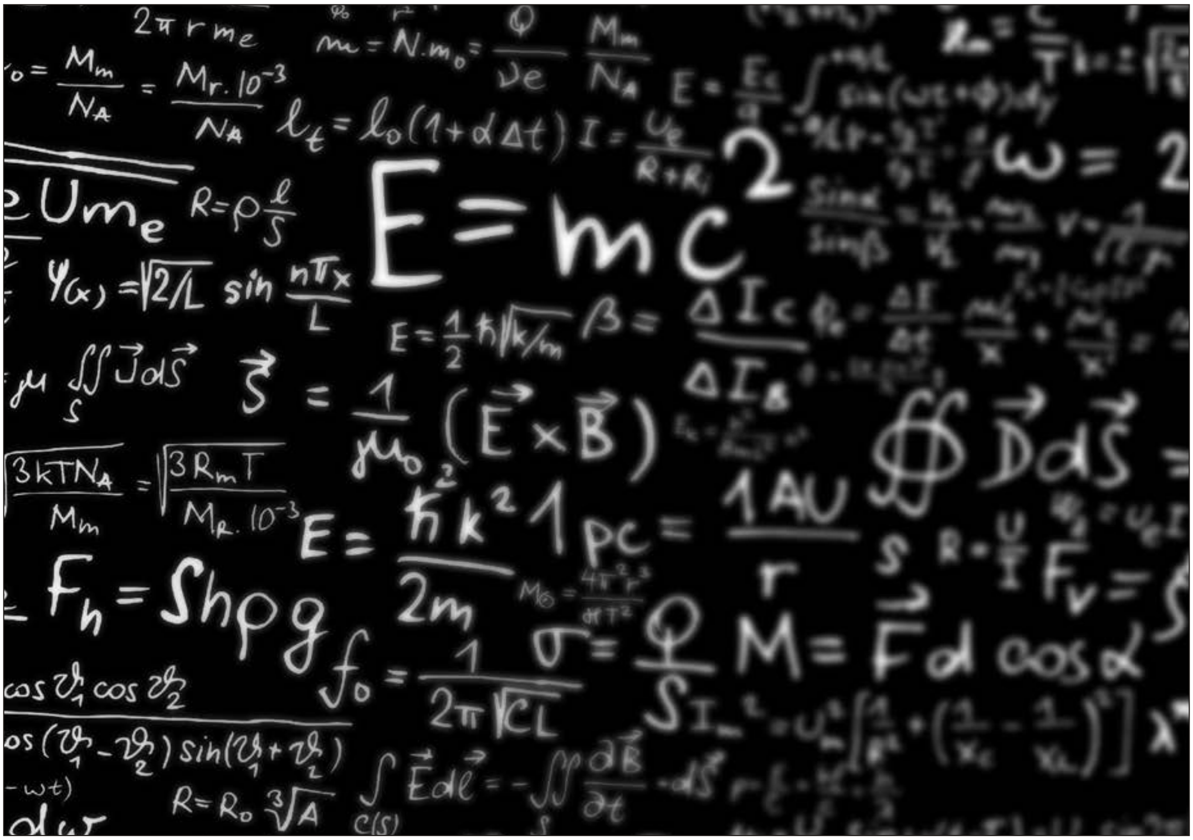

Asian Journal of Mathematics & Statistics

ISSN 1994-5418

science alert

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Asian Journal of Mathematics and Statistics 4 (3): 109-112, 2011
ISSN 1994-5418 / DOI: 10.3923/ajms.2011.109.112

Two-Dimensional Sofic Systems and Shift of Finite Type using Allowable Block

Abdulkafi A. Al-Refaei
Faculty of Applied Sciences, Thamar University, Yemen

ABSTRACT

A two-dimensional sofic system has been defined by using the notion of allowable block. This definition is an extension of the original definition in the one-dimensional case. It is shown that the present definition is equivalent to using the notion of symbolic factors of subshift of finite types and to point out some of the phenomena which arise in the transition from classical shift of finite type $X \subset A^{Z^2}$ to two-dimensional shift of finite type $X \subset A^{Z^2}$ where, A is the finite alphabet. The rigidity properties of certain two dimensional shift of finite type and two dimensional sofic system has been discussed. Some examples are presented to illustrate this notation.

