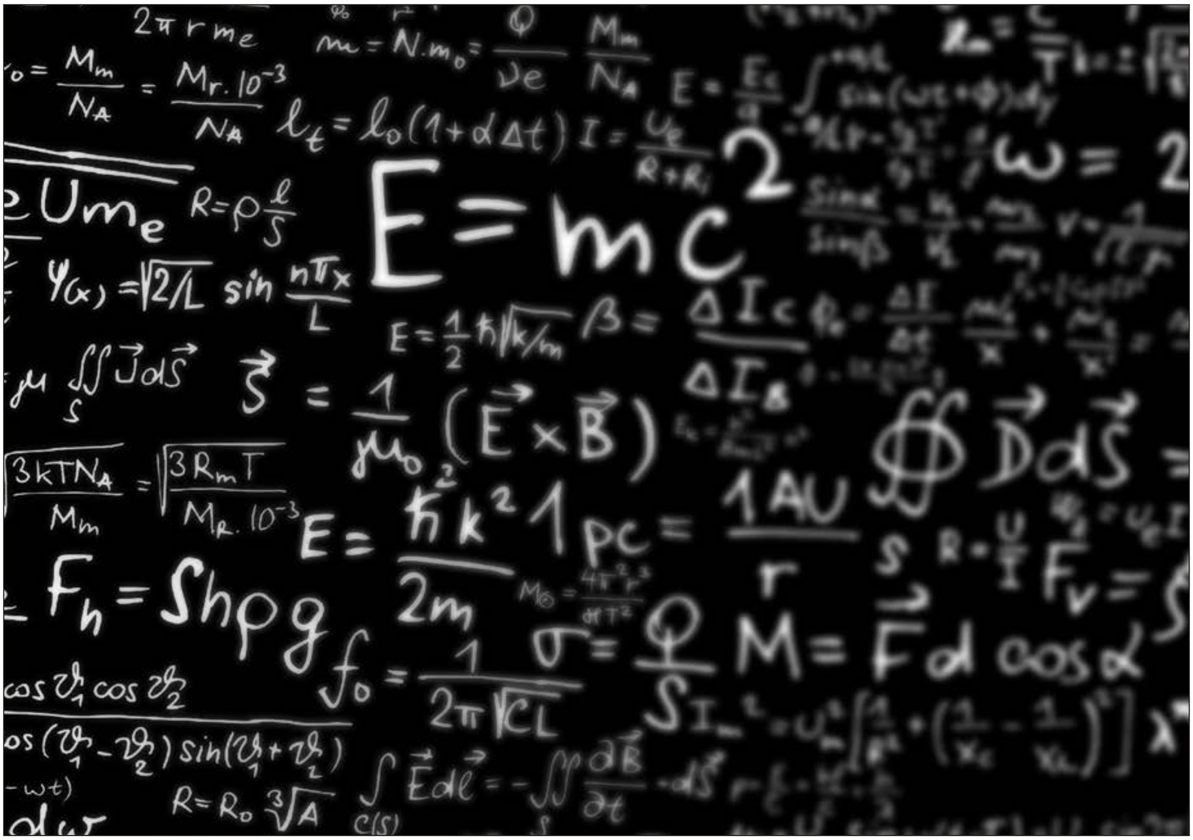

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Boolean Centre of Pre A*-Algebra

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ABSTRACT

This manuscript illustrates the essential congruence θ_x on a Pre A*-algebra A and arrive at a variety of properties of these. Also it bear out certain properties of the operations $\Gamma_x(p, q)$ and Φ_x. It has been confirmed that θ is a factor congruence on A if and only if $\theta = \theta_x$ for some $x \epsilon B(A)$. Further it was proved that the centre B (A) of a Pre A*-algebra A with 1 is isomorphic with the Boolean centre $\mathbb{B}(A)$ of A.

