Parametric Study of Minimum Energy Loss Culvert

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Abstract: This study extends the authors previous results on finite element simulation of flow through minimum energy loss culverts. Parameters of the model are varied to determine their effect on the energy loss. The influence of each of the parameters on the energy loss is displayed graphically. As culvert depth increases from 0 to Ld, the energy loss in the culvert increases as a result of reduction in outlet velocity.

Keywords: Finite element, minimum energy loss culvert, energy loss, parametric study, Laplace

INTRODUCTION

The design procedure for a minimum energy loss culvert has always been based on the assumption of a rectangular section for the culvert for both the sub-critical and critical flow conditions (i.e., general equations for flow through a rectangular section (Apelt, 1983).

This assumption implies that the flow through minimum energy loss culverts is straight. But the geometry of a minimum energy loss culvert is not rectangular and the flow cannot be assumed to be straight; therefore there is need for a design method that can analyze complex geometries.

Minimum energy loss culvert designs are based on the assumption that the flow at design discharge will be hydraulically smooth, i.e., they have virtually no head loss (Apelt, 1983). Johnson and Apelt (1987) tested a series of modeled outlet fans. They found in preparatory runs that the head losses are too small to measure and therefore designed on the assumption of no head loss. This assumption of near zero head loss has led to many years of controversy between opponents and proponents of minimum energy design. With finite element method, the minimum head loss can be determined. Minimum energy loss culverts involve a complex geometry and the analytical method of building models is extremely costly. Finite element method uses mathematical model to simulate the behavior of these complex physical systems. It can easily handle discontinuous geometrical shapes as material discontinuity. A sketch of a minimum energy loss culvert is shown in Fig. 1: Water flows in through the inlet lip and flows out through the outlet lip.

Finite element method has now been widely accepted for all kinds of structural applications. Fluid mechanics applications are currently being developed for studying tidal motions, thermal and chemical transport, and diffusion problems as well as fluid-structural interactions.

The concept of minimum energy design was first detailed by Cottman and McKay (1990), based on Bernoulli’s theorem. Although the method is based on hydraulic principles, which have long been recognized, its practical utilization has attracted significant controversy. Proponents of the design principle put forward the following benefits (Apelt, 1983):

- The throat or barrel of the waterway has a minimum width, thus reducing construction costs and impact on the stream environment.
- The flow through the structure is streamlined and therefore has reduced turbulence, which, in turn, reduces the erosion potential of the flow and minimizes the need for surface protection.
- The minimization of energy losses results in little or no adverse effect on upstream flood levels.

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Fig. 1: Sketch of a minimum energy loss culvert

In a later study, Cade and Keller (1995) introduced an energy loss model which correctly accounts for the effect of varying flow geometry and roughness on energy loss throughout the whole minimum energy loss structure. Additionally, Cade and Keller (1995) introduced the idea of using the natural shape of river meanders as a model for the design of the fans walls, based on the minimum stream power concept of Chang (1983). Akanmu and Gambo (2004) developed a two dimensional model of a minimum energy loss culvert by simulating the flow through the culvert and also presented the effect of sediment accumulation on the flow through such culverts.

The purpose of the present study is to parametric study of a two dimensional model of a minimum energy loss culvert using finite element method (Fig. 1).

MATERIALS AND METHODS

The Governing Equation

A finite element program based on Laplace equation written in FORTRAN programming language was used for the simulation (Connor and Breebria, 1977). The program uses a 3 node triangular element with two degrees of freedom per node.

\[ Hx \frac{d^2U}{dx^2} + Hy \frac{d^2U}{dy^2} = 0 \]  \hspace{1cm} \text{(1)}

where:

\( HX \) and \( HY \) = The diffusion coefficients of water.

\( U \) = The velocity potential of flow.

Finite Element Program

The finite element computer program consists of the following steps:

- Reading and checking of data
- Generation of element matrices
Fig. 2: Finite element model of minimum energy loss culvert

Fig. 3: Mesh of minimum energy loss culvert

- Assemblage of global matrices
- Application of boundary conditions
- Solution of system of equations
- Printing of nodal velocity values

**Mesh Generation**

The model (Fig. 2) developed in a previous study (Akanmu and Gambo, 2007) is used for this study. The mesh of the minimum energy loss culvert model (Fig. 3) was generated using the GID 8 software professional software. The aspect ratio of each element was set as 1. Due to the limitation of the code used, the number of nodes and elements used was limited to 855 and 3766, respectively.