Key words: Sofic system, shift of finite type, allowable block, subshift, dynamic system

INTRODUCTION

We shall give some definitions that are natural extensions to the two-dimensional case of similar notions that are well-known in the one-dimensional case (Lind and Marcus, 1995; Kitchens, 1998). We begin with the dynamical system and its subsystems which are of interest to us.

Modeling and simulation of thermal dynamic system has been studied by Bhatti *et al*. (2006). Whereas Shabbak *et al*. (2011) proposed multivariate control chart which is less sensitive to the sustained shift in mean process. Another related work has been proposed by Abu-Shawiesh *et al*. (2009) and Goegebeur *et al*. (2005).

Huffman code has been widely used in text, image and video compression. For example, it is used to compress the result of quantization stage in JPEG (Hashemain, 1995). The simplest data structure used in the Huffman decoding is the Huffman tree. Array data structure (Chen *et al*., 1999) has been used to implement the corresponding complete ternary tree for the Huffman tree.

Let A be a finite set and the set A^{Z^2} be the set of all maps x: $Z^2 \rightarrow A$. For every we denoted by $\pi_E : A^{Z^2} \rightarrow A^E$ the projection $\pi_E(x) = x|_E$, $x \in A^{Z^2}$, where, $x|_E$ is the restriction of x: $Z^2 \rightarrow A$ to E if $x \in A^{\mathbf{Z}^2}$ and $n \in Z^2$ then x_n is the n-th coordinate of x, i.e., the value of x at n. We define, for every $m \in Z^2$, a homomorphism σ_m of A^{Z^2} by setting $(\sigma_m x)_n = x_{m+n}$.

A closed subset $X \subset A^{Z^2}$ is shift invariant (or a subshift of A^{Z^2}) if σ_m (X) = X, $\forall m \in Z^2$. If there exist a finite set $F \subset Z^2$ and a set $P \subset A^F$ such that $X = X_{(F,P)} = \left\{x \in X : \pi_F(\sigma_m x) \in P..\text{for,every}.m \in Z^2\right\}$. The set $P \subset A^F$ is the collection of permissible (or allowed) words of X.

Example 1: Let, F = {(0,0), (1,0), (0,1)}$\subset Z^2$ and P = {$(t_{(0,0)})$, $(t_{(1,0)})$, $(t_{(0,1)}) \in$TF: $t_{(0,0)}+t_{(1,0)}+t_{((0,1))} = 0$}. Here the alphabet set A = {0, 1} and addition is W.r.t modulo 2 arithmetic. Then:

$$X_{(F,P)} = \left\{x \in A^{Z^2} : x_{(m_1,m_2)} + x_{(m_1+1,m_2)} + x_{(m_1,m_2+1)} = 0, \forall (m_1, m_2) \in Z^2\right\} \quad (1)$$

Given 2 directed graph G_H and G_V with the same set of edges ε we now define the edge shift:

$$X = X_{(G_H, G_V)} = \left\{ \begin{array}{llll} e \in \varepsilon^{Z^2}: & \text{horizontal and} & \text{vertical} & \text{coordinates} \\ \text{are} & \text{governed} & \text{by } G_H \text{ and } G_V & \text{respctivly} \end{array} \right\} \tag{2}$$

Definition 2: Let $X = X_{(GH, GV)}$ be an edge shift. Also let $\lambda:\varepsilon \rightarrow A$ be a map where A is some finite set, a sofic shift S with presentation (G_H, G_V, λ) is defined by $S = \{\lambda^\infty(\xi): \xi \in X(G_H, G_V)\}$ and $\lambda^\infty(\xi)_{(i, j)} = \lambda(\xi_{(i, j)})$.

Definition 3: Let X_1, X_2 is topological space and Z^2 acts on X_1, X_2 by a commuting homomorphism. For each $n \in Z^2$, the action is written as n.x where $x \in X_1$ to represent $n \cdot x = f_1^{n_1} \circ g_1^{n_2}(x)$. Whenever $n = (n_1, n_2)$. Similarly for $x \in X_2$, $n \cdot x = f_2^{n_1} \circ g_2^{n_2}(x)$, we say X_2 is a factor of X_1 if there exist a continuous onto map $\phi: X_1 \rightarrow X_2$ such that $\phi(n.x) = n.\phi(x)$.

