Non-Newtonian Liquid Flow Through Small Diameter Piping Components

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Abstract: The present study has been carried out to evaluate the frictional pressure drop across the different piping components for non-Newtonian liquid flow. Empirical correlations in term of the various physical and dynamic variables of the system have been developed for prediction of the frictional pressure drop across the each piping component.

Key words: Non-Newtonian, piping components, frictional pressure drop

Introduction

Newtonian fluids exhibit a direct proportionality between shear stress and shear rate in the laminar flow region. Non-Newtonian fluids exhibit a non-linear shear stress-shear rate dependence. A majority of the non-Newtonian fluids are to be found as pseudoplastic, in nature, such as rubber solutions, adhesives, polymer melts or solutions, greases, starch suspensions, cellulose acetate, solutions used in rayon manufacturing, mayonnaise, soap, detergent slurries, paper pulp, napalm, paints, certain pharmaceutical dispersions, biological fluids, dilute suspensions of inert, unsolvated solids etc. It displays, on arithmetic coordinates, the concave downward flow curve relationship, on logarithmic coordinates these materials exhibit flow curves having slopes between zero and unity.

Alves et al. (1952) pointed out that the curve showed a point of inflection and the slope approaches a value of unity at extremely low as well as at very high shear rates. Metzner and Reed (1955) showed that the flow curve was a straight line on logarithmic coordinates over 10 to 100 fold range of shear rates, sometimes having slopes appreciably less than 0.10. For such straight-line regions the flow curve defined by the power law model is

\[
\tau = K \left( \frac{d\gamma}{dr} \right)^n
\]

(1)

The flow behavior index, n is the slope of the logarithmic plot, which ranges from unity towards zero with increasing pseudoplasticity; consistency index K characterizes the consistency or thickness of a fluid.

Pipe fittings like valves, bends, elbows, tees, reducers, expander etc. the integral part of any piping system. Flow through piping components is more complex than the straight pipes. The problem of determining the pressure losses in fittings is important in design and analysis of the fluid machinery. Forcing a fluid through pipe fittings consumes energy provided by the drop in pressure across the fittings. The friction between the fluid and the fitting wall causes this pressure drop. The problem of predicting pressure losses in pipe fittings is much more uncertain than for the pipe because,
The mechanism of flow is not clearly defined. At least two types of losses are superposed: skin friction and loss due to change in flow direction and
There are very few experimental data available in the literature.

There are two approaches for analysis of the pressure drop across the pipe fittings equivalent length ($L_e$) and velocity head. In the equivalent length method the fitting is treated as a piece of straight pipe of some physical length, i.e., equivalent length ($L_e$) that has the same total loss as the fitting. The main drawback of this simple approach is that the equivalent length for a given fitting is not constant but depends on Reynolds number and roughness as well as size and geometry. In the case of the other method, the velocity head is the amount of potential energy (head) necessary to accelerate a fluid to its flowing velocity. The number of velocity head ($H$) in a flowing fluid can be calculated directly from the velocity of the fluid ($V$) as

$$H = \frac{V^2}{2g}$$

Flow through a piping component in a pipeline also causes a reduction in static head, which may be expressed in terms of velocity head and the resistance coefficient, $K$ as

$$H = K \frac{V^2}{2g}$$

$K$ is thus defined as the number of velocity heads lost due to the piping component. The main drawback of this method is that it also depends on Reynolds number.

Hooper\(^1\) developed two-$K$ method to predict the head loss in pipe fittings. He defined $K$ as

$$K = \frac{K_1}{Re} = K_e \left(1 + \frac{1}{D_i}ight)$$

where, $K_1 = K$ for the fitting at $Re = 1.0$

$K_e = K$ for the fitting at $Re = \infty$

He also tabulated the values of $K_1$ and $K_e$ for some standard elbows of different angles, tees and values of different types and dimensions of particular openings. Later he reported the mathematical expression for $K_e$ based on inlet velocity head for reducer, expander and orifices of different shape and dimension. Fairhurst (1983) found that the resistance coefficient, $K$ for water flow through different valves is comparable with the values presented by Miller (1978). Hoang and David (1984) reported that the pressure drop for relatively sharp return bends increases by a factor of approximately 20 compared to that in pipe of equivalent length of the bend. Norstabo (1985) and Sookpassong et al. (1986) observed that the resistance coefficient for valves varies from 2.1 to 7.0, independently of Reynolds number. Edwards et al. (1985) observed that the loss coefficient depends on the Reynolds number. Mandal and Das (2001) reported that the $K$ value is strongly dependent on the Reynolds number similar to the earlier report of Kittredge and Rowley (1957) and Ito (1960). This study deals with the experimental investigation of non-Newtonian liquid flow through piping components in laminar flow condition. Since most pseudoplastic liquids are highly viscous in nature, laminar flow is of the greatest practical interest, (Joshi and Bergles, 1980).
Materials and Methods

The study was carried out at Department of Chemical Engineering, University of Calcutta during 2002-2003.

