in to Eq. 11-14 and collecting the terms of different powers of$\in$, in the lowest order, we obtain:

$$U_{d1} = \frac{\phi_1}{V_0} \tag{16}$$

$$N_{d1} = \frac{\phi_1}{V_0^2} \tag{17}$$

Where:

$$Zd1 = [1\text{-}\beta_n+\mu_i(1\text{-}\beta_n+\sigma)\text{-}\frac{1}{V_0^2}]\phi_1 \tag{18}$$

$$V_0^2 = \frac{\Gamma}{\Gamma[1-\beta_n+\mu_i(1-\beta_n+\sigma)+\mu R(1+\beta)]} \tag{19}$$

Where:

$$\Gamma = \mu P\alpha^2-\mu P\alpha-\mu P\alpha-\mu Q\beta^2+\frac{3}{2}\mu Q\beta^3+\mu R\beta$$

The next order in ϵ,O(ϵ^2) yields a system of equations that leads to Burgers equation as follows:

$$\frac{\partial N_{d1}}{\partial\tau}-V_0\frac{\partial N_{d2}}{\partial\xi}+\frac{\partial U_{d2}}{\partial\xi}(N_{d1}U_{d1}) = 0 \tag{20}$$

$$\frac{\partial N_{d1}}{\partial \tau} - V_0 \frac{\partial N_{d2}}{\partial \xi} + U_{d1} \frac{\partial U_{d1}}{\partial \xi} = -\frac{\partial \phi_2}{\partial \xi} - Z_{d1} \frac{\partial \phi_2}{\partial \xi} \tag{21}$$

$$\begin{aligned}
&[(1-\beta_n)\{1+\mu_i\}+\sigma\mu_i]\phi_2 + \left[\frac{(1-\beta_n)}{2} + \mu_i\left\{\frac{(1-\beta_n)}{2} - \frac{\sigma^2}{2}\right\}\right]\phi_1^2 = Z_{d2} + Z_{d1}N_{d1} + N_{d2} \\
&-Z_0\frac{\partial Z_{d1}}{\partial y} = \mu\left[-P\alpha + P\alpha^2 - Q\beta^2 + \frac{3Q\beta^3}{2}RB\right]Z_{d2} + \mu\left[\frac{P\alpha^2}{2} - \frac{1}{2}Q\beta^2 + \frac{3}{2}\mu Q\beta^3\right]Z_{d1}^2
\end{aligned} \tag{22}$$

$$+\mu R(1+\beta)\phi_2 + \frac{1}{2}\mu R(1+\beta)\phi_1^2 + \mu R\beta Z_{d1}\phi_1 \tag{23}$$

Making using of Eq. 16-23 we eliminate N_{d2}, U_{d2}, Z_{d2} and ϕ_2 to obtain the following equation:

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} = B\frac{\partial_2 \phi_1}{\partial \xi^2} \tag{24}$$

where, the nonlinear coefficient A and the dissipation coefficient B are given by:

$$A = \frac{\lambda V_0^3}{2} \tag{25}$$

$$\begin{aligned}
\lambda = &-2\left[\frac{(1-\beta_n)}{2} + \mu_i\left\{\frac{(1-\beta_n)}{2} - \frac{\sigma^2}{2}\right\}\right] + \frac{3}{V_0^2}[1-\beta_n+\mu_i(1-\beta_n+\sigma)] - \frac{\mu R}{\Gamma}(1+\beta) - \frac{2\mu R\beta}{\Gamma} \\
&[1-\beta_n+\mu_i(1-\beta_n+\sigma)] - \frac{\mu}{\Gamma}(P\alpha^2 - \phi\beta^2 + 3\phi\beta^3)\left[1-\beta_n+\mu_i(1-\beta_n+\sigma) - \frac{1}{V_0^2}\right]^2
\end{aligned} \tag{26}$$

$$B = \frac{V_0^4}{2\Gamma}\left[1-\beta_n+\mu_i(1-\beta_n+\sigma) - \frac{1}{V_0^2}\right] \tag{27}$$

The Burgers equation which describes the nonlinear propagation of the DA shock waves in the dusty plasma under consideration is given as Eq. 24. It can be observed that, the right hand side of Eq. 24 which represent the dissipative term is due to the presence of non thermal parameter (β_n), the ratio of electron and ion temperature (σ) and the charge fluctuating positive dust (μ_i).

