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Spline and Overlap Techniques for Analyzing SLR Data

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ABSTRACT

Spline and Overlap techniques are used for analyzing Satellite Laser Ranging (SLR) data. We applied the two techniques for fitting various SLR-data for 30 samples. It is found that the polynomial degree and the corresponding standard deviation of the two techniques are much better than that obtained when fitting the total number of data, in addition to a slight advantage for Spline technique over the overlap technique. However, it is found that about 11/30 samples represents that the overlap technique is better than the Spline technique which may refers to the distribution of the data points inside the interval. Furthermore, the effect of the satellite signatures on the fitting accuracy is investigated. The study clarified that the precision of the data fitting for the low signatures satellites are much better than those obtained for high signature satellites.

Key words: Spline technique, overlap technique, fitting, Chebyshev polynomial, SLR data, standard deviation

INTRODUCTION

For orbital computations, simultaneous observations are needed to be carried out by the individual observing stations (Batrakov, 1961; Degnan, 1994; Cech *et al.*, 1998). If the different observational data, obtained from the individual stations, are not carried out at the same time we would deal with functions which either is not known at every value of the independent variable within an interval or their expressions are so complicated that the evaluation of the function is prohibitive.

So far, we are dealing, in fact, with data fitting. This can be subjected to the methods of interpolation. Mathematically, we are concerned with a function $y = f(x)$, whose values x_i, $(i = 0, 1, 2, \ldots, n)$ of the independent variable x and it is required to adopt a function $p(x)$, to replace the function $f(x)$ which assumes the same values as $f(x)$ for x_i and from which other values can be easily calculated to a desired degree of accuracy. In dealing with SLR data, the interpolating polynomials may not be suitable for use as an approximation. Particularly, when fitting large number of data points using high degree polynomial, the fitting may suffer from noticeable error due to the nature of approximation (Johnson and Riess, 1982). For this reason, either the Spline or overlapping techniques are used as an attractive alternative.

In this article, we present two techniques for fitting SLR data, the Spline technique and overlapping technique. We apply these two techniques on the SLR data of 7 artificial satellites taken from Helwan SLR station.

SPLINE TECHNIQUE FOR MATCHING CONTINUOUS PROGRESSIVELY INTERVALS

Spline technique is defined as an artificially constructed sequence of polynomials matched one to another at the end points of the intervals at which the polynomials are defined (Nmadu *et al.*, 2009; Hama-Salh, 2010). This technique is used in order to join lower degree polynomials which leads to a good fitting (De Boor, 1978; Nurnberger, 1989).

Starting with observational measurements y_i = 1, 2, ..., n taken at time t_i = 1, 2, ..., n. These data are divided into groups. Two consecutive groups, y_{ij}, i = 1, 2 and j = 0, 1, ..., n_i are selected. The closed time intervals $[t_{i0}, t_{in}]$ are mapped into the closed intervals $[-1, 1]_i$, using the following Xtransformation:

$$x_{ij} = \frac{2t_{ij} - t_{i0} - t_{in}}{t_{in} - t_{i0}}, \qquad j = 0, 1, ..., n$$

Two independent Chebyshev polynomials (Taiwo and Abubakar, 2011; Zhang *et al.*, 2012) are constructed for y_{ij} as functions of variable x are given in the following form:

$$p_i(x) = \sum_{k=0}^{m_i} \tilde{a}_{ik} f_{ik}(x)$$

where, $f_{ik}(x)$ are the Chebyshev polynomials as given by Hanna (2002).

The functions y_i, i = 1, 2 are two independent approximating polynomials representing two successive intervals of one set of the data. These two constructed polynomials are, in general, disjoined. The value of the polynomial at the first data point of the second subinterval may not be matched with the value of the polynomial at the end data point of the first subinterval. The combined adjustment method, given below, is to construct the proper Spline.