When dealing with shift spaces, Curtis- Hedlund-Lyudon,s theorem says that the factor map is a sliding block code, i.e., there exist positive integers m, n, r. S and $\Phi: B_{(n+m+1)x(s+r+1)}(X_1) \rightarrow A(X_2)$:

$$\Phi(X)_{[i,j]} = \varphi \left\{ \begin{array}{ccc} x(i-m, j+s) & x(i, j+s)..... & x(i+n, j+s) \\ \vdots & \vdots & \vdots \\ x(i-m, j).... & x(i, j)..... & x(i+n, j) \\ \vdots & \vdots & \vdots \\ x(i-m, j-r).... & x(i, j-r)..... & x(i+n, j-r) \end{array} \right\} \tag{3}$$

Here $B_{(n+m+1)\times(S+r+1)}$ (X1) denotes the rectangular allowable blocks of size $(n+m+1)\times(s+r+1)$ and $A(X_2)$ is the alphabet set of X_2. From the above work and by Al-Refaei *et al*. (2007) we get this result.

Proposition 4: Every edge shift is a shift of finite type.

Proof: Given $X = X(G_H, G_V)$, fined F and P such that $X = X_{(F, P)}X$ has alphabet $\varepsilon = \varepsilon(G_H) = \varepsilon(G_V)$. Take $P = \{ef : e, f \in \varepsilon, t_H(e) = i_H(f) \quad \text{or} \quad t_V(e) = i_V(f)\}$

Take:

$$F = \left\{ \begin{array}{cc} & (1,1) \\ (*,0) & (1,0) \end{array} \right\} \text{and} \quad P_H = \left\{ \begin{array}{cc} * & * \\ e & f \end{array} : t_H(e) = i_H(f) \right\} \quad \text{and} \quad P_V = \left\{ \begin{array}{cc} * & f \\ * & e \end{array} : \quad t_V(e) = i_V(f) \right\}$$

Then $P = P_H \cup P_V$. In fact, it is a matrix subshift with transition matrix H and V defined as follows:

H and V is of size $|\varepsilon|\times|\varepsilon|$ record ε so that each are $\{e_1, e_2, e_3, \ldots, w_{|\varepsilon|}\}$.

The define:

$$H_{(i,j)} = \begin{cases} 1 & t_H(e_i) = i_H(e_j) \\ 0 & \text{otherwise} \end{cases}$$

Similarly for V.

Definition 5: Let X = X (G_H, G_V) be an edge shift. Also let λ:$\varepsilon\rightarrow$A be a map where *A* is some finite set. A sofic shift *S* with presentation (G_H, G_V, λ) is defined by S = {$\lambda\infty$ (ξ): $\xi\in$X (G_H, G_V)} and:

$$\lambda^{\infty}(\xi)_{(i,j)} = \lambda\left(\xi_{(i,j)}\right) \tag{4}$$

Proposition 6: If S a sofic system then S is a factor of shift of finite type.

Proof: Let (G_H, $G_{GV}\lambda$) be presentation of S .By definition there exist edge shift X = X(G_H, G_V) and a map λ^{∞}: X$\rightarrow$S which is onto. Also λ^{∞} commutes with the respective shift maps on X and S. Thus S is a factor of X via λ^{∞}.

The result follows since *X* is a shift of finite type.

Proposition 7: Let X be shift of finite type and Y is shift space ϕ: X$\rightarrow$Y be factor map. Then Y is a sofic system.

Proof: First by going to higher block system one can find an edge shift X' which is isomorphic to X. Let Φ be the associated map that induced Φ: X$\rightarrow$Y. If we denote by ε the set of edges arising from the graphs of the edge shift X', then one can define a map λ: $\varepsilon\rightarrow$A (Y) by λ' (e) = Φ (e), $\forall e\in\varepsilon$. It is then an easy matter to check for commutatively so that λ and the respective graphs is the presentation of the sofic system.

Theorem 8: A two-dimensional shift space A^{Z^2} is two-dimensional sofic system if and only if a factor of two-dimensional shift of finite type.

Proof: Let is sofic shift and $X \subseteq A^{Z^2}$ be presentation of X The 1×1-block map λ induces a sliding block code λ:$\chi_{(GH,\, GV)}\rightarrow\chi_S$ that is onto by definition of.