RESULTS AND DISCUSSION

Our expression for Z_{d1} as Eq. 18 agrees with what is obtained by Paul *et al.* (2009) when nonthermal parameter β_n is set to zero and σ set to 1. We strongly feel that the last term in the denominator for Eq. 18 should be [$+\mu R(1+\beta)$] as against what is obtained by Paul *et al.* (2009) as [$-\mu R(1+\beta)$]. Likewise, the last term for f as ($-\mu R\beta$) reported by Paul *et al.* (2009) should be ($+\mu R\beta$) as in our report for Γ. Equation 19 which gives the linear desperation relation for DA waves is greatly altered by the presence of the electron nonthermal parameter, ratio of electron and ion temperature, as well as the positive dust charge fluctuation. For stationary shock wave solution of Eq. 24, we set $\zeta = \xi - U_0\tau'$ and $\tau' = \tau$ to obtain the equation:

$$-U_0\frac{\partial\phi_1}{\partial\zeta}+A\phi_1\frac{\partial\phi_1}{\partial\zeta} = B\frac{\partial^2\phi_1}{\partial\zeta^2} \tag{28}$$

The latter equation can be integrated, using the conduction that ϕ is bounded as w$\zeta\rightarrow$ or by the application of Tanh method (Malfliet, 1992, 2004; Malfliet and Hereman, 1996a, b) to yield:

$$\phi_1 = \phi_0\left\{1-\tanh\left(\frac{\zeta}{\Delta_{sh}}\right)\right\} \tag{29}$$

Where:

$$\phi_0 = \frac{U_0}{A}$$

and:

$$\Delta_{sh} = \frac{2B}{U_0}$$

Equation 29 represents a monotonic shock-like solution with the shock speed, the shock height and the shock thickness given by U_0, ϕ_0 and Δ_{sh}, respectively. It is obvious from Eq. 29 that, the presence of electron non- thermal parameter significantly modifies the shock wave amplitude and its width.

To see the influence of the non-thermal parameter on the DA shock waves, we chose σ as 1.5 and vary β_n. The following parameters; $U_0 = 0.1$ m sec^{-1}, $P = 5.00\times10^{17}$ m^{-2}sec^{-1}, $Q = 1.07\times10^{31}$ m^{-2} sec^{-1}, $R = 2.48\times10^{13}$ m^{-2} sec^{-1}, $V_0 = 0.8$, $\beta = 1.2\times10^{-9}$ c^2 kg^{-1} m^{-3} sec^{-2}, $\mu = 2.5\times10^{-12}$ m^2 sec^{-1} corresponding to the mesosphere event has been chosen from Paul *et al.* (2009). Figure 1 shows that as the β_n increases; the positive shock width decreases, while the amplitude of the positive shock width varies as μ_i increases.

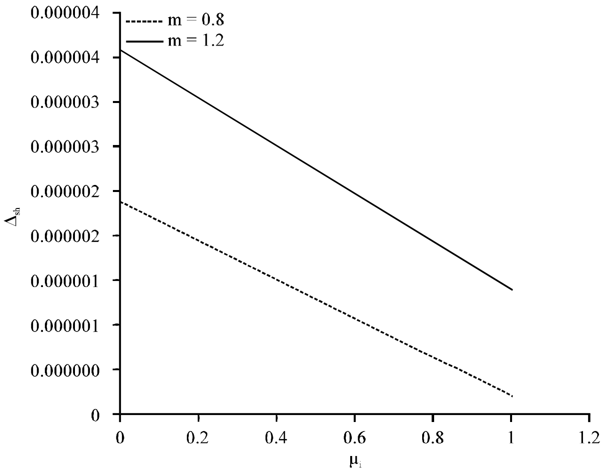

Fig. 1: Variation of positive shock thickness (Δ_{sh}) potential profile with non-thermal electron parameter (β_n) for different values of μ_i

