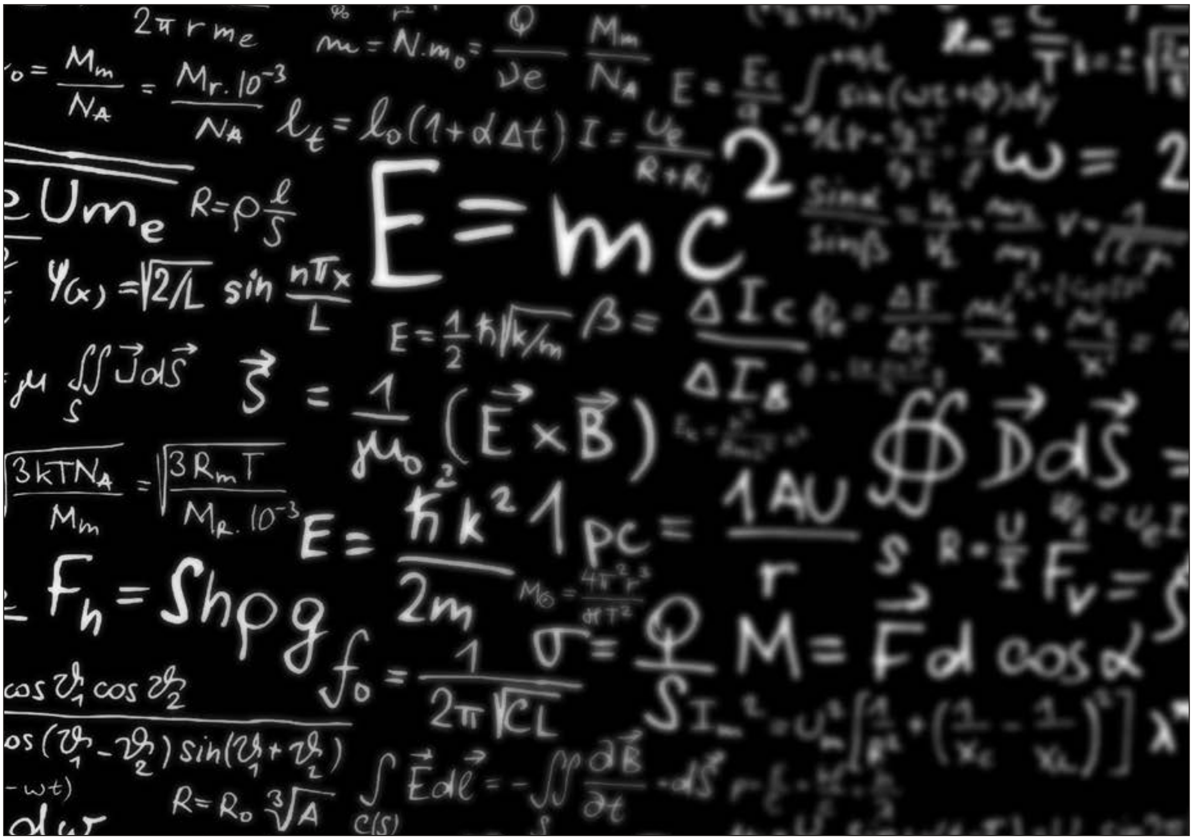

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Partial Orders Fascinating on Pre A*-Algebra

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ABSTRACT

This study characterizes the distinct partial orders $\leq_*$ and $\leq_\oplus$ taking place on a Pre A*-algebra A. As a consequence achieve a result that $x\leq_* y$ if and only if $y^\sim\leq_\oplus x^\sim$. The Hasse diagrams are sketched for certain Pre A*-algebras with respect to $\leq_*$ as well as $\leq_\oplus$. Further derived equivalent conditions for a Pre A*-algebra A to be a Boolean algebra or a trivial Pre A*-algebra in terms of the partial orders $\leq_*$ and $\leq_\oplus$.