Key words: Pre A*-algebra, congruence, centre, boolean algebra, factor congruence, balanced congruence, boolean centre

INTRODUCTION

In an outline study, Manes (1989) introduced the concept of Ada (Algebra of disjoint alternatives) $(A, \wedge, \vee, (-)'(-)_\pi, 0, 1, 2)$, (where A is a nonempty set; $\wedge$ and $\vee$ are two binary operations on A; (-)' and $(-)_\pi$ are unary operations on A and 0, 1, 2 are distinguished elements in A). Which is however different from the definition of the Ada of his later paper (Manes, 1993). While the Ada of the earlier draft seems to be based on extending the If-Then -Else concept more on the basis of Boolean algebra and the later concept is based on C-algebra $(A, \wedge, \vee, ')$ introduced by Guzman and Squier (1990).

Koteswara Rao (1994) firstly introduced the concept A*-Algebra $(A, \wedge, \vee, (-)^{\sim} (-)_\pi, 0, 1, 2)$ and studied its equivalence with Ada, C-algebra and Ada's connection with 3- Ring. Further he made an effort on the If-Then-Else structure over A*-algebra and introduced the concept of Ideal of A*-algebra. Venkateswara Rao (2000) introduced the concept of Pre A*-algebra $(A, \wedge, \vee, ')$ as the variety generated by the 3-element algebra A = {0, 1, 2} which is an algebraic form of three valued conditional logic. Satyanarayana *et al.* (2010) generated semilattice structure on Pre A*-Algebras. Venkateswara Rao and Srinivasa Rao (2009) defined a partial ordering on a Pre A*-algebra A and studied its properties. Satyanarayana *et al.* (2010) derive necessary and sufficient conditions for pre A*-algebra A to become a Boolean algebra in terms of the partial ordering. Srinivasa Rao (2009) studied the structural compatibility of Pre A*-algebra with Boolean algebra.

This study perceives a fundamental congruence θ_x on a Pre A*-algebra and confer its various properties. Also it establishes certain properties of operations Γ_x (p, q) and Φ_x. It has been proved that θ is a factor congruence on A if and only if $\theta = \theta_x$ for some $x \epsilon B(A)$. Further it has been derived that the centre B(A) of a Pre A*-algebra A with 1 is isomorphic with the Boolean centre $\mathbb{B}(A)$ of A.

PRELIMINARIES

Definition 1: Boolean algebra is an algebra (B, ∧, ∨, (-)', 0, 1, 2) with two binary operations, one unary operation (called complementation) and two nullary operations which satisfies:

(i) (B, ∧, ∨) is a distributive lattice
(ii) $x \wedge 0 = 0$, $x \vee 1 = 1$
(iii) $x \wedge x' = 0$, $x \vee x' = 1$

We can prove that $x'' = x$, $(x \vee y)' = x' \wedge y'$, $(x \wedge y)' = x' \vee y'$, for all x, y∈B.

Here, we concentrate on the algebraic structure of Pre A*-algebra and state some results which will be used in the later text.

Definition 2: An algebra (A, ∧, ∨, (-)˜) where A is non-empty set with 1, ∧, ∨ are binary operations and (-)˜ is a unary operation satisfying:

(a) $x^{\sim\sim} = x \quad \forall x \in A$
(b) $x \wedge x = x, \quad \forall x \in A$
(c) $x \wedge y = y \wedge x, \quad \forall x, y \in A$
(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim} \quad \forall x, y \in A$
(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \quad \forall x, y, z \in A$
(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \quad \forall x, y, z \in A$
(g) $x \wedge y = x \wedge (x^{\sim} \vee y), \quad \forall x, y \in A$ is called a Pre A*-algebra

Example 1: 2 = {0, 1, 2} with operations ∧, ∨, (-)˜ defined below is a Pre A*-algebra.

∧	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

∨	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	x˜
0	1
1	0
2	2

Note 1: The elements 0, 1, 2 in the above example satisfy the following laws:

(a) $2^{\sim} = 2$
(b) $1 \wedge x = x$ for all x ∈ 3
(c) $0 \vee x = x$ for all x ∈ 3
(d) $2 \wedge x = 2 \vee x = 2$ for all x ∈ 3.

