An Approximate Formula Relating the Two Transcendental Numbers $e$ and $\pi$

M.J. Buckram and E.A. Castro
Department of Chemistry, INIFTA, Faculty of Exact Sciences, La Plata National University, Suc 4, C.C. 16, Diag. 113 and 64, La Plata 1900, Buenos Aires, Argentina

Abstract: This study describes an approximate mathematical formula that relates the two ubiquitous transcendental mathematical constants $e$ and $\pi$ to each other through the use of only four other integers, which include the first four prime numbers 2, 3, 5 and 7. The relation is useful for understanding the connection between $e$ and $\pi$. It may provide a fruitful starting point for the generation of further relations between the two transcendental constants including potentially new series and product representations of them relating them together. The relation potentially provides some insight and inspiration into the comprehension of Eddington’s fundamental theory of the universe, with the simplicity of the connection between these fundamental constants and their ubiquity in scientific and mathematical formulae.

Key words: Transcendental, prime, number theory, approximate formula, Eddington

Introduction

The mathematical constants $e$ and $\pi$ are ubiquitous in mathematical and scientific formulae. In addition they are known as the transcendental numbers as they are infinitely, non-repeating continued fractions (Beckmann, 1971; Blattner, 1997; Muir, 1994). A formula has been devised in this paper where these two transcendental mathematical constants are related to each other by only four other integers: 3, 4, 5 and 7. In fact, by factoring these numbers, in this relation, we can see that the first 4 prime numbers: 2, 3, 5 and 7 are the basis for this fundamental connection between $e$ and $\pi$.

Mathematical Formula

The $e\pi$ relation, which is only approximate and returns an equality to within $\sim 99\%$ of the true value, is shown as Eq. 1 below:

$$e \pi = \frac{3 \times 4 \times 5}{7}$$  \hspace{1cm} (1)

It is interesting in this context that two transcendental numbers can be related to each other to within $\sim 99\%$ accuracy through the use of only four natural numbers, including, when factored, the first 4 prime numbers 2, 3, 5 and 7.

Equation 2 shows a reformulation of the $e\pi$ relation given in Eq. 1 to emphasize the strange nature of the equation:

$$1 \times 2.3333333333333333 \ldots \times e \pi = 4 \times 5$$  \hspace{1cm} (2)

Corresponding Author: E.A. Castro, Department of Chemistry, INIFTA, Faculty of Exact Sciences, La Plata National University, Suc 4, C.C. 16, Diag. 113 and 64, La Plata 1900, Buenos Aires, Argentina

113
In this case we see that 1 multiplied by the continued fraction represented as \( 2^{1/\phi} \), multiplied by \( e \) and \( \pi \), leads to the product of 4 and 5. It is as if the integer sequence 1, 2, 3, 3, 3, 3,... is upgraded to the integer sequence 4, 5 by the insertion of the mathematical factor \( e \pi \). It is also very curious that \( e \) and \( \pi \) are indeed connected together by the first 5 counting numbers: 1, 2, 3, 4 and 5 in this formulation.

It has been suggested to the authors from various sources, including inspiration from the fundamental theory of the universe as formulated by S. Eddington Sir (1959) that relation 2 implies that there are only 5 dimensions in the universe with the three spatial dimensions, represented in 2 as the numbers 1, 2 and 3 related to two time dimensions, represented in 2 as the numbers 4 and 5 through a mathematical reciprocity.

References