Key words: Pre A*-algebra, poset, semilattice, center, Boolean algebra

INTRODUCTION

Birkhoff (1948) made an input to the perception of partial ordering. E.G. Manes (1989) initiated the concept of Ada (algebra of disjoint alternatives) $(A,\wedge,\vee,(-)',(-)_\pi,0,1,2)$. This was be different from characterization of Ada of his afterward paper (Manes, 1993). The Ada of the former sketch gives the impressions that to be founded on broadening the If-Then-Else concept more on the foundation of Boolean algebra and the in a while notion is based on C-algebra $(A,\wedge,\vee,')$ bring in by Guzman and Squir (1990).

The concept of A*-algebra $(A,\wedge,\vee,(-)^\sim,(-)_\pi,0,1,2)$, (where A is a nonempty set, $\wedge$, $\vee$ and * are binary operations; $(-)^\sim$ and π are unary operations and 0, 1, 2 are distinguished elements), was originated by Koteswara Rao (1994). He deliberated the equivalence of A*-algebra with Adas and C-algebra and identified its connections with 3-Ring. Further he derived an If-Then-Else structure over A*-algebra and Ideal of A*-algebra. Later on Venkateswara Rao (2000) bring in the concept of Pre A*-algebra $(A,\wedge,\vee,(-)^\sim)$ as the assortment produced by the 3-element algebra A = {0, 1, 2} which is an algebraic form of three valued conditional logic. Satyanarayana *et al*. (2010) engendered a semilattice configuration on Pre A*-Algebras. Venkateswara Rao and Srinivasa Rao (2009) classified a partial ordering on a Pre A*-algebra and originated the properties as a poset. Satyanarayana *et al*. (2010) draw from the necessary and sufficient conditions for Pre A*-algebra to become a Boolean algebra in terms of the partial ordering.

In this study, we define the distinct partial orders $\leq_*$ and $\leq_\oplus$ on a Pre A*-algebra A and obtain $x\leq_* y$ if and only if $y^\sim\leq_\oplus x^\sim$. The Hasse diagrams are sketched for certain Pre A*-algebras with respect to $\leq_*$ as well as $\leq_\oplus$. Further we obtain equivalent conditions for a Pre A*-algebra A to be a Boolean algebra or a trivial Pre A*-algebra in terms of the partial orders $\leq_*$ and $\leq_\oplus$.

PRELIMINARIES

Definition 1: Boolean algebra is an algebra $(B,\wedge,\vee,(-)',0,1)$ with two binary operations, one unary operation (called complementation) and two nullary operations which satisfies:

(i) $(B,\wedge,\vee)$ is a distributive lattice
(ii) $x\wedge 0 = 0, x\vee 1 = 1$
(iii) $x\wedge x' = 0, x\vee x' = 1$

We can prove that $x'' = x$, $(x\vee y)' = x'\wedge y'$, $(x\wedge y)' = x'\vee y'$ for all $x,y\in B$.

Here, we concentrate on the algebraic structure of Pre A*-algebra and state some results which will be used in the later text.

Definition 2: An algebra $(A,\wedge,\vee,(-)^\sim)$ where A is non-empty set with $1,\wedge,\vee$ are binary operations and $(-)^\sim$ is a unary operation satisfying:

(a) $x^{\sim\sim} = x \quad \forall x\in A$
(b) $x\wedge x = x, \quad \forall x\in A$
(c) $x\wedge y = y\wedge x, \quad \forall x,y\in A$
(d) $(x\wedge y)^\sim = x^\sim\vee y^\sim \quad \forall x,y\in A$
(e) $x\wedge(y\wedge z) = (x\wedge y)\wedge z, \quad \forall x,y,z\in A$
(f) $x\wedge(y\vee z) = (x\wedge y)\vee(x\wedge z), \quad \forall x,y,z\in A$
(g) $x\wedge y = x\wedge(x^\sim\vee y), \quad \forall x,y\in A$ is called a Pre A*-algebra

Example 1: $3 = \{0, 1, 2\}$ with operations $\wedge,\vee(-)^\sim$ defined below is a Pre A*-algebra:

∧	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

∨	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	$x^\sim$
0	1
1	0
2	2

Note: The elements 0, 1, 2 in the above example satisfy the following laws:

(a) $2^\sim = 2$
(b) $1 \wedge x = x$ for all $x \in 3$
(c) $0 \vee x = x$ for all $x \in 3$
(d) $2 \wedge x = 2 \vee x = 2$ for all $x \in 3$

Example 2: $2 = \{0, 1\}$ with operations $\wedge, \vee, (-)^\sim$ defined below is a Pre A*-algebra:

∧	0	1
0	0	0
1	0	1

∨	0	1
0	0	1
1	1	1

x	$x^\sim$
0	1
1	0

Note:

- $(2,\wedge,\vee,(-)^\sim)$ is a Boolean algebra. So every Boolean algebra is a Pre A* algebra
- The identities 2(a) and 2(d) imply that the varieties of Pre A*-algebras satisfies all the dual statements of 2(b) to 2(g).