Example 2: 2 = {0, 1} with operations ∧, ∨, (-)˜ defined below is a Pre A*-algebra.

∧	0	1
0	0	0
1	0	1

∨	0	1
0	0	1
1	1	1

x	x˜
0	1
1	0

Note 2:

(i) (2, ∧, ∨, (-)˜) is a Boolean algebra. So, every Boolean algebra is a Pre A* algebra
(ii) The identities (a) and (d) imply that the varieties of Pre A*-algebras satisfies all the dual statements of (b) to (g) of definition 2

Note 3: Let A be a Pre A*-algebra then A is Boolean algebra if and only if $x \vee (x \wedge y) = x$, $x \wedge (x \vee y) = x$ (absorption laws holds).

Lemma 1: Every Pre A*-algebra satisfies the following laws.

(a) $x \vee (x^{\sim} \wedge x) = x$
(b) $(x \vee x^{\sim}) \wedge y = (x \wedge y) \vee (x^{\sim} \wedge y)$
(c) $(x \vee x^{\sim}) \wedge x = x$
(d) $(x \vee y) \wedge z = (x \wedge z) \vee (x^{\sim} \wedge y \wedge z)$

Definition 3: Let A be a Pre A*-algebra. An element $x \in A$ is called central element of A if $x \vee x^{\sim} = 1$ and the set $\{x \in A / x \vee x^{\sim} = 1\}$ of all central elements of A is called the centre of A and it is denoted by B (A). Note that if A is a Pre A*-algebra with 1 then $1, 0 \in B(A)$. If the centre of Pre A*-algebra coincides with {0, 1} then we say that A has trivial centre.

Theorem 1: Venkateswara Rao and Srinivasa Rao (2009): Let A be a Pre A*-algebra with 1, then B (A) is a Boolean algebra with the induced operations $\wedge$, $\vee$, $(-)^{\sim}$.

Lemma 2: Venkateswara Rao and Srinivasa Rao (2009): Let A be a Pre A*-algebra with 1:

(a) If $y \in B(A)$ then $x \wedge x^{\sim} \wedge y = x \wedge x^{\sim}, \forall x \in A$
(b) $x \wedge (x \vee y) = x \vee (x \wedge y) = x$ if and only if $x, y \in B(A)$

Congruences on Pre A*-algebra

Definition 4: Srinivasa Rao (2009): Let A be a Pre A*-algebra and θ be binary relation on A. Then θ is said to be an equivalence relation on A if θ satisfies the following:

(i) Reflexive: $(x, x) \in \theta$, for all $x \in A$
(ii) Symmetric: $(x, y) \in \theta \Rightarrow (y, x) \in \theta$, for all $x, y \in A$
(iii) Transitive: $(x, y) \in \theta$ and $(y, z) \in \theta \Rightarrow (x, z) \in \theta$, f or all $x, y, z \in A$

We write $x \, \theta \, y$ to indicate $(x, y) \in \theta$.

Definition 5: Srinivasa Rao (2009): A relation θ on a Pre A*- algebra $(A, \wedge, \vee, (-)^{\sim})$ is called congruence relation if:

(i) θ is an equivalence relation
(ii) θ is closed under $\wedge$, $\vee$, $(-)^{\sim}$

Lemma 3: Srinivasa Rao (2009): Let $(A, \wedge, \vee, (-)^{\sim})$ be a Pre A*-algebra and let $a \in A$. Then the relation $\theta_a = \{(x, y) \in A \times A / a \wedge x = a \wedge y\}$ is:

(a) a congruence relation
(b) $\theta_a \cap \theta_{a^{\sim}} = \theta_{a \vee a^{\sim}}$
(c) $\theta_a \cap \theta_b \subseteq \theta_{a \vee b}$
(d) $\theta_a \cap \theta_{a^{\sim}} \subseteq \theta_{a \wedge a^{\sim}}$

We will write $x\ \theta_a y$ to indicate $(x, y) \in \theta_a$.