Hence X = χ_S is factor of edge shift χ_{GH} and χ_{GV} which has finite type. Conversely, suppose that $X \subseteq A^{Z^2}$ a shift space for which there is two-dimensional shift of finite type Y and a factor map ϕ: Y$\rightarrow$X let ϕ have memory m(r) and anticipation m (s) then ϕ is induced by Φ on $B_{m(r)+n(s)+1}$(Y) by increasing m(r) if necessary .We can assume that Y is [m(r)+n(s)]-step. Define Ψ: Y$\rightarrow$$Y^{[m(r)+n(s)+1]}$ by $\Psi(y)_{[i,\, j]}$ = y [I-m, j+s and I+n, j+s].

Ψ Almost the same the higher block map $B_{m(r)+n(s)+1}$ except that. We use coordinates from y starting at index I-m, j+s rather than i, j. Since y is (m(r)+n(s))-step so, there is graphs G_H, G_V with edges named by block $B_{m(r)+n(s)+1}$ (Y) So That $Y_{[m(r)+n(s)+1]} = X_{(GHGV)}$. Define a labeling λ on G_H, G_V by λ (e) = Φ (e). We will show that S = ($\chi_{(GH,\, GV)}$, λ) is presentation of X proving that X is Sofic observe that Fig. 1 is commutes for if y$\in$Y, then ϕ (y)[i, j] = Φ ($y_{[i-m,\, j+s \text{ and } I+n,\, j+s]}$).

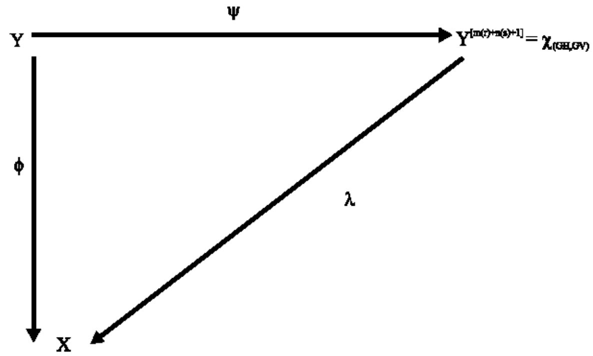

Fig. 1: Two dimensional shift

While $\lambda\ (\Psi\ (y)_{[i,\ j]}) = \lambda\ (\Psi\ (y)_{[i,\ j]})$ since Ψ is conjugacy, the image of ϕ and of, λ are equal hence $X = \phi\ (y) = = \lambda(\chi_{GHV,GV})$ so that S is presentation of X.

REFERENCES

Abu-Shawiesh, M.O., F.M. Al-Athari and H.F. Kittani, 2009. Confidence interval for the mean of a contaminated normal distribution. J. Applied Sci., 9: 2835-2840.

Al-Refaei, A., S.C. Dzul-Kifli and M.S. Noorani, 2007. Characterizing two-dimensional sofic systems. Jpn. J. Algebra Number Theory Appl., 11: 159-167.

Bhatti, M.A., L.C. Xi and Y. Lin, 2006. Modeling and simulation of dynamic systems. J. Applied Sci., 6: 950-954.

Chen, H., Y. Wag and Y. Lan, 1999. A memory-efficient and fast Huffman decoding algorithm. Inform. Process. Lett., 69: 119-122.

Goegebeur, Y., V. Planchon, J. Beirlant and R. Oger, 2005. Quality assessment of pedochemical data using extreme value methodology. J. Applied Sci., 5: 1092-1102.

Hashemain, R., 1995. Memory efficient and high-speed search huffman coding. IEEE Trans. Commun., 43: 2576-2581.

Kitchens, B.P., 1998. Symbolic Dynamics: One-Sided, Two-Sided and Countable State Markov Shifts. Springer, New York, ISBN-13: 978-3540627388.

Lind, D. and B. Marcus, 1995. An introduction to Symbolic Dynamics and Coding. Cambridge University Press, Cambridge, London, ISBN-13: 978-0521559003.

Shabbak, A., H. Midi and M.N. Hassan, 2011. The performance of robust multivariate statistical control charts based on different cutoff-points with sustained shift in mean. J. Applied Sci., 11: 56-65.