Note: Let A be a Pre A*-algebra then A is Boolean algebra if $x \vee (x \wedge y) = x$:
$x \wedge (x \vee y) = x$ (absorption laws holds)

Lemma 1: According to Venkateswara Rao and Srinivasa Rao (2009), every Pre A*-algebra satisfies the following laws:

(a) $x \vee (x^{\sim} \wedge x) = x$
(b) $(x \vee x^{\sim}) \wedge y = (x \wedge y) \vee (x^{\sim} \wedge y)$
(c) $(x \vee x^{\sim}) \wedge x = x$
(d) $(x \vee y) \wedge z = (x \wedge z) \vee (x^{\sim} \wedge y \wedge z)$

Definition 3: Let A be a Pre A*-algebra. An element $x \in A$ is called central element of A if $x \vee x^{\sim} = 1$ and the set $\{x \in A / x \vee x^{\sim} = 1\}$ of all central elements of A is called the centre of A and it is denoted by B (A). Note that if A is a Pre A*-algebra with 1 then $1, 0 \in B$ (A). If the centre of Pre A*-algebra coincides with {0, 1} then we say that A has trivial centre.

Theorem 1 (Venkateswara Rao and Srinivasa Rao, 2009): Let A be a Pre A*-algebra with 1, then B (A) is a Boolean algebra with the induced operations $\wedge, \vee, (-)^{\sim}$.

Lemma2 (Venkateswara Rao and Srinivasa Rao, 2009): Let A be a Pre A*-algebra with 1:

(a) If $y \in B(A)$ then $x \wedge x^{\sim} \wedge y = x \wedge x^{\sim}, \forall x \in A$
(b) $x \wedge (x \vee y) = x \vee (x \wedge y) = x$ if and only if $x, y \in B(A)$

PARTIAL ORDERS ON PRE A*-ALGEBRA

Lemma 3: Let A be a Pre A*-algebra define a relation $\leq_*$ on A by $x \leq_* y$ if and only if $x \wedge y = x$ then $(A, \leq_*)$ is a partial order set.

Proof: Since $x \wedge x = x$, $x \leq_* x$, for all $x \in A$
Therefore, $\leq_*$ is reflexive.
Suppose $x \leq_* y$, $y \leq_* z$, for all $x, y, z \in A$ then $x \wedge y = x$ and $y \wedge z = y$
Now $x \wedge z = (x \wedge y) \wedge z = x \wedge (y \wedge z) = x \wedge y = x$
That is $x \leq_* z$, this shows that $\leq_*$ is Transitive.
Let $x \leq_* y$ and $y \leq_* x$ for all $x, y \in A$ then $x \wedge y = x$ and $y \wedge x = y \Rightarrow x = y$
This shows that $\leq_*$ is anti symmetric. Therefore $(A, \leq_*)$ is a poset.

Lemma 4: Let A be a Pre A*-algebra define a relation $\leq_{\oplus}$ on A by $x \leq_{\oplus} y$ if and only if $x \vee y = y$. Then $(A, \leq_*)$ is a partial order set.

Proof: Since $x \vee x = x$, $x \leq_{\oplus} x$, for all $x \in A$
Therefore, $\leq_{\oplus}$ is reflexive.
Suppose $x \leq_{\oplus} y$, $y \leq_{\oplus} z$, for all $x, y, z \in A$ then $x \vee y = y$ and $y \vee z = z$
Now $x \vee z = x \vee (y \vee z) = (x \vee y) \vee z = y \vee z = z$

That is $x \leq_{\oplus} z$, this shows that $\leq_{\oplus}$ is Transitive.
Let $x \leq_{\oplus} y$ and $y \leq_{\oplus} x$ for all x, y∈A then $x \vee y = y$ and $y \vee x = x$
Which implies $x = y$
This shows that $\leq_{\oplus}$ is anti symmetric. Therefore $(A, \leq_{\oplus})$ is a poset.