In order to join the two constructed interpolating polynomials y_i, i = 1, 2 for the two successive intervals, the coefficients $\tilde{a}_{2k}$ $(k = 0, 1, ..., m_2)$ have to be determined for the constraint (Hanna *et al.*, 2004):

$$y_2(-1) = y_1(1)$$

Where:

$$y_2(-1) = \sum_{j=0}^{m_2} \tilde{a}_{2j} f_{2j}(-1)$$

$$y_1(1) = \sum_{j=0}^{m_1} \tilde{a}_{1i} f_{1i}(1)$$

and then the coefficients can be obtained in the form:

$$\tilde{a}_{2k} = a_{2k} + \frac{b_{2k}[y_1(1) - \sum_{s=0}^{m_2} \tilde{a}_{2s} f_{2s}(-1)]}{\sum_{s=0}^{m_2} b_{2s} f_{2s}(-1)}$$

where, $\tilde{a}_{2k}$ are the coefficients of the 2nd polynomial (Hanna *et al.*, 2004) as a result of applying the proper Spline.

THE OVERLAPPING TECHNIQUE

The overlapping method is based on using the overlapped intervals of observations. The reason that it has not been used in orbit computations is connected probably with the fact that the statistical characteristics of the normal points obtained by the overlapping method have not yet been studied and their strict use is difficult (Bjorck and Yuan, 1999). We have not the aim to investigate these characteristics completely but we shall restrict ourselves to a more particular problem. This problem is how to form a normal point corresponding to the all-available measurements (for example, laser measurements of the range during one passage of an artificial Earth's satellite in the zone of visibility of a tracking station by using normal points obtained with overlapping subintervals.

Let the interval of time L, covering one passage of the satellite over the station, be divided into a number of subintervals L_i (i = 1, 2, ... , m) successively matched to one another. Assume that we have a sufficiently good preliminary orbit of a satellite. The discrepancy between the observed and the calculated ranges are small and can be smoothed by a polynomial of a low degree on both of the subintervals L_i and the whole interval L. The measurements are considered to be independent and of equal accuracy.

Let the first intermediate normal point be formed as an arithmetic mean of the differences between the observed and computed ranges at the united interval containing L_1 and L_2. The second arithmetic mean is formed by an analogous way at the united interval including L_2 and L_3 and so on until the united interval L_{m-1} and L_m. We suppose also that in a quite similar way the normal point at the united interval L_1 and L_m has been determined.

Analysis of the variations within and between the overlapping intervals will be found and discussed by Hanna (2002) and Samwel *et al.* (2005).

COMPARATIVE STUDIES ON THE OVERLAP AND SPLINE TECHNIQUES

In the present study, both Spline and overlapping techniques are applied for 30 samples of ranging data for satellites AJISAI (Aj), TOPEX (TP) and BEACON-C (BC) (Ibrahim *et al.*, 2011). The number of data points in these samples range from 153 to 1482. Table 1 summarized the results obtained.

The name of satellite and the number of data points are represented in the first column. Second column shows the polynomial degree and the corresponding Standard Deviation (SD) obtained in case of fitting total number of data. Third and forth columns represent the polynomial degree and the corresponding standard deviation obtained using Spline technique and overlapping technique, respectively.

As represented in Table 1, the average polynomial used for fitting the whole data is of 8th degree while it is of 4th degree using both spline and overlapping technique. In addition the average standard deviation reached in case of fitting the whole data is about 882.7 mm while it is of about 1.2 mm for spline technique and 3.97 mm for overlapping technique which clarify that the used polynomial and the corresponding standard deviation of the two techniques, spline and overlap techniques, are much better than that obtained when fitting the total number of data.

Also, it is found that the average polynomial degrees and average standard deviations of both Spline and overlapping techniques are comparable with slight merit for spline technique over the

Table 1: Results of fitting SLR data. The standard deviation and the corresponding degree of the used polynomials are given