Lemma 4: Srinivasa Rao (2009): Let A be a Pre A*-algebra with 1 and $x, y \in B(A)$. Then

(1) $\theta_x \cap \theta_y = \theta_{x \vee y}$
(2) $\theta_x \circ \theta_y \subseteq \theta_{x \wedge y}$
(3) $\theta_x \vee \theta_y \subseteq \theta_{x \wedge y}$

Theorem 2: Srinivasa Rao (2009): Let Abe a Pre A*-algebra, then A $=\{\theta a / a \in A\}$ is a Pre A*-algebra, is called quotient Pre A*-Algebra, whose operations are defined as:

(i) $\theta_a \wedge \theta_b = \theta_{a \wedge b}$
(ii) $\theta_a \vee \theta_b = \theta_{a \vee b}$
(iii) $(\theta_a)^{\sim} = \theta_{a\sim}$

Lemma 5: Let A be a Pre A*-algebra and $x, y \in A$. Then, $\theta_x \subseteq \theta_y$ if and only if $y = x \wedge y$.

Proof: Suppose that $\theta_x \subseteq \theta_y$.
Since $x \wedge y = x \wedge x \wedge y$, we have $(y, x \wedge y) \in \theta_x$ and therefore, $(y, x \wedge y) \in \theta_y$.
(Supposition)
$\Rightarrow\quad y \wedge y = y \wedge x \wedge y$
$\Rightarrow\quad y = x \wedge y$
Conversely suppose that $y = x \wedge y$
Let $(p, q) \in \theta_x \Rightarrow x \wedge p = x \wedge q$
Now $y \wedge p = x \wedge y \wedge p$ (Supposition)
$= y \wedge x \wedge p$
$= y \wedge x \wedge q$
$= y \wedge q$ (Supposition)
Therefore, $(p, q) \in \theta_y$ and hence, $\theta_x \subseteq \theta_y$.

Lemma 6: Let A be a Pre A*-algebra and $x, y \in A$. Then $\theta_x \subseteq \theta_y$ and $\theta_y \subseteq \theta_{x \wedge y}$.

Proof: We know that $x \wedge (x \wedge y) = x \wedge y$, by above Lemma we have $\theta_x \subseteq \theta_{x \wedge y}$.
Also, we know that $y \wedge (x \wedge y) = x \wedge y$, by above Lemma we have $\theta_y \subseteq \theta_{x \wedge y}$.

Lemma 7: Let A be a Pre A*-algebra and $x, y \in B(A)$. Then $\theta_{x \vee y} \subseteq \theta_x$.

Proof: Let $(p, q) \in \theta_{x \vee y}$. Then $(x \vee y) \wedge p = (x \vee y) \wedge q$
Now $x \wedge p = x \wedge (x \vee y) \wedge p$ (By Lemma 2)
$= x \wedge (x \vee y) \wedge q$
$= x \wedge q$
Therefore $(p, q) \in \theta_x$ and hence, $\theta_{x \vee y} \subseteq \theta_x$.
Note that it is similar way $\theta_{x \vee y} \subseteq \theta_y$.

Theorem 3: Let A be a Pre A*-algebra with 1 and $x \in A$. Then θ_x is the smallest congruence on A containing (1, x).

Proof: We know that θ_x is a congruence of A and clearly $(1,x) \in \theta_x$.
Let θ be a congruence on A and $(1,x) \in \theta$.
Suppose that $(p, q) \in \theta_x \Rightarrow x\wedge p = x\wedge q$
Since $(1,x) \in \theta$, we have $(1\wedge p, x\wedge p)$ and $(1\wedge q, x\wedge q) \in \theta$; that is $(p, x\wedge p)$ and $(q, x\wedge q) \in \theta$. Therefore $(p, q) \in \theta$ and hence $\theta_x \subseteq \theta$.