The schematic diagram of the experimental setup consisting of valves is shown Fig. 1. It consists of a liquid storage tank, test section, control and measuring systems for flow rates, pressure and other accessories. The test section consisted of around 4.5 m long horizontal upstream straight pipe, pipe fitting section and long downstream straight pipe. The main idea of putting long upstream and downstream portions of the fittings is to achieve fully developed flow conditions to facilitate the measurement of pressure drop across the fitting. The pipe fitting was connected to the upstream and downstream portion with the help of flanges. Before the test section a 0.5-0.7 m long Perspex tube was incorporated in the system. The rest of the test section was fabricated from mild steel. The test section was fitted horizontally with the help of leveling gauge. The inner diameter of the test section pipe varies from 1/4-1 inch.

The liquid storage tank was a cylindrical vessel of 0.45 m³ capacity and was fitted with a propeller type stirrer to prepare non-Newtonian liquids. A cooling coil incorporated in the liquid tank controlled the liquid temperature. The test liquid was circulated from the tank by means of a centrifugal pump to the test section. Its flow rate was controlled by bypass valves and was measured by a set of rotameters connected in parallel. The liquid was discharged in the separator and was returned to the tank.

The test section was provided with pressure taps (piezometric ring) at various points in the upstream section and downstream section, sometimes on the pipe fitting like elbow, bends etc. The static pressure at different points was measured by means of U-tube manometers containing mercury beneath water. Arrangement for purging non-Newtonian liquids in manometer line was also provided. All experiments were carried out in the laminar flow region.

Four aqueous solutions of SCMC (sodium salt of carboxy methyl cellulose, high viscous grade, Loba Chemie Pvt. Ltd., Bombay, India) of approximate concentration s 0.2-0.8 kg m⁻¹ were used as

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Fig. 1: Schematic diagram of the experimental setup
Table 1: Physical properties of the SCMC solutions

<table>
<thead>
<tr>
<th>Concentration of SCMC solution (kg m⁻³)</th>
<th>Flow behavior index n'</th>
<th>Consistency index K (N s² m⁻²)</th>
<th>Density ρ (kg m⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.9013</td>
<td>0.0142</td>
<td>1001.49</td>
</tr>
<tr>
<td>0.4</td>
<td>0.7443</td>
<td>0.1222</td>
<td>1002.13</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6605</td>
<td>0.3416</td>
<td>1002.87</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6015</td>
<td>0.7112</td>
<td>1003.83</td>
</tr>
</tbody>
</table>

the non-Newtonian liquid. Adding small amounts of formalin prevented biological degradation of the SCMC solutions. Rheological properties and density of the solutions were measured experimentally by pipeline viscometer and by specific gravity bottle respectively. Rheological and physical properties of the test liquids are given in Table 1.

**Results and Discussion**

Fluids obey the power law, Metzner and Reed (1955) developed the basic relationship for relating pressure drop to flow rate by means of geometric parameters and the two physical properties of the fluid K' and n' as

\[
\frac{\Delta P}{4L} = K' \left(\frac{8V}{D}\right)^{n'}
\]  

where,

\[
K' = K \left(\frac{3n' + 1}{4n'}\right)
\]

\[
n' = \frac{d}{d} \left[ \ln \left(\frac{\Delta P}{4L}\right) \right] = n
\]

n' or n is the power law exponent, slope of the line from a plot of \(\Delta P/4L\) vs. \(8V/D\) on logarithmic coordinates. They also correlated laminar flow data as

\[
f = \frac{16}{\text{Re}}
\]

where,

\[
\text{Re} = \frac{VD\rho}{8\pi^2\eta^2 V^{\frac{n' + 1}{2}} K'}
\]

**Evaluation of Pressure Drop Across the Pipefitting**

Variations of the static pressure along the tube are schematically shown in Fig. 2 for liquid flow through valve. The curve a-b-c-d-e-f is the static pressure distribution in the straight upstream portion, valve portion and the straight downstream portion of the test section. The curve a-b-c' and d'-e'-f are
the extended portions of the pressure distribution curve in the upstream and downstream portion of the fully developed flow region, respectively. The pressure loss due to valve, $\Delta P$, was obtained from the difference in static pressure of the upstream fully developed flow and the downstream fully developed flow region. Thus, $\Delta P_c$ includes the frictional pressure drop for the flowing fluid flow through a passage of length equal to the axis of the valve and the additional pressure loss due to irreversibility. The typical static pressure distribution curves for non-Newtonian liquid flow through valves are shown in Fig. 3.
Correlations

The total pressure drop across the pipe fitting for non-Newtonian liquid flow is the same as the frictional pressure drop because the hydrostatic head component and the accelerational component are both negligible.