Data		Fitting whole data		Fitting data using spline technique		Fitting data using overlapping technique	
Satellite name	No. of points	Poly. degree	Standard deviation (mm)	Poly. degree	Standard deviation (mm)	Poly. degree	Standard deviation (mm)
AJ 2000/09/03	1285	9	1534.00	4	0.875	4	2.850
AJ 2005/04/24	669	11	70.38	5	10.770	5	24.290
AJ 2000/09/04	636	9	139.00	5	1.007	5	0.145
AJ 2000/09/09	552	11	81.99	5	7.480	5	4.308
AJ 2005/07/10	303	9	501.10	5	11.780	5	6.840
AJ 2000/09/11	291	5	34.01	4	0.635	4	4.270
AJ 2005/08/21	249	9	804.80	5	10.287	5	12.970
AJ 2005/08/23	243	9	48.64	5	0.646	5	17.180
AJ 2000/10/17	219	9	1618.80	5	3.475	5	15.660
AJ 2000/10/22	213	11	20.47	5	1.041	5	1.690
TP 2000/04/15	1482	9	94.07	4	0.050	4	0.087
TP 2000/10/25	1055	9	222.30	4	0.191	4	0.740
TP 2000/09/10	985	9	1044.10	4	6.892	4	0.890
TP 2000/10/26	744	9	1967.00	5	0.183	5	0.035
TP 2005/06/16	568	9	7.86	5	0.091	4	2.260
TP 2005/06/17	536	8	329.15	4	0.519	5	2.690
TP 2005/08/02	492	11	22.26	5	0.415	5	0.645
TP 2005/06/14	492	9	170.50	5	1.463	4	3.680
TP 2005/08/06	468	9	97.32	5	0.378	4	4.700
TP 2000/09/11	460	9	63.15	5	0.701	5	0.260
BC 2000/09/04	915	10	75.84	4	0.150	4	1.040
BC 2000/09/03	775	9	325.40	4	0.520	4	0.380
BC 2000/11/06	489	8	103.60	5	2.560	4	0.290
BC 2000/09/10	426	9	1556.00	5	11.380	5	8.670
BC 2000/11/05	384	11	124.70	6	1.390	5	34.250
BC 2000/10/29	354	8	33.86	5	1.370	5	0.020
BC 2000/11/08	309	6	1.98	4	1.260	4	17.020
BC 2000/11/04	237	4	1.12	3	0.050	3	0.029
BC 2005/08/13	177	9	92.61	5	0.110	5	0.870
BC 2005/05/16	153	7	231.40	4	1.530	4	5.090
Average		8	882.70	4	1.2025	4	3.970

overlap technique. The average standard deviation using Spline technique is 1.2 mm with polynomial of 4th degree, while it is about 3.97 mm with polynomial of 4th degree using overlapping technique.

For more clarity, Fig. 1 represents the results obtained for one of the samples under study. The data taken in the present case consist of 552 data points for satellite Beacon-C (6503201) which was observed in 4/9/2000 from Helwan SLR station. Figure 1a represents the range and best fitting for the whole number of data. Figure 1b and c represent the range and best fitting of the second subinterval, using Spline technique and overlapping technique, respectively.

As it is clarified in Fig. 1, the polynomial, used for fitting the whole data is of 10th degree with corresponding standard deviation of 75.84 mm while the polynomials used for fitting data using both Spline and overlapping technique are of the 4th degree and the corresponding average standard deviations are about 0.15 mm for Spline technique and about 1.04 mm for overlapping technique.