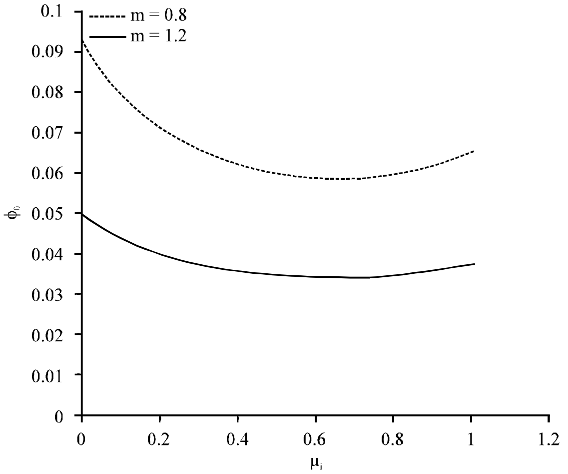

Fig. 2: Variation of amplitude ϕ_0 of the positive shock wave with (β_n) for different values of μ_i

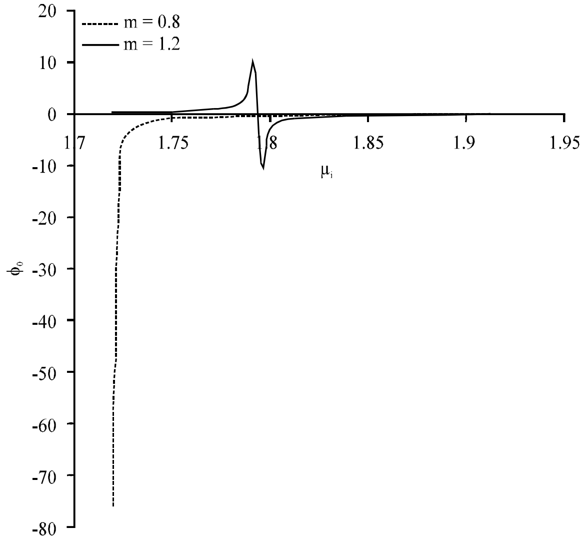

Fig. 3: Variation of amplitude ϕ_0 of the negative shock wave with β_n for different values of μ_i

The variation of amplitude (ϕ_0) of the positive shock height waves are presented in Fig. 2. From this Fig. 2, it can be observed that, the amplitude of the positive shock height decrease with increase in β_n up to 0.5 and after this point it increases. Also, we can observe that, when μ_i the amplitude of the shock height wave, is smaller than what is obtained for $\mu_i = 0.8$ due to the fact that the dust charge fluctuation is a sources of dissipation DA waves.

Figure 3 demonstrate the variation of negative amplitude of the shock height with β_n. It shows that the shock height increases with β_n up to 1.75 after which, there is no variation of the shock height with β_n for $\mu_i = 0.8$. When $\mu_i = 1.2$, the amplitude of the negative shock height suddenly increases with β_n from 1.78 and then drops to exactly the same amplitude on the negative axis. After which the negative amplitude of the shock height increase with β_n to infinity. Since space plasma are more realistically modeled by making use of non-thermal velocity distributions as discussed by Maharaj *et al.* (2006), we have seen that the non-thermal electron distribution function significantly modifies the result obtained by Paul *et al.* (2009).

CONCLUSION

We have extended the recent work of Paul *et al.* (2009) to see under what conditions the electron non-thermality effect can be expected to modify the results of their analysis. We have shown here that, the basic feature; of the non-linear DA waves are modified by both presence of the non-thermal electron and the charge fluctuating dust in dusty plasmas. Present results are summarized as follows:

- The width of the DA shock structures decreases as the non- thermal parameter increases, while the amplitude of the shock structures width varies as μ_i increases. This is due to the fact that dust charge fluctuation is a source of dissipation and lead to the development of DA shock waves in the dust plasma
- It is also shown that, the positive amplitude of the shock height decreases with increase in β_n up to a point and then increases. While, when the charge fluctuation (μ_i) is increased to 1.2, the negative shock height exhibits the occurrence of kink as in Fig. 3. The findings in this paper are important in understanding nonlinear DA wave phenomena in space plasmas

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