From the literature review it is clear that the measured K values for different piping components depend not only on the Reynolds number but also on the non-Newtonian characteristics of the fluids and also on the characteristics of the pipefitting.

The determination of frictional pressure drop is not possible by theoretical analysis because the wall friction and the friction due to the change in the flow direction cannot be specified quantitatively. Therefore, the alternative correlating method is the dimensional analysis. By dimensional analysis correlations have been developed to predict the frictional pressure drop across the fitting as a function of the various parameters. The variables involved are the flow variables, liquid properties, geometric properties of the conduit and fitting. The final correlations for different fitting are as follows:

Orifice:

\[
\frac{\Delta P}{\rho V^2} = 0.601 \text{Re}^{0.046} \left( \frac{D_b}{D_i} \right)^{4.379} \tag{10}
\]

Valves:

- Gate valve

\[
\frac{\Delta P}{\rho V^2} = 1.905 \text{Re}^{0.193} \alpha^{1.867} \tag{11}
\]

- Glove valve

\[
\frac{\Delta P}{\rho V^2} = 8.266 \text{Re}^{0.160} \alpha^{0.89} \tag{12}
\]

for, \( 45^\circ < \text{Re} < 216.5 \)
\( 0.25 < \alpha < 1.0 \)

Elbow:

\[
\left( \frac{f_{\text{eq}}}{f} - 1 \right) = 7.939 \times 10^{-2} \text{De}^{0.316} \left( \frac{\beta}{135} \right)^{-0.335} \tag{13}
\]

for, \( 40 < \text{Re} < 2000 \)
\( 30 < \text{De} < 2150 \)
\( 45^\circ < \alpha < 135^\circ \)
Bend:

\[
\left( \frac{f_b}{f_t} - 1 \right) = 2.569 \times 10^4 \text{De}^{0.508} \left( \frac{\beta}{180} \right)^{-0.56}
\]

for, \( 2 < \text{De} < 1150 \)  
\( 45^\circ < \beta < 180^\circ \)

**Conclusions**

Correlations in terms of physical and dynamic variables of the system are developed for predicting the frictional pressure drop across the piping components for the flow of non-Newtonian pseudoplastic liquids.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>D</td>
<td>Diameter, m</td>
</tr>
<tr>
<td>De</td>
<td>Dean number, Re(D/De)^{1.5}</td>
</tr>
<tr>
<td>f</td>
<td>Fanning friction factor, dimensionless</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity, m s(^{-2})</td>
</tr>
<tr>
<td>h, H</td>
<td>Head, m</td>
</tr>
<tr>
<td>K</td>
<td>Resistance coefficient, dimensionless</td>
</tr>
<tr>
<td>K', K''</td>
<td>Consistency index, Ns(^{n'}) m(^{-2})</td>
</tr>
<tr>
<td>n, n'</td>
<td>Flow behavior index, dimensionless</td>
</tr>
<tr>
<td>\Delta P</td>
<td>Pressure drop, N m(^{-2})</td>
</tr>
<tr>
<td>\Re</td>
<td>Reynolds number, ( \frac{VD_p}{8\pi D^{1.5} V^2 n K'} )</td>
</tr>
<tr>
<td>\nu</td>
<td>Velocity, m s(^{-1})</td>
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</table>

**Greek Letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>\alpha</td>
<td>Ratio of the valve opening to the full opening of the valve, dimensionless</td>
</tr>
<tr>
<td>\beta</td>
<td>Bend angle, deg.</td>
</tr>
<tr>
<td>\rho</td>
<td>Density, kg m(^{-3})</td>
</tr>
<tr>
<td>\tau</td>
<td>Shear stress, N m(^{-2})</td>
</tr>
</tbody>
</table>

**Subscripts**

<table>
<thead>
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<th>Definition</th>
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<tbody>
<tr>
<td>b</td>
<td>Bend</td>
</tr>
<tr>
<td>c</td>
<td>Curvature</td>
</tr>
<tr>
<td>o</td>
<td>Orifice</td>
</tr>
<tr>
<td>s</td>
<td>Straight tube</td>
</tr>
<tr>
<td>t</td>
<td>Tube</td>
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<tr>
<td>eb</td>
<td>Elbow</td>
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**References**