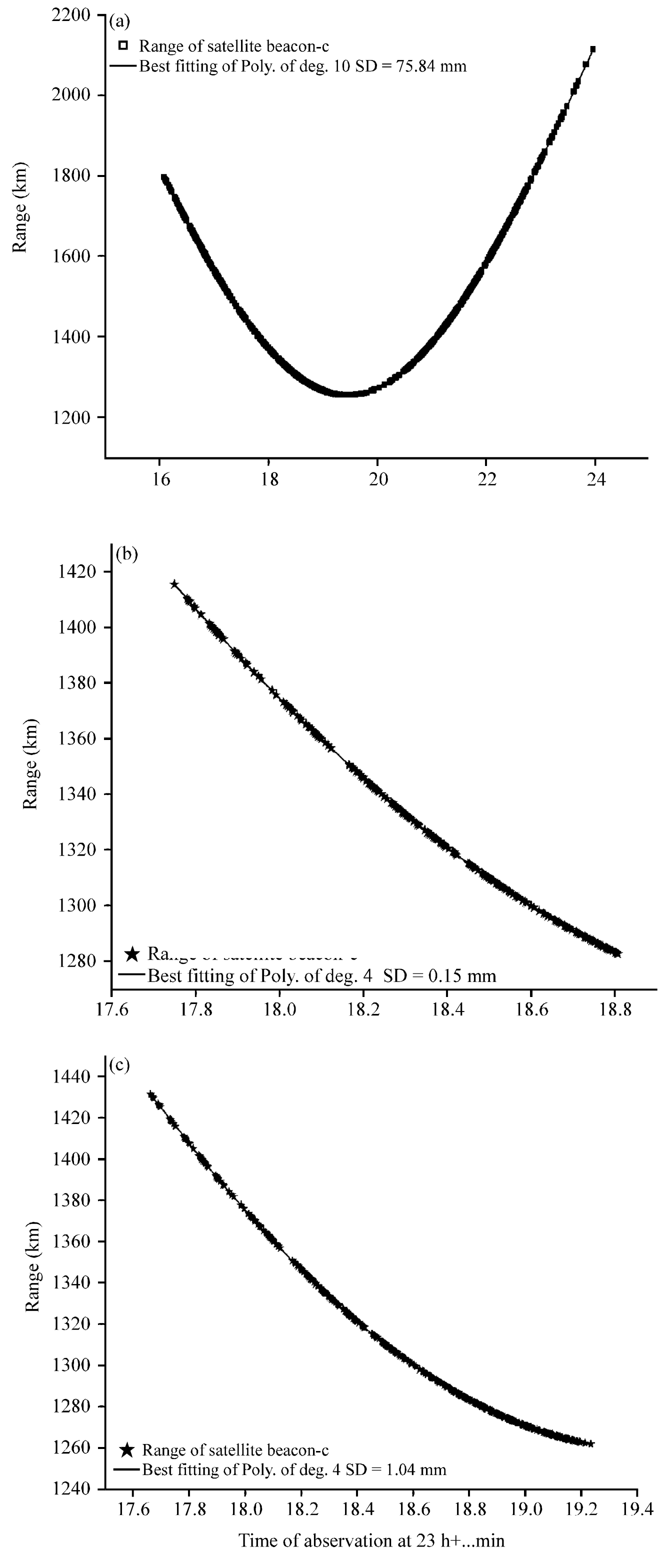

Fig. 1(a-c): The range and best fitting of satellite Beacon-C for (a) Fitting whole data, (b) Fitting using Spline technique and (c) Fitting using overlapping technique

