The Effect of $\beta$ in Kaiser Window on the SNR of MST Radar Signals

1G.H. Reddy, 2Y. Venkatarami Reddy and 3S.N. Reddy  
1Department of ECE, Priyadarshini College of Engineering, Nellore, India  
2Department of ECE, JNT University, Hyderabad, India  
3Department of ECE, SVU College of Engineering, Tirupati, India

Abstract: In this study, the effect of parameter $\beta$ in Kaiser Window on the SNR values of MST radar echoes is investigated. Use is made of six sets of multibeam observations of the lower atmosphere made by the Indian Mesosphere-Stratosphere-Troposphere (MST) radar. The effects of data weighting with the variation in the parameter $\beta$ of the Kaiser window applied to the inphase and quadrature components of the radar echo samples prior to Fourier transformation are presented. It is observed that the increase of $\beta$, increases the SNR in the higher bins and a good improvement is reported. At the same time in the lower bins a slight decrease in SNR is observed. From these observations, it is concluded that the Kaiser window can be used with $\beta$ greater than or equal to 6, to taper the data for spectral analysis in place of a rectangular window. The results also show that, the improvement of SNR of noisy data due to the effect of side lobe reduction and demands for the design of optimal windows.

Keywords: Rectangular window, noise, doppler spectra, dft, spectral analysis, main lobe, side lobe reduction

INTRODUCTION

Harmonic analysis with the Discrete Fourier Transform (DFT) plays a central role in radar signal processing. The significance of using data weighting windows with the DFT (Harris, 1978; Kay, 1988; Marple, 1987) plays an important role in resolving the frequency components of the signal buried under the noise. Since the use of an in-appropriate window can lead to corruption of the principal spectral parameters it is instructive to consider the criteria by which the choice of data weighting window to be used is made (Hooper, 1999). This paper presents the effect of $\beta$ in Kaiser window on the SNR of radar returns and proposed an optimum value of $\beta$ with which data may be weighted using Kaiser window.

The Data Weighting Windows Windowing

It is well known (Harris, 1978; Kay, 1988; Marple, 1987) that the application of FFT to a finite length data gives rise to leakage and picket fence effects. Weighting the data with suitable windows can reduce these effects. However the use of the data windows other than the rectangular window affects the bias, variance and frequency resolution of the spectral estimates (Kay, 1988; Marple, 1987). In general, the variance of the estimate increases with the use of a window. An estimate is to be consistent if the bias and the variance both tend to zero as the number of observations is increased. Thus, the problem associated with the spectral estimation of a finite length data by the FFT techniques is the problem of establishing efficient data windows or data smoothing schemes.

Corresponding Author: G.H. Reddy, Department of ECE, Priyadarshini College of Engg.,  
A.K. Nagar (Post) Nellore-524 004, India. Tel: +91 861 2344 956

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Data windows are used to weight time series of the in-phase and quadrature components of the radar return samples prior to applying the DFT. The observed Doppler spectra therefore represent convolutions of the Fourier transforms of the original signals with those of the data weighting windows, projected onto the discrete (angular) frequencies (Harris, 1978).

**Spectral Leakage**

For signal frequencies, observed through the rectangular window, which do not correspond exactly to one of the sampling frequencies, the pattern is shifted such that non-zero values are projected onto all sampling frequencies. This phenomenon of spreading signal power from the nominal frequency across the entire width of the observed spectrum is known as spectral leakage (Harris, 1978).

The radar returns considered to be composed of a quasi-monotonic (atmospheric) signal superimposed on a background of white noise. As might be expected, since the signal does not correspond exactly to one of the sampling frequencies, the forms of the signal portions of the spectra follow those of the envelopes of the side lobe maxima. Spectral leakage from the signal therefore exceeds noise level, evaluated by the method (Hildebrand and Sekhon, 1974) and a corresponding underestimate of signal-to-noise ratio.

**The Kaiser Window**

It is a relatively simple approximation of the prolate spheroidal functions. For discrete time the Kaiser window is expressed as (Alan and Schafer, 1988, Kay, 1988):

\[
    w[n] = \begin{cases} 
        \frac{I_0(\beta \sqrt{1 - \left(\frac{n-N/2}{N/2}\right)^2})}{I_0(\beta)} & 0 \leq n \leq N-1 \\
        0 & \text{else}
    \end{cases}
\]

Where:
- \(N\) = The window length, controls the main lobe width
- \(\beta\) = Controls the amplitude for the side lobes
- \(I_0(\cdot)\) = The modified zeroth-order Bessel function

In contrast to the other windows, the Kaiser window has two parameters: The length of the sequence \(N\) and a shape parameter \(\beta\). As the length of the window is fixed to 512 data points here, the shape parameter \(\beta\) can be varied. As the parameter \(\beta\) increases the side lobe level of the frequency response decreases (Alan and Schafer, 1988). In this paper we have investigated the effect of \(\beta\) on the SNR of MST radar data. The results are presented in Fig. 1a-f and 2a-f.