In any lattice $(L, \wedge, \vee)$ we know that $x \wedge y = x$ if only if $x \vee y = y$, but it is not true in the case of a Pre A*-algebra. In general the partial orders $\leq_*$ and $\leq_{\oplus}$ on Pre A*-algebra are not equal. Infact none of them is contained in the other.
For in the Pre A*-algebra A, we have that
(a) $2 \leq_* 1$ (since $2 \wedge 1 = 2$), but $2 \not\leq_{\oplus} 1$ (since $2 \vee x = 2 \neq 1$)
(b) $0 \leq_{\oplus} 2$ (since $0 \vee 2 = 2$), but $0 \not\leq_* 2$ (since $0 \wedge 2 = 2 \neq 0$)

Now we prove the following lemma, which shows that $\leq_*$ and $\leq_{\oplus}$ are dually related in the following sense.

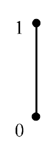
Lemma 5: Let A be a Pre A*-algebra and x, y∈A. Then $x \leq_* y$ if and only if $y^{\sim} \leq_{\oplus} x^{\sim}$.

Proof: For x, y∈A,
we have $x \leq_* y$ if and only if $x \wedge y = x$ if and only if $(x \wedge y)^{\sim} = x^{\sim}$ if and only if $x^{\sim} \vee y^{\sim} = x^{\sim}$
if and only if $y^{\sim} \leq_{\oplus} x^{\sim}$.

Now we give the Hasse diagrams of the partial orders $\leq_*$ and $\leq_{\oplus}$ in certain Pre A*-algebras.

Example 3: In the Pre A*-algebra B = {0,1} 0 covered by 1 with respect to $\leq_*$ as well as $\leq_{\oplus}$. Hence, the Hasse diagram of $(B, \leq_*)$ and $(B, \leq_{\oplus})$ are as follows:

$(B, \leq_*)$



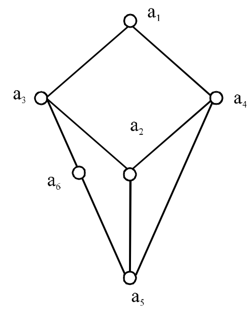
$(B, \leq_{\oplus})$

Example 4: In the Pre A*-algebra A={0,1,2}. Hence the Hasse diagram of $(A, \leq_*)$ and $(A, \leq_{\oplus})$ are as follows:

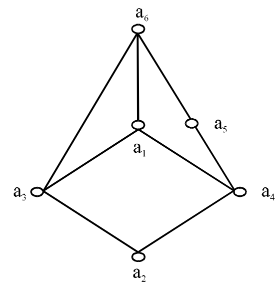
$(A, \leq_*)$

$(A, \leq_{\oplus})$

Example 5: We have B×A = {$a_1 = (1,1)$, $a_2 = (0,0)$, $a_3 = (1,0)$, $a_4 = (0,1)$, $a_5 = (0,2)$, $a_6 = (1,2)$} is a Pre A*-algebra under point wise operation having four central elements, two non-central elements and no element is satisfying the property that $a^{\sim} = a$. the Hasse diagram of $(B \times A, \leq_*)$ and $(B \times A, \leq_{\oplus})$ are as follows:

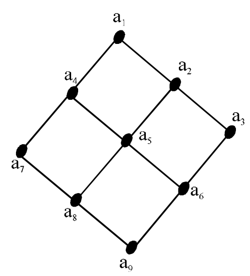


$(B\times A, \leq_*)$

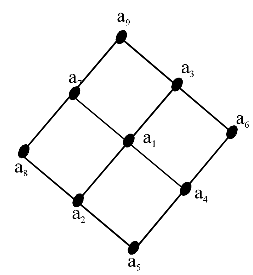


$(B\times A, \leq_\oplus)$

Example 6: We have $A\times A = \{a_1 = (1,1),\ a_2 = (1,0),\ a_3 = (1,2),\ a_4 = (0,1),\ a_5 = (0,0),\ a_6 = (0,2),\ a_7 = (2,1),\ a_8 = (2,0),\ a_9 = (2,2)\}$ is a Pre A*-algebra under point wise operation and $A\times A$ is having four central elements and remaining are non central elements. The Hasse diagram is of the poset $(A\times A, \leq_*)$ and $(A\times A, \leq_\oplus)$ are as follows.



