Modeling and Simulation of Vortex Induced Vibration on the Gas Transporting GRP Pipeline Riser

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ABSTRACT
This research work investigates the dynamics characteristics of the offshore riser pipeline due to vortex flow and to develop a model that could predict its vortex induced responses. A riser made of Glass-fiber Reinforced Plastic (GRP) is used for this study. The mathematical model is derived based on Hamilton’s principle and modeled by using a finite-element method. The riser is subjected to sea wave and current and the hydrodynamic forces are calculated using Morison’s equation. A direct integration method namely Newmark integration scheme is proposed to solve the equation of motion. A MATLAB program code was developed to obtain the simulation results. The model used is validated by comparing the results with the analytical method and it shows a very good agreement. The simulation results are obtained to illustrate the dynamic characteristic of GRP pipe when subjected to combined sea wave and current. The effect of vortex shedding frequency and internal flow velocity on the response of riser are investigated and discussed in this study.

Key words: Glass-fiber reinforced plastic pipe, vortex induced vibration, finite element

INTRODUCTION
Subsea riser is one of the most important structure in offshore design. It is developed to convey materials such as fluid, oil and gas from the seabed to production facilities at sea level and vice versa. This slender structure exhibits complex dynamic behavior due to dynamic loading from sea current and wave.

One of the major challenges facing the offshore industry is the phenomenon of Vortex Induced Vibration (VIV). Each time a vortex is shed, there is a resultant forces acting on the body of the riser which can cause the vibration and this is called vortex induced vibration (Bathe, 1996). If the excitation frequency of the vortices is close to natural frequency of the structure, resonance will happen which in turn cause fatigue damage or even collisions with other risers.

Investigation of the dynamic responses of the subsea riser has been conducted through various approaches such as numerical and experimental in the last few decades (Chakrabarti et al., 2002). The prediction method of subsea riser becomes very significant to the offshore structure design. Without an accurate prediction method, small damages of the offshore riser could lead to adverse effect to the environment.

In most studies dealing with the VIV problems, some of the researchers focus on the modeling, simulation and analysis, while others conducted experiments to establish experimental data which can be used to benchmark prediction tools such as semi empirical and Computational Fluid
Dynamics (CFD). The approach is based on the virtual work-energy functional to develop the structural model. Nonlinear equation of motion due to the effect of a nonlinear Morison type term coupled in axial and transverse displacements is derived through the Hamilton's principle. The dynamic responses of the marine risers are determined using the finite element method for which the Newmark average acceleration method is implemented for direct numerical integration. Their results show that internal flow rate and hydrodynamic drag force have a major effect on the displacement amplitude. It has also been reported that the increment of natural frequencies of risers is due to top tension and modulus of elasticity (Graves and Dareing, 2004). This method is adopted in this research study.

The conventional alloy steel pipes used in offshore field are Nowadays, being replaced with the Glass Reinforced Plastics (GRP) pipes. GRP offers many advantages compared to steel material. These include high corrosion resistance, long service life, low weight, low cost and reduced maintenance (Kaewunruen et al., 2005). GRP is composite element made of E-glass fibres as a reinforcing component with epoxy polymer matrix. In order to determine the dynamics response of GRP pipes, analytical or computer-based method should be used. The model of the pipe should reflect the actual behavior of the pipe system, based on the behavior of GRP (Kaewunruen et al., 2008).

There seems to be a limited number of works dealing with GRP pipe for numerical method. Some of them were focusing on composite riser pipeline for this study. Valenzuela and Moore (1987) has conducted technical feasibility study between drilling riser made of composite materials and steel (Leklong et al., 2008). They concluded that the composite riser has smaller dynamic amplification factor compare with steel riser with the same buoyancy compensation factor. This is due to a difference in mass between these two materials.

Another paper presented vortex-induced vibration of composite riser using CFD by Ljustina et al. (2004). Their study revealed some interesting features regarding VIV of a composite riser. They found that for a composite riser, with a smaller mass ratio, the amplitude of vibration tends to be higher than that of a geometrically similar steel riser. The number of modes was also found to affect the vibration amplitude. Besides the mass ratio, the effects of other parameters such as top tension and damping were also being observed and studied.

The objective of this study is to investigate the dynamics characteristics of the riser pipeline due to the vortex flow and to predict the vortex induced responses. The mathematical model is governed to stand for the dynamic behavior of marine riser when subjected to both transverse and axial forces. Computational techniques by using MATLAB commercial software are developed in order to numerically solve the finite element. The results are presented and finally the conclusions are drawn based on the results presented. By conducting VIV analysis and modeling for GRP pipe, the understanding and dynamic response prediction capabilities for GRP pipe can be increased.

**FORMULATION**

*Equation of motion:* In this study, the riser is modeled as a vertical pipe with uniform internal flow. It is assumed to be submerged in the ocean and subjected to external hydrodynamic forces from both wave and current.