Parameters in terms of the culvert length (Lc) for a unit inlet flow (1 m sec⁻¹):

\[ \Theta_a = 12.5L_c \text{ (rad.)} \]
\[ \Theta_b = 4.81L_c \text{ (rad.)} \]
\[ H_1 = 7.125L_c \text{ (m)} \]
\[ H_2 = 7.125L_c \text{ (m)} \]
\[ L_a = 0.1L_c \text{ (m)} \]
\[ H_a = 8.375L_c \text{ (m)} \]
\[ L_c = 0.4m \]

**Parametric Study**

The verified finite element program was used to study the influence of each of the parameters of the culvert on the energy loss. The parameters investigated in this study are inlet and outlet slope,
culvert length, barrel length, inlet and outlet depth. While studying the influence of each parameter, some of parameters were kept constant at their values while others had to change because of the change in the parameter under consideration.

**Influence of Inlet slope on Energy Loss**
Analyses were undertaken on the model culvert to determine the influence of the variation in the inlet slope on the energy loss in the finite element model. The values of $\Theta_i$ were varied at interval of 0.2 $\Theta_i$ from 0 to $\Theta_i$.

**Influence of Outlet Slope on Energy Loss**
The outlet slope of the model was taken as $\Theta_o$. The value of $\Theta_o$ was varied at intervals of 0.2$\Theta_o$ from 0 to $\Theta_o$ and the effect of this value on other parameters was noted.

**Influence of the Culvert length on Energy Loss**
The culvert length (Lc) was varied at intervals of 2.5Lc from Lc to Lc and the effect on the energy loss in the model was noted. The effect on other parameters was also noted.

**Influence of Culvert depth on Energy Loss**
The culvert depth was varied at intervals of 0.2Hc from 0 to Hc and the effect on the energy loss in the model was noted. The effect on other parameters was also noted.

**Influence of Barrel Length on Energy loss**
The barrel length was varied at intervals of 0.2Lb from 0 to Lb and the effect on the energy loss in the model was noted. The effect on other parameters was also noted.

**RESULTS AND DISCUSSION**

The results obtained from the parametric study are represented in graphical form. The graphs show the influence of each of the parameters on the energy loss in the model.

A decrease in the inlet slope from $\Theta_i$ to 0.8 $\Theta_i$ will result in a decrease in the energy loss in the minimum energy loss culvert model. It can also be deduced that the decrease in the inlet slope causes an increase and decrease at regular intervals, thereby causing a sinusoidal motion (Fig. 4).

An increase in the outlet slope from $\Theta_o$ to 0.2 $\Theta_o$ causes a decrease and subsequent decrease in the energy loss (Fig. 5).

![Inlet slope vs Energy loss](image)

Fig. 4: Influence of inlet slope on energy loss
Fig. 5: Influence of outlet slope on energy loss

Fig. 6: Influence of culvert length on energy loss

Fig. 7: Influence of culvert depth on energy loss
Fig. 8: Influence of barrel length on energy loss

An increase in the culvert length from \( L_b \) to 5.2\( L_b \), causes a reduction in the energy loss and a subsequent increase in culvert length from 5.2 \( L_b \) to \( L_b \), causes an increase in energy loss in the model (Fig. 6).

An increase in the culvert depth from \( L_d \) causes an increase in the energy loss in the model (Fig. 7).

An increase in the barrel length from \( L_b \) to 0.043\( L_b \), causes an increase in the energy loss (Fig. 8).

It can thus be deduced from the parametric study that variations in the parameters considered in the study, will adversely affect the outlet flow through the culvert.

CONCLUSIONS

A finite element program for flow through minimum energy loss culvert has been developed. The program has been used to conduct a parametric study of a two dimensional model. The advantage of this study is that engineers can use the parametric study graphs for checking the performance of practical minimum energy loss culverts.

NOMENCLATURE

- \( H_i \) = Inlet Height
- \( H_o \) = Outlet Height
- \( L_b \) = Barrel Length
- \( L_c \) = Culvert Length
- \( \Theta_i \) = Inlet Slope
- \( \Theta_o \) = Outlet Slope
- \( L_d \) = Culvert Depth

REFERENCES

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