$(A\times A, \leq_*)$



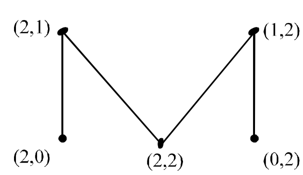
$(A\times A, \leq_\oplus)$

We can observe that in each of the above examples the Pre A*-algebra becomes a join semilattice with respect to $\leq_*$ and meet semilattice with respect to $\leq_\oplus$. But in general it is not true which is clarified in the following example.

Example 7: Let $C = \{(x,y)\in A\times A / x = 2 \text{ or } y = 2 \text{ or both}\}$.Then first observe that C is a sub algebra of the Pre A*-algebra $A\times A$ with point wise operation.
Let $(x,y)\in C$. Then $x = 2$ or $y = 2$ or both.
Therefore $x^\sim = 2$ or $y^\sim = 2$ or both and hence, $(x,y)^\sim = (x^\sim, y^\sim)\in C$.
Let $(x_1,y_1), (x_2,y_2)\in C$. Then $x_1 = 2$ or $y_1 = 2$ or both and $x_2, = 2$ or $y_2 = 2$ or both.
Now $(x_1,y_1) \vee (x_2,y_2) = (x_1\vee x_2, y_1\vee y_2)$

$$= \begin{cases} (2, y_1 \vee y_2) & ,\text{if } x_1 = 2 \\ (x_1 \vee x_2, 2) & ,\text{if } y_1 = 2 \\ (2,2) & ,\text{if } x_1 = y_1 = 2 \end{cases}$$

Therefore, $(x_1,y_1) \vee (x_2,y_2) \in C$ and hence C is a sub algebra of Pre A*-algebra $A\times A$.
Now the Hasse diagram of $(C, \leq_*)$ is as follows.



In Boolean algebra, minimal and maximal elements with respect to $\leq_*$ are unique. In fact, they are the least and greatest elements. Whereas, in Pre A*-algebra minimal and maximal elements are not unique; that is, the least and the greatest elements may not exist in A.

Here we can observe that (2,0), (2,2), (0,2) are the minimal elements and (2,1), (1,2) are maximal elements and there are no least and greatest elements. Hence the Pre A*-algebra C is neither join semilattice nor meet semilattice with respect to $\leq_*$.

Lemma 6: Let A be a Pre A*-algebra. Then
(1) If A has 2 , then 2 is minimal with respect to $\leq_*$
(2) If A has 2, then 2 is maximal with respect to $\leq_\oplus$

Proof: (1) Suppose A has 2 and $x \in A$.
Now $x \leq_* 2 \Rightarrow x \wedge 2 = x$
$\Rightarrow 2 = x$
Therefore, 2 is minimal with respect to $\leq_*$.
(2) Suppose A has 2 and $x \in A$.
Now $2 \leq_\oplus x \Rightarrow x \vee 2 = x$
$\Rightarrow 2 = x$
Therefore, 2 is maximal with respect to $\leq_\oplus$.

Lemma 7: Let A be a Pre A*-algebra $a \in A$. Then a and $a^\sim$ are minimal elements in A with respect to $\leq_*$ if and only if a = 2.

Proof: Suppose a and $a^\sim$ are minimal elements in A with respect to $\leq_*$.
We have $a \wedge (a \wedge a^\sim) = a \wedge a^\sim$
$\Rightarrow a \wedge a^\sim \leq_* a$
Also $(a^\sim \wedge a) \wedge a^\sim = a^\sim \wedge a$
$\Rightarrow a^\sim \wedge a \leq_* a^\sim$
Since a and $a^\sim$ are minimal elements we have $a \wedge a^\sim = a$ and $a^\sim \wedge a = a^\sim$ and hence $a = a^\sim$.
Therefore a = 2.
Conversely suppose that a = 2.
We have $a^\sim = a = 2$. Lemma b it follows a and $a^\sim$ are minimal elements in A with respect to $\leq_*$.