Theorem 4: Let A be a Pre A*-algebra with 1 and $x, y \in A$ then the following are equivalent:

(1) $x, y \in B(A)$
(2) $\theta_{x\vee y} \subseteq \theta_y$
(3) $\theta_{x\vee y} \subseteq \theta_x \cap \theta_y$
(4) $\theta_{x\vee y} = \theta_x \cap \theta_y$

Proof: (1)⇒(2) Suppose that $x, y \in B(A)$
Let $(p, q) \in \theta$. Then $(x\vee y)\wedge p = (x\vee y)\wedge q$
Now $y\wedge p = y\wedge(y\vee x)\wedge p$ (Since $x, y \in B(A)$ by Lemma 2)
$= y\wedge(x\vee y)\wedge q = y\wedge q$
Therefore, $(p, q) \in \theta_y$ and hence, $\theta_{x\vee y} \subseteq \theta_y$.
(2)⇒(3) Suppose that $\theta_{x\vee y} \subseteq \theta_x$.
By symmetry we have, $\theta_{x\vee y} \subseteq \theta_x$.
Therefore, $\theta_{x\vee y} \subseteq \theta_x \cap \theta_y$.
(3)⇒(4) Suppose that $\theta_{x\vee y} \subseteq \theta_x \cap \theta_y$.
We know that, $\theta_x \cap \theta_y \subseteq \theta_{x\vee y}$ (By Lemma 3(c))
Therefore, $\theta_{x\vee y} = \theta_x \subseteq \theta_y$
(4)⇒(1) Suppose that $\theta_{x\vee y} = \theta_x \cap \theta_y$
Then $(1, x\vee y) \in \theta_{x\vee y} = \theta_x \cap \theta_y$ (Supposition)
Therefore $(1, x\vee y) \in \theta_x$ and $(1, x\vee y) \in \theta_y \Rightarrow x\wedge 1 = x\wedge(x\vee y)$ and $y\wedge 1 = y\wedge(x\vee y)$
⇒ $x = x\wedge(x\vee y)$ and $y = y\wedge(x\vee y)$
⇒ $x, y \in B(A)$ (By Lemma 2)

Definition 6: Srinivasa Rao (2009): Let A be a Pre -A* algebra with 1. For any $x, p, q \in A$, define $\Gamma_x(p, q) = (x\wedge p)\vee(x^{\sim}\wedge q)$.

Now we prove certain properties of $\Gamma_x(p, q)$ which will be used later text.

Lemma 8: Let A be a Pre -A* algebra and $x, p, q \in B(A)$. Then:

(1) $\Gamma_x(p, q) = (x^{\sim}\vee p)\wedge(x\vee q)$
(2) $\Gamma_x(p, q)^{\sim} = \Gamma_x(p^{\sim}, q^{\sim})$

Proof:
(1) $\Gamma x(p, q) = (x\wedge p)\vee(x^{\sim}\wedge q)$
$= [(x\wedge p)\vee x^{\sim}]\wedge[(x\wedge p)\vee q]$ (By distributive law)
$= [x^{\sim}\vee p]\wedge[(x\vee q)\wedge(x^{\sim}\vee p\vee q)]$ (Definition 2 (g) and Lemma 1(d))
$= [(x^{\sim}\vee p)\wedge(x\vee q)\wedge(x^{\sim}\vee p)]\wedge[(x^{\sim}\vee p)\wedge(x\vee q)\wedge q)]$ (By distributive law)
$= [(x^{\sim}\vee p)\wedge(x\vee q)]\wedge[(x^{\sim}\vee p)\wedge(x\vee q)\wedge q)]$
$= (x^{\sim}\vee p)\wedge(x\vee q)$ (Lemma 2 (b))

(2) By (1) we have Γ_x (p, q) = $(x^{\sim}\vee p) \wedge(x\vee q)$.
Now $\Gamma_x(p, q)^{\sim} = [(x^{\sim}\vee p) \wedge(x\vee q)]^{\sim}$
$= (x^{\sim}\vee p)^{\sim}\vee(x\vee q)^{\sim}$
$= (x \wedge p^{\sim})\vee(x^{\sim}\wedge q^{\sim})$
$= \Gamma_x(p^{\sim}, q^{\sim})$

Lemma 9: Let A be a Pre -A* algebra and x, p, q $\in$ A. Then Γ_x (p, q) = $\Gamma_{x\sim}$ (q, p).