**Kaiser-Window Applied to Atmospheric Radar Signals**

Wind profile detection of a MST Radar signal meant the measurement of Dopplers of the signal due to scattering of the atmospheric elements. Atmospheric Radar signal meant the signal received by the Radar due to the back scattering property of the atmospheric layers, stratified or turbulent. The back-scattered signal from the atmospheric layers is very small in terms of power with which it was emitted. The received back-scattered signals otherwise called as Radar returns are associated with Gaussian noise. The noise dominates the signal as the distance between the Radar and the target increases and this leads to a decrease in Signal to Noise ratio. This makes the detection of the signal difficult. Doppler profile information is obtained from the power spectrum using Fast Fourier Transform. Frequency characteristics of the back-scattered signals of the Radar s are analyzed with power spectrum, which specifies the spectral characteristics of a signal in frequency domain.
Fig. 1: Average SNR for the MST Radar data collected from NARL, Gadanki, India on 8th July 2002.

The specifications of the data are shown in Table 1. The SNR analysis is performed on MST Radar data corresponding to the lower stratosphere obtained from the NARL, Gadanki, India, on 8th July 2002. The Radar was operated in Zenith X, Zenith Y, North, South, West and East with an angle of 10° from the vertical, direction. The data obtained from the six directions are used to carry on the analysis. The complete implementation of the scheme using C++ and Matlab, to study the effect of β on the SNR of the radar returns can be put as follows.

- Compute the Kaiser window with β = 0 using Eq. 1.
- Taper the radar data with the Kaiser window parameters specified in step (a).
- Perform the Fourier analysis of the above tapered data (Appendix A)
Table 1: Specifications of the MST Radar, India data on which the analysis is performed

<table>
<thead>
<tr>
<th>Lower Stratosphere (up to 30 km) MST Radar, Gadanki, India</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of range bins</td>
</tr>
<tr>
<td>No. of FFT points</td>
</tr>
<tr>
<td>No. of coherent integrations</td>
</tr>
<tr>
<td>No. of incoherent integrations</td>
</tr>
<tr>
<td>Inter pulse period</td>
</tr>
<tr>
<td>Pulse width</td>
</tr>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>Period of observation</td>
</tr>
</tbody>
</table>

- Pulse width: 16 μ sec
- Range resolution: 150 m
- Inter pulse period: 1000 μ sec
- No of beams: 6 (E100y, W100y, Zy, Zx, N100x, S100x)
- No of FFT points: 512
- No of incoherent integrations: 1
- Maximum doppler frequency: 3.9 Hz
- Maximum doppler velocity: 10.94 m sec⁻¹
- Frequency resolution: 0.063 Hz
- Velocity resolution: 0.176 m sec⁻¹

E100y = East West polarization with off-zenith angle of 10°; W100y = East West polarization with off-zenith angle of 10°; N100x = North South polarization with off-zenith angle of 10°; S100x = North South polarization with off-zenith angle of 10°.

• Compute the SNR using the procedure (Anandan, 1987; Hooper, 1999) (Appendix B)
• Compute the average of SNR minima’s (Mean of - dB SNR Values)
• Compute the average of SNR maxima’s (Mean of + dB SNR Values)
• Update the value of β and repeat the steps b-f.

RESULTS AND DISCUSSION

The SNR computation (Hildebrand and Selch, 1974; Anandan, 2000, 1987) (Appendix B) for the six sets of Radar data is carried on and presented in Fig. 1a-f and 2a-f. From the Fig. 1a-f, it is inferred that when β = 0, which corresponds to the rectangular window, the average SNR in the negative side is minimum and increases with the increase in β steadily. The SNR increases slowly when β is varied from 4 to 6. There is no appreciable change in average SNR value for further increase in β.

On the other hand in all the six sets of data, the average positive SNR has not suffered any major change when β is increased. More over, a slight decrease in average positive SNR is observed when β is increased beyond 6.

From the Fig. 2a-f it is also seen that for the lowermost 50 bins the average SNR in the negative and positive sides are not improved appreciably. Moreover there is slight and marginal decrease in both SNR’s is observed. For the middle 50 bins and the uppermost 50 bins the increase in average negative SNR values is almost 5-6 dB when β is around 6. Further improvement is also seen when β is increased beyond 6. This result is important since the back-scattered signal from the middle and uppermost bins is very week and improvement in SNR is highly desirable in spectral estimation.