The extended Hamilton’s principle is used to derive the equation of motion. Hamilton’s principle with the presence of external forces, states that:

\[ \delta \int_0^T (L + W) \, dt = \delta \int_0^T (T - P + W) \, dt = 0 \]  

(1)
where, $L$ is the Lagrangian of the system, $T$ is the kinetic energy, $W$ is the strain energy and $W$ is the virtual work done by external loads on the body.

Total kinetic energy, $T$ consists of kinetic energy caused by the riser movement and the internal fluid flow movement. Total strain energy, $W$ consists of the strain energy due to axial and flexural deformation. The virtual work done due to external forces such as the effective weight, hydrodynamic forces and inertia forces are taken into account (Morooka and Tsukada, 2009).

After substituting the equation of kinetic energy, strain energy and external loads into Eq. 1, the general differential equations of motion in transverse and in-line direction is:

$$
[m + m_r] \frac{\partial^2 u_{x,y}}{\partial t^2} + 2m_r \nu \frac{\partial u_{x,y}}{\partial x} + m_r \nu V \frac{\partial^2 u_{x,y}}{\partial z^2} + EI \frac{\partial^4 u_{x,y}}{\partial z^4} = F_{x,y}
$$

(2)

also, the governing equation of motion in vertical direction is:

$$
[m + m_r] \frac{\partial^2 w}{\partial t^2} + 2m_r \nu \frac{\partial w}{\partial x} + m_r \nu V \frac{\partial^2 w}{\partial z^2} - EA \frac{\partial^4 w}{\partial z^4} = F_0
$$

(3)

where, $u_{x,y}$ is the displacement at $x$ and $y$-axis and $w$ is the displacement of the riser at $z$-axis. $m_r$ and $m_r$ are the mass of riser and internal fluid inside the riser, $EI$ is the riser bending stiffness, $EA$ is the riser axial rigidity and $V$ is the internal flow velocity.

Meanwhile the external fluid forces in the right-hand side of Eq. 2 and 3 are defined using Morison equation of lift and drag forces. Hydrodynamic drag force is modeled by the Morison equation as follows:

$$
F_D = \frac{1}{2} \rho C_D (U - u) |U - u| + \rho C_m |U - u|
$$

(4)

in which $U$, $C_D$, $C_m$ are wave velocity, drag coefficient and added mass coefficient, respectively. The lift force which is the force induced by vortex shedding in transverse direction is described as below:

$$
F_L = \frac{1}{2} \rho U^2 C_L \sin(\omega t)
$$

(5)

where, $U$ is the instantaneous velocity, $U = U_m \sin(\omega t)$, $C_L$ is the lift coefficient and $\omega$ is the vortex shedding frequency. The angular frequency of vortex shedding is determined by using this equation:

$$
\omega = \frac{s U}{D}
$$

(6)

Equation 2 and 3 are used as a mathematical model for this study.

**Finite element method:** The governing partial differential equation presented in Eq. 2 and 3 cannot be solved exactly for arbitrary riser problems and load patterns. Hence, a numerical method is required, such as the Finite Element Method (FEM).

To improve the accuracy, the riser structure is discretized into 61 nodes connected by 60. As mentioned earlier, the riser is clamped at both ends to the platform leg. It is assumed that the riser
is clamped at 20 m each. Each node is described with four degrees of freedom, axial displacement, w and transverse displacement, u are both dependent variables which are functions of z.

The equation of motion defined in Eq. 2 and 3 can be linearized into matrix formulation as stated below:

\[
[M] \ddot{\mathbf{D}} + [G] \dot{\mathbf{D}} + [K] \mathbf{D} = \mathbf{R}
\]

(7)

where, [M] is a mass matrix, [G] is a gyroscopic matrix, [K] is a stiffness matrix of structure. The term in right side \( \mathbf{R} \) is external force vectors acting on the system and varies with time. \( \dot{\mathbf{D}} \) is the nodal displacement vector and the dot notation is used to indicate derivative with respect time t (PETRONAS, 2010).

**Numerical simulation:** In order to calculate the response of the riser, the riser governing equation of motion is solved using the Newmark integration method which is conditionally stable (Rakshit et al., 2008). At each time step, the displacements, velocities and accelerations are computed and obtained. The MATLAB code is developed to perform the numerical simulation and solve the equation.

**RESULTS AND DISCUSSION**

The subsea riser in Fig. 1 is used as an example for this study. As shown in Fig. 1, the riser is clamped to the side of the fixed platform by riser clamped to avoid buckling. The water depth is 62.5 m. The length of the riser between the clamps is approximately 20 m.

Instead of using carbon-steel riser, Glassfiber Reinforced Plastic (GRP) riser is chosen to be the case study for this research. The specification of the GRP riser model is tabulated in Table 1 and taken based on the specification of the gas transmission pipeline riser at RDPA platform connecting with the OGT, Kerteh (15:00, 16:00).