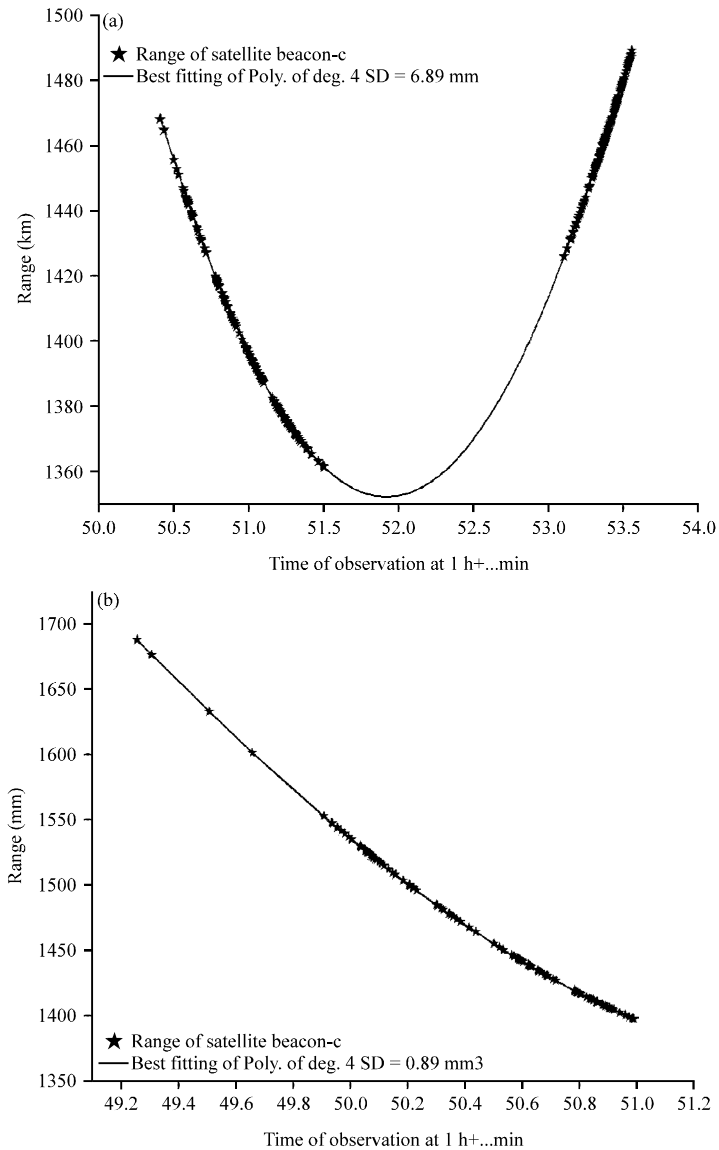

Fig. 2(a-b): The range and best fitting of satellite TOPEX for (a) Fitting data using Spline technique and (b) Fitting data using overlapping technique

However, it is found that about 11/30 cases have slight merit for overlap technique over the Spline technique as represented in Fig. 2 which be explained by the existence of gaps within the data points. So, generally, the distribution of the data points in each interval may adversely affect the degree of the used polynomial and the corresponding standard deviation obtained.

For more clarification, Fig. 2 represents the results obtained for one of the samples under study. The data taken in the present case consist of 985 data points for satellite Beacon-C (6503201) which was observed in 10/9/2000 from Helwan SLR station. Figure 2a represents the range and best fitting of the second subinterval, using Spline technique, Fig. 2b represents the range and best fitting of the second subinterval, using the overlapping technique.

As it is clarified from Fig. 2, the data is fitted using polynomial of 4th degree with corresponding average standard deviation of about 6.89 mm using spline technique and of 0.89 mm, using the overlapping technique which clarify how adversely the distribution of data can affect on the fitting process.

SATELLITE SIGNATURE EFFECT

According to the Satellite Laser Ranging (SLR) technique, the incident laser beam on a retro reflector onboard an artificial satelhte is reflected in a path that is parallel to the initial incident one but in an opposite direction. The size of the geodetic satellites has become one of the limiting factors of the laser ranging data precision and is known as satellite signature effect. The most visible geodetic satellites are AJISAI (perigee height of 1485 km), TOPEX (perigee height of 1350 km) and BEACON–C (perigee height of 927 km). Also, the number and type of the retro reflectors as well as their orbital trajectory vary significantly from satellite to another, leading to different expected optical responses in which AJISAI has the most favorable one (Villoresi *et al.*, 2008). Besides, the spread of retro reflectors due to multiple reflectors on the satellites is now recognized as key error factor and known as target effect (Appleby, 1992). Furthermore, it is known that the large array of corner cubes reflectors on the satellites, made them possible to obtain a fairly large amount of range data. However, the varying size of the array causes a severe target signature effect (Neubert, 1994; Otsubo *et al.*, 1999). These previous satellite signature effect mentioned above have a great influence on the precision of the SLR data obtained. AJISAI (AJ), TOPEX (TP) and BEACON-C (BC) are taken as examples of high signature satellites. ERS-2 (ES) and STARLETTE (ST) satellites are taken as examples for low signature satellites.

The characteristics of satellites under study are represented in Table 2. Column 1 represents the satellite name, while the corresponding ID is indicated in column 2. Column 3, 4 and 5 represent the retro-reflector array shape, diameter and reflectors of each satellite. Finally, column 6 represents the satellite altitude in kilometers.

In the present study, a comparison between the SLR data precisions for both low and high satellite signatures is performed. The precision of the SLR data is expressed by the used polynomial degree for data fitting and the corresponding standard deviation as represented in Table 3.