Noting the above observations, it is concluded that the Kaiser window can be used with β greater than or equal to 6 to taper the data for spectral analysis in place of a rectangular window. The results also suggest that, the effect of side lobe reduction in the improvement of SNR of noisy data and demands for the design of optimal windows.
Fig. 2: Continued
Fig. 2: Average SNR for Lower, middle and upper bins data collected from NARL, Gadanki, India on 8th July 2002

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APPENDIX A

The Algorithm presented here to compute the power spectrum estimation is based on the procedure described in the references (Anandan, 2000, 1987).

Power Spectrum Estimation

Step 1
Read the raw data (complex time samples) X(n)

Step 2
Normalize the Pre-Processed data

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Step 3
Taper theNormalized data with Kaiser window described in Eq. 1

\[(d(n) = x(n) * w(n))\]    \hspace{1cm} (A1)

Step 4
Compute the Normalized FFT of \(d(n)\)

\[D(i) = \frac{1}{N} \sum_{k=1}^{N} d(n) \exp \left(-2\pi i k / N\right)\]    \hspace{1cm} (A2)

Step 5
Compute the Power Spectrum \(P_i\)

\[P_i = |D(i)|^2 = 0, \ldots, \ldots, N - 1\]    \hspace{1cm} (A3)

Step 6
Perform Incoherent Integration (i.e. averaging of the power spectrum number of times).

\[\overline{P_i} = \frac{1}{M} \sum_{k=1}^{M} P_{ik} \text{ i = 0, \ldots, \ldots, N - 1}\]    \hspace{1cm} (A4)

Where, \(M\) is the number of spectra integrated. In this work \(M = 1\) is taken.

Step 7
Power spectrum cleaning is performed to remove Clutter/DC value or zero mean velocities using the basic operation as:

\[\overline{P_{n/2}} = \frac{\left(\frac{P_{n+1}}{2} + \frac{P_{n-1}}{2}\right)}{2}\]    \hspace{1cm} (A5)

\(N/2\) corresponds to zero frequency

APPENDIX B

Noise Level Estimation

The Algorithm presented here for Noise level estimation is based on the procedure described in the references (Anandan, 1987, 2000).

Step 1
Reorder the spectrum \((P_i, i = 0, \ldots, N - 1)\) in ascending order and form the sequence \((A_i, i = 0, \ldots, N - 1)\) and \(A_i < A_j\) for \(i < j\)

Step 2
Compute
\[ P_n = \sum_{i=0}^{n} \frac{A_i}{(n+1)} \]  
(B1)

\[ Q_n = \sum_{i=0}^{n} \frac{A_i^2}{(n+1)} - P_n^2 \]  
(B2)

and if \( Q_n > 0 \),

\[ R_n = \frac{P_n^2}{(Q_n * M)} \text{ for } n = 0, \ldots, N - 1 \]

Where, \( M = 1 \), the number of spectra that were averaged.

**Step 3**

Compute the Noise Level.

\[ \text{Noise level (L)} = P_k \text{ Where } k = \min \left| n \text{ such that } R_n > 1 \right| \text{ if no } n \text{ meets the above criterion} \]

**Step 4**

Reorder the spectrum to its correct index of frequency in the following manner.

<table>
<thead>
<tr>
<th>Spectral index</th>
<th>Ambiguous frequency</th>
<th>Maximum negative frequency</th>
<th>Zero frequency</th>
<th>Maximum positive frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\ldots</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\ldots</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 5**

Subtract noise level \( L \) from spectrum to form \( \tilde{p}_i \)

**Step 6**

- Find the index of the peak value in the spectrum,

\[ i.e., \tilde{p}_i \geq \tilde{p}_i \text{ for all } i = 0, \ldots, N - 1 \]

- Find the minimum index of positive point corresponding to the valley point of the detected peak Doppler spectrum.
i.e., \( \bar{p}_i \geq 0 \) for all \( m \leq 1 \leq 1 \)

- Find \( n \) the maximum index of positive point corresponding to the valley point of the detected peak Doppler spectrum.

\[ i.e., p_i \geq 0 \quad \text{for all} \quad 1 \leq i \leq 1 \]

**Step 7**
Compute the zeroth moment or the total power in the Doppler spectrum.

\[ M_0 = \sum_{i=1}^{n} p_i \]  \hspace{1cm} (B5)

**Step 8**
Compute the Signal to Noise Ratio as

\[ \text{Signal to Noise Ratio (SNR) = 10log} \left( \frac{M_1}{N \cdot L} \right) \text{ dB} \]  \hspace{1cm} (B6)

Where:
\( N \) = The number of FFT points.

**Step 9**
Repeat the Procedures described in Appendix-A and Appendix-B for all the bins of MST radar data.

**Step 10**
Compute the Mean of -dB SNR Values.

**Step 11**
Compute the Mean of +dB SNR Values.

**Step 12**
Update \( \beta \) value and repeat the steps in Appendix A and B.

**Step 13**
Plot the Mean of -dB SNR Values and Mean of +dB SNR Values with respect to \( \beta \).

**REFERENCES**