Proof: Γ_x (p, q) = $(x\wedge p)\vee(x^{\sim}\wedge q)$
$= (x^{\sim\sim}\wedge p)\vee(x^{\sim}\wedge q)$
$= (x^{\sim}\wedge q) \vee(x^{\sim\sim}\wedge p)$
$= \Gamma_{x\sim}(q, p)$.

Definition 7: Let A be a Pre -A* algebra and x$\in$A. Define:
$\Phi_x = \{(p, q) \in A\times A \,/\, \Gamma_x(p, q) = p\}$.

Theorem 5: Let A be a Pre -A* algebra and x$\in$B(A). Then:
(1) $\Phi_x\subseteq\theta_{x\sim}$
(2) Φ_x is transitive relation on A.

Proof:
(1) Let p, q $\in$ A and (p, q) $\in\Phi_x$, that is $\Gamma_x(p, q)= p$
$\Rightarrow(x\wedge p)\vee(x^{\sim}\wedge q) = p$
Now $x^{\sim}\wedge p = x^{\sim}\wedge\{(x\wedge p\vee(x^{\sim}\wedge q))\}$
$= \{x^{\sim}\wedge x\wedge p\} \vee\{x^{\sim}\vee x^{\sim}\wedge q\}$
$= (0\wedge p) \vee(x^{\sim}\wedge q)$
$= 0\vee(x^{\sim}\wedge q)$ (provided $p \neq 2$)
$= x^{\sim}\wedge q$
If p=2 then $x^{\sim}\wedge p$ =2 to get the required result q should be 2.
Therefore $x^{\sim}\wedge p = x^{\sim}\wedge q$, hence (p, q) $\in \theta_{x\sim}$
Thus, $\Phi_x\subseteq\theta_{x\sim}$

(2) Let p, q, r $\in$A and (p, q), (q, r) $\in \Phi_x$, that is $\Gamma_x(p, q) = p$ and $\Gamma_x(q, r) = q$.
Then by (1) we have (p, q), (q, r)$\in \theta_{x\sim}$
$\Rightarrow x^{\sim}\wedge p = x^{\sim}\wedge q$ and $x^{\sim}\wedge q = x^{\sim}\wedge r$
Now $\Gamma_x(p, r) = (x\wedge p)\vee(x^{\sim}\wedge r)$
$= (x\wedge p)\vee(x^{\sim}\wedge q)$
$= \Gamma x(p, q)$
$= p$

Therefore (p, r) $\in \Phi_x$ and hence Φ_x is transitive relation on A.

Theorem 6: Let A be a Pre A*-algebra induced by a Boolean algebra and θ be congruence on A. Then θ_x is a factor congruence on A if and only if $\theta = \theta_x$ for some x$\in$A.

Proof: Suppose that $\theta = \theta_x$ for some $x\epsilon B(A)$.
Then $x^{\sim}\epsilon B(A)$ and $\theta_x \cap \theta_{x\sim} = \theta_{x\vee x\sim} = \theta_1 = \Delta_A$
and $\theta_x \text{ o } \theta_{x\sim} = \theta_{x\wedge x\sim} = \theta_0 = A\times A$
Thus, θ_x is a factor congruence on A
Conversely suppose that θ is a factor congruence on A
Then there exist a congruence β on A such that $\theta \cap \beta = \Delta_A$ and $\theta \text{ o } \beta = A\times A$.
Now we show that $\theta = \theta_x$
Suppose that $(p,q)\ \epsilon\theta_x$ then $x\wedge p= x\wedge q$.
Since $(x,1)\ \epsilon\theta$ we have $(x\wedge p,\ 1\wedge p)$, $(x\wedge q,\ 1\wedge q)\ \epsilon$ that is $(x\wedge p,\ p)$, $(x\wedge q,\ q)\ \epsilon\theta$ which imply that $(p,q)\ \epsilon\theta$.
Hence, $\theta_x \subseteq \theta$
Suppose $(p,q)\ \epsilon\theta$. Then $(x\wedge p,\ x\wedge q)\ \epsilon\theta$.
Since $(0,x)\ \epsilon\beta$ we have $(0\wedge p,\ x\wedge p)$, $(0\wedge q,\ x\wedge q)\ \epsilon\beta$ that is $(0,\ x\wedge p)$, $(0,\ x\wedge q)\ \epsilon\beta$ which implies that $(x\wedge p,\ x\wedge q)\ \epsilon\beta$.
Therefore $(x\wedge p,\ x\wedge q)\ \epsilon\beta\cap \theta = \Delta_A$ and hence $x\wedge p = x\wedge q \Rightarrow (p,\ q)\epsilon\theta_x$
Hence, $\theta\subseteq\theta_x$
Thus $\theta = \theta_x$.
Hence, θ is a factor congruence on A if and only if $\theta = \theta_x$ for some $x\epsilon B(A)$.