Lemma 8: Let A be a Pre A*-algebra $a \in A$. Then a and $a^\sim$ are maximal elements in A with respect to $\leq_\oplus$ if and only if a = 2.

Proof: Suppose a and $a^\sim$ are maximal elements in A with respect to $\leq_\oplus$.
We have $a \vee (a \vee a^\sim) = a \vee a^\sim$
$\Rightarrow a \leq_\oplus a \vee a^\sim$
Also $(a^\sim \vee a) \vee a^\sim = a^\sim \vee a$
$\Rightarrow a^\sim \leq_\oplus a^\sim \vee a$
Since a and $a^\sim$ are maximal elements we have $a \vee a^\sim = a$ and $a^\sim \vee a = a^\sim$ and hence $a = a^\sim$.
Therefore, a = 2.
Conversely suppose that a = 2.
We have $a^\sim = a = 2$, by Lemma 6, it follows a and $a^\sim$ are maximal elements in A with respect to $\leq_\oplus$.

Lemma 9: Let A be a Pre A*-algebra and a, $b\in A$. Then $a\vee b$ is the upper bound of {a, b} with respect to $\leq_*$ if and only if $a\vee b$ is the least upper bound of {a ,b} with respect to $\leq_*$.

Proof: Suppose $a\vee b$ is the upper bound of {a, b} with respect to $\leq_*$.
Let $k\in A$ be an upper bound of {a, b} with respect to $\leq_*$.
Then $a\leq_* k$ and $b\leq_* k$ that is $a\wedge k = a$, $b\wedge k = b$.
Now $(a\vee b)\wedge k = (a\wedge k)\vee(b\wedge k)$

$= a\wedge b$

Therefore, $a\vee b\leq_* k$.
Hence, $a\vee b$ is the least upper bound of {a, b} with respect to $\leq_\oplus$.
Converse is clear.

Lemma 10: Let A be a Pre A*-algebra and a, $b\in A$. Then $a\wedge b$ is the lower bound of {a, b} with respect to $\leq_\oplus$ if and only if $a\wedge b$ is the greatest lower bound of {a, b} with respect to $\leq_\oplus$.

Proof: Suppose $a\wedge b$ is the lower bound of {a ,b} with respect to $\leq_\oplus$.
Let $k\in A$ be an lower bound of {a ,b} with respect to $\leq_\oplus$.
Then $k\leq_\oplus a$ and $k\leq_\oplus b$ that is $a\vee k = a$, $b\vee k = b$.
Now $(a\wedge b)\vee k = (a\vee k)\wedge(b\vee k)$

$= a\wedge b$

Therefore, $k\leq_\oplus a\wedge b$.
Hence, $a\wedge b$ is the greatest lower bound of {a, b} with respect to $\leq_\oplus$.
Converse is clear.

Theorem 2: Let A be a Pre A*-algebra induced by a Boolean algebra then

(i) $x\wedge x^{\sim}$ is minimal with respect to $\leq_*$, for all $x\in A$.
(ii) $x\vee x^{\sim}$ is maximal with respect to $\leq_\oplus$, for all $x\in A$.

Proof: (i) Let $x,y\in A$.
Now $y\leq_* x\wedge x^{\sim} \Rightarrow x\wedge x^{\sim}\wedge y = y$

$\Rightarrow x\wedge x^2 = y$

Therefore, $x\wedge x^{\sim}$ is minimal with respect to $\leq_*$, for all $x\in A$.
(ii) Let $x,y\in A$.
Now $x\vee x^{\sim}\leq_\oplus y \Rightarrow x\vee x^{\sim}\vee y = y$

$\Rightarrow x\vee x^{\sim} = y$

Therefore, $x\vee x^{\sim}$ is maximal with respect to $\leq_\oplus$, for all $x\in A$.

Theorem 3: Let A be a Pre A*-algebra then the following are equivalent.