As a result of the high degree of satellite signature of the Ajisai, Topex and Beacon-C in comparable with the satellite signatures of ERS-2 and Starlette, it becomes clear that the average precision of the fitting of the data taken for the satelhtes STARLETTE and ERS-2 are much better than those obtained for the satellites AJISAI, TOPEX and BEACON-C. The average precision of the high signature satellites is about 470.45 mm while it is 21.43 mm for the low signature satelhte.

Table 2: Characteristics of the used satellites

Satellite name	Satellite ID	RRA Shape	RRA diameter	RRA reflectors	Altitude (km)
AJISAI	8606101	circular	215 cm	1436(+318 Mirrors)	1490
TOPEX	9205201	Annulus array	150 cm array	192 corner cubes	815
BEACON-C	6503201	pyramid	64 pannel	160 corner cubes	927
ERS-2	9502101	hemspherical	20 cm	9 corner cubes	750
STARLETTE	7501001	sphere	24 cm	60 corner cubes	815

Table 3: Polynomial degree and standard deviation of low and high satellite signatures

High signature satellites	Poly. Deg.	SD (mm)	Low signature satellites	Poly. Deg.	SD (mm)
AJ 2000/09/03	9	1534.00	ST2000/09/04	8	12.450
AJ 2005/04/24	11	70.38	ST 2000/11/05	7	2.230
AJ 2000/09/04	9	139.00	ST 2000/11/8	7	21.500
AJ 2000/09/09	11	81.99	ST 2000/09/05	10	26.810
AJ 2005/07/10	9	501.10	ST 2000/04/15	9	7.280
AJ 2000/09/11	5	34.01	ST 2000/11/01	10	9.662
AJ 2005/08/21	9	804.80	ST 2000/09/01	7	1.002
AJ 2005/08/23	9	48.64	ST 2000/11/02	9	80.030
AJ 2000/10/17	9	1618.80	ST 2000/10/29	10	44.570
AJ 2000/10/22	11	20.47	ST 2005/08/14	6	1.204
TP 2000/04/15	9	94.07	ES 2000/10/25	10	80.670
TP 2000/10/25	9	222.30	ES 2000/6/17	9	8.038
TP 2000/09/10	9	1044.10	ES 2000/6/27	7	6.395
TP 2000/10/26	9	1967.00	ES 2005/04/24	11	50.290
TP 2005/06/16	9	7.86	ES 2000/09/01	8	1.790
TP 2005/06/17	8	329.15	ES 2000/11/01	8	81.190
TP 2005/08/02	11	22.26	ES 2005/06/17	9	28.150
TP 2005/06/14	9	170.50	ES 2000/02/26	6	76.610
TP 2005/08/06	9	97.32	ES 2000/06/26	5	0.493
TP 2000/09/11	9	63.15	ES 2000/06/21	5	50.130
BC 2000/09/04	10	75.84			
BC 2000/09/03	9	325.40			
BC 2000/11/06	8	103.60			
BC 2000/09/10	9	1556.00			
BC 2000/11/05	11	124.70			
BC 2000/10/29	8	33.86			
BC 2000/11/08	6	1.98			
BC 2000/11/04	4	1.12			
BC 2005/08/13	9	92.61			
BC 2005/05/16	7	231.40			
Average	8	882.70	Average	7	31.290

CONCLUSION

In the present study, two different methods are used for analyzing SLR data. These are the Spline and overlapping techniques. It is concluded that the polynomial degree and the corresponding standard deviation of the two techniques are much better than that obtained when fitting the total number of data. In addition, it is found that the average polynomial degrees and the corresponding average standard deviations of both Spline and overlapping techniques are comparable with slight advantage for Spline technique over the overlap technique.

However, it is found that about 11/30 samples under study represent that the overlap technique is better than the spline technique which may refers to the distribution of the data points inside the intervals.

Furthermore, the effect of the satellite signatures on the SLR data precision is examined. It is found that the precision of the data fitting for the low signatures satellites are much better than those obtained for high signature satellites.

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