Definition 8: Let A be a Pre A*-algebra and $\alpha\epsilon Con(A)$. Then α is called factor congruence if there exist $\beta\epsilon Con(A)$ such that $\alpha\cap\beta = \Delta_A$ and $\alpha o\beta= A\times A$. In this case β is called direct complement of α.

Definition 9: A congruence β on Pre A*-algebra A is called balanced if $(\beta\vee\theta)\cap(\beta\vee\theta^{\sim}) = \beta$ for any direct factor congruences θ and any of its direct complement $\theta^{\sim}$ on A.

Theorem 7: Let A be a Pre A*-algebra with 1 and $x\epsilon B(A)$. Then θ_x is balanced.

Proof: Let θ_x is a congruence on Pre A*-algebra A. Let Ψ be a factor congruence A and $\Psi^{\sim}$ be its complement. Then by theorem (b) there exist $y, z\epsilon B(A)$ such that $\Psi = \theta_y$ and $\Psi^{\sim} = \theta_z$.
Now $(\theta_x\vee\Psi)\cap(\theta_x\vee\Psi^{\sim}) = (\theta_x\vee\theta_y)\cap (\theta_x\vee\theta_z)$
$=\theta_{x\vee y}\cap\theta_{x\vee z}$
$=\theta_{(x\vee y)\vee(x\vee z)}$
$=\theta_{x\vee(y\vee z)}$
$= \theta_x\vee\theta_{y\vee z}$
$= \theta_x \vee (\theta_y\cap\theta_z)$
$= \theta_x \vee(\Psi\cap\Psi^{\sim})$
$= \theta_x\vee\Delta_A$ (Since Ψ and $\Psi^{\sim}$ are complements)
$= \theta_x$
Hence, θ_x is balanced.

Therefore, the set of balanced congruence which admit a balanced complement is precisely the set $\mathbb{B}(A) = \{\theta_x/x\epsilon B(A)\}$ and hence, $\mathbb{B}(A)$ is the Boolean centre of A.

Theorem 8: Let A be a Pre A*-algebra with 1. Then the Boolean centre $\mathbb{B}(A) = \{\theta_x \ /\ x\epsilon B(A)\}$ is a Boolean algebra and the map $x\rightarrow\theta_{x\sim}$ is an isomorphism of B(A) into $\mathbb{B}(A)$.

Proof: It follows from Lemma 4, theorem 6 and theorem 7.

CONCLUSION

This manuscript point ups the essential congruence θ_x on a Pre A*-algebra and reach your destination at a variety of properties of these. Also it corroborate certain properties of the operations Γ_x(p, q) and Φ_x. It has been long-established that θ is a factor congruence on A if and only if $\theta = \theta_x$ for some x ϵB(A). If A is a Pre A*-algebra with 1 and xϵA, then obtained that θ_x is the smallest congruence on A containing the ordered pair (1, x). Additionally it was ensuing that the centre B(A) of a Pre A*-algebra A with 1 is isomorphic with the Boolean centre $\mathbb{B}$(A) of A.

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