(1) A is Boolean algebra
(2) $\leq_*\subseteq\leq_\oplus$
(3) $\leq_\oplus\subseteq\leq_*$
(4) $\leq_* =\leq_\oplus$

Proof: (1) $\Rightarrow$(2) suppose that A is Boolean algebra and $x,y \in A$

Let $x \leq_* y$ $\Rightarrow x \wedge y = x$

$\Rightarrow (x \wedge y) \vee y = x \vee y$

$\Rightarrow y = x \vee y$ (since A is Boolean algebra $(x \wedge y) \vee y = y$)

$\Rightarrow x \leq_\oplus y$

Therefore, $\leq_* \subseteq \leq_\oplus$.

(2) $\Rightarrow$(3) Suppose that $\leq_* \subseteq \leq_\oplus$ and $x,y \in A$

Let $x \leq_\oplus y \Rightarrow y^\sim \leq_* x^\sim$ (Lemma 5)

$\Rightarrow y^\sim \leq_\oplus x^\sim$ (by supposition)

$\Rightarrow x \leq_* y$ (Lemma 5)

Therefore, $\leq_\oplus \subseteq \leq_*$.

(3) $\Rightarrow$ (4) Suppose that $\leq_\oplus \subseteq \leq_*$ and $x,y \in A$

Let $x \leq_* y \Rightarrow y^\sim \leq_\oplus x^\sim$ (Lemma 5)

$\Rightarrow y^\sim \leq_* x^\sim$ (by supposition)

$\Rightarrow x \leq_\oplus y$ (Lemma 5)

Therefore, $\leq_* \subseteq \leq_\oplus$ and hence, $\leq_* = \leq_\oplus$.

(4) $\Rightarrow$ (1) Suppose that $\leq_* = \leq_\oplus$ and $x, y \in A$.

We have $x \vee (x \vee y) = x \vee y$

Therefore, $x \leq_\oplus x \vee y$ and hence by (4) we have $x \leq_* x \vee y$, that is $(x \vee y) \wedge x = x$ (absorption law holds)

Therefore A is Boolean algebra.

It is natural to expect that $\leq_*$ and $\leq_\oplus$ may become dual to each other in a Pre A*****-algebra. We conclude this paper by proving the following theorem which states that the Pre A*****-algebra becomes trivial if the partial orders $\leq_*$ and $\leq_\oplus$ are dual to each other.

Theorem 4: Let A be a Pre A*****-algebra. Let $\leq_*^\sim$ and $\leq_\oplus^\sim$ be the duals of $\leq_*$ and $\leq_\oplus$, respectively. Then the following are equivalent.

(1) A is trivial

(2) $\leq_* \subseteq \leq_\oplus^\sim$

(3) $\leq_\oplus^\sim \subseteq \leq_*$

(4) $\leq_\oplus \subseteq \leq_*^\sim$

(5) $\leq_*^\sim \subseteq \leq_\oplus$

(6) $\leq_\oplus = \leq_*^\sim$

(7) $\leq_* = \leq_\oplus^\sim$

Proof: (1) $\Rightarrow$ (2) is clear.

(2) $\Rightarrow$ (3) Suppose that $\leq_* \subseteq \leq_\oplus^\sim$ and $x, y \in A$.

Let $x \leq_\oplus^\sim y \Rightarrow y \leq_\oplus x$

$\Rightarrow x^\sim \leq_* y^\sim$

$\Rightarrow x^\sim \leq_\oplus^\sim y^\sim$ (Supposition)

$\Rightarrow y^\sim \leq_\oplus x^\sim$

$\Rightarrow x \leq_* y$

Therefore $\leq_\oplus^\sim \subseteq \leq_*$

(3) $\Rightarrow$ (4) Suppose that $\leq_\oplus^\sim \subseteq \leq_*$ and $x, y \in A$.

Let $x \leq_\oplus y$ $\Rightarrow y \leq_\oplus^\sim x$

$\Rightarrow y \leq_* x$ (Supposition)

$\Rightarrow x \leq_*^{\sim} y$

Therefore, $\leq_{\oplus} \subseteq \leq_*^{\sim}$

(4) $\Rightarrow$ (5) Suppose that $\leq_{\oplus} \subseteq \leq_*^{\sim}$ and $x, y \in A$.

Let $x \leq_*^{\sim} y \Rightarrow y \leq_* x$

$\Rightarrow x^{\sim} \leq_{\oplus} y^{\sim}$

$\Rightarrow x^{\sim} \leq_*^{\sim} y^{\sim}$ (Supposition)

$\Rightarrow y^{\sim} \leq_* x^{\sim}$

$\Rightarrow x \leq_{\oplus} y$

Therefore $\leq_*^{\sim} \subseteq \leq_{\oplus}$

(5) $\Rightarrow$ (6) Suppose that $\leq_*^{\sim} \subseteq \leq_{\oplus}$ and $x, y \in A$.

Let $x \leq_{\oplus} y \Rightarrow y^{\sim} \leq_* x^{\sim}$

$\Rightarrow x^{\sim} \leq_*^{\sim} y^{\sim}$

$\Rightarrow x^{\sim} \leq_{\oplus} y^{\sim}$ (Supposition)

$\Rightarrow y \leq_* x$

$\Rightarrow x \leq_*^{\sim} y$

Therefore, $\leq_{\oplus} \subseteq \leq_*^{\sim}$

Hence, $\leq_{\oplus} = \leq_*^{\sim}$

(6) $\Rightarrow$ (7) Suppose that $\leq_{\oplus} = \leq_*^{\sim}$ and $x, y \in A$.

Let $x \leq_* y \Leftrightarrow y \leq_*^{\sim} x$

$\Leftrightarrow y \leq_{\oplus} x$ (Supposition)

$\Leftrightarrow x \leq_{\oplus}^{\sim} y$

Therefore, $\leq_* = \leq_{\oplus}^{\sim}$

(7) $\Rightarrow$ (1) Suppose that $\leq_* = \leq_{\oplus}^{\sim}$ and $x \in A$.

We have $x \wedge (x \wedge x^{\sim}) = x \wedge x^{\sim}$

$\Rightarrow x \wedge x^{\sim} \leq_* x$

$\Rightarrow x \wedge x^{\sim} \leq_{\oplus}^{\sim} x$ (Supposition)

$\Rightarrow x \leq_{\oplus} x \wedge x^{\sim}$

$\Rightarrow x \vee (x \wedge x^{\sim}) = x \wedge x^{\sim}$

$\Rightarrow x = x \wedge x^{\sim}$ (Lemma 1)

and by symmetry $x^{\sim} \wedge x = x^{\sim}$

Thus $x = x^{\sim}$ for all $x \in A$ and hence $x = 2$.

Therefore A is trivial.

CONCLUSION

This document differentiates the dissimilar partial orders $\leq_*$ and $\leq_{\oplus}$ taking place on a Pre A*-algebra. As a outcome achieve a result that $x \leq_* y$ if and only if $y^{\sim} \leq_{\oplus} x^{\sim}$. The Hasse diagrams are drafted for certain Pre A*-algebras with respect to $\leq_*$ as well as $\leq_{\oplus}$. Additionally derived equivalent conditions for a Pre A*-algebra A to be a Boolean algebra or a trivial Pre A*-algebra in terms of the partial orders $\leq_*$ and $\leq_{\oplus}$. It has been proved a result that if A is a Pre A*-algebra and by defining a relation $\leq_*$ on A by $x \leq_* y$ if and only if $x \wedge y = x$, then $(A, \leq_*)$ is a partially order set. Similarly if A is a Pre A*-algebra and by define a relation $\leq_{\oplus}$ on A by $x \leq_{\oplus} y$ if and only if $x \vee y = y$ then $(A, \leq_*)$ is a partially order set. If A is a Pre A*-algebra and $a \in A$, then proved that a and $a^{\sim}$ are minimal elements in A with respect to $\leq_*$ if and only if $a = 2$. If A is a Pre A*-algebra $a \in A$, then derived that a and $a^{\sim}$ are maximal elements in A with respect to $\leq_{\oplus}$ if and only if a = 2. If A is a Pre A*-algebra and a, $b \in A$, then established that $a \vee b$ is the upper bound of {a, b} with respect to $\leq_{*}$ if and only if $a \vee b$ is the least upper bound of {a ,b} with respect to $\leq_{*}$. If A is a Pre A*-algebra and a, $b \in A$, then observed that $a \wedge b$ is the lower bound of {a, b} with respect to $\leq_{\oplus}$ if and only if $a \wedge b$ is the greatest lower bound of {a, b} with respect to $\leq_{\oplus}$.

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