![Fig. 1: Schematic representation of subsea riser](image)
Table 1: Input data for GRP riser model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riser density (GRP)</td>
<td>1880 (kg m⁻³)</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>2.8 × 10⁹ (N m⁻²)</td>
</tr>
<tr>
<td>Length of the riser (each clamped)</td>
<td>20 (m)</td>
</tr>
<tr>
<td>Diameter of riser</td>
<td>0.700 (m)</td>
</tr>
<tr>
<td>Thickness of riser</td>
<td>0.0181 (m)</td>
</tr>
<tr>
<td>Internal flow density (gas)</td>
<td>895 (kg m⁻³)</td>
</tr>
</tbody>
</table>

Table 2: Comparison of present work and analytical results

<table>
<thead>
<tr>
<th>Mode</th>
<th>Present work (Hz)</th>
<th>Analytical result (Hz)</th>
<th>Percent error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.7914</td>
<td>6.7914</td>
<td>0.00000</td>
</tr>
<tr>
<td>2</td>
<td>27.1566</td>
<td>27.1657</td>
<td>0.00350</td>
</tr>
<tr>
<td>3</td>
<td>61.1249</td>
<td>61.1238</td>
<td>0.00044</td>
</tr>
<tr>
<td>4</td>
<td>108.6746</td>
<td>108.6628</td>
<td>0.01030</td>
</tr>
<tr>
<td>5</td>
<td>169.8308</td>
<td>169.7857</td>
<td>0.02507</td>
</tr>
</tbody>
</table>

Numerical validation of model: The verification of the mathematical model is required to ensure the model produces an accurate numerical solution of the governing equation in Eq. 2 and 3. The validation of the proposed model is carried out by comparing the fundamental frequency of marine riser using both analytical and the proposed model.

By assuming the boundary condition as simply supported at both ends, the analytical values of natural frequencies are calculated using Eq. 1 as presented by Sumer and Fredsoe (1997). Table 2 lists the natural frequencies for the first five modes when internal velocity is assumed to be zero. The percentage error between both methods is less than 0.04% and therefore shows a very good agreement. Therefore, this mathematical model is used to conduct the analysis:

\[
\omega = \frac{1}{2\pi} \sqrt{\frac{\pi^2 EI}{L} \left( \frac{\pi^2 T}{L} \right)^2} \]

Effect of vortex shedding frequency: In the case of transverse vibration, the investigation of the vortex response of riser was undertaken by determining the effect of vortex shedding and internal flow velocity. Here, the wave frequency is assumed to be 10 rad sec⁻¹ and wave velocity is 10 m sec⁻¹. Since the Reynolds number for wave velocity is more than 3 × 10⁶, the Strouhal number and lift coefficient are assumed to be 0.3 and 0.2, respectively (Valenzuela and Moore, 1987).

Figure 2 depicts a time series of lift force acting on the riser when the shedding frequency is 8 rad sec⁻¹. In this figure, there are several different peaks which are induced by vortex shedding. The peaks in the lift force are related with the existence of vortices acting on the riser (Vijayaraghavan et al., 2010).

The effect of vortex shedding frequency is studied. The natural frequency of a subsea riser when the internal velocity is 10 m sec⁻¹ is equal to 15.348 rad sec⁻¹. By changing the value of vortex shedding frequency, some phenomenon of VIV are captured. The time series of transverse displacement (A/D), with different frequency ratios, \( f / f_s = 0.5, 0.9, 1.0 \) and 1.1 are shown in Fig. 3. In Fig. 3a, the amplitude of A/D is quite small which is less than 0.01, compared to the others. This figure indicates a weak vibration induced by the vortex shedding. Then, when the
Fig. 2: Effect of time on the lifting force of riser for 10 sec

Fig. 3(a-d): Amplitude of transverse displacements (A/D) at frequency ratio ($f_l/f_n$) (a) 0.5, (b) 0.9, (c) 1.0 and (d) 1.1

Vortex shedding is increased to be near to the natural frequency, the beating effect is captured and the amplitude of A/D is increased to 0.035. However, when the vortex shedding is equal to the natural frequency where frequency ratio is 1.0, the response is in resonance state. In this state, both vibration frequency and vortex shedding frequency are locked in the natural frequency of the structure. During the lock-in, the large amplitude may develop and undergo a great number of stress cycles that can lead to fatigue damage. When the frequency ratio is further increased to 1.1, the beating effect appears again with the amplitude of A/D is 0.05. Both phenomenon, resonance and beating effect show two dynamic characteristics of a structure due to vortex flow.

In Fig. 4, the variation of maximum amplitude of transverse displacement is plotted with different ratios from 0.1 to 2.0. The figure shows that at frequency ratio of 1.0, it
Fig. 4: Maximum transverse displacement with different frequency ratio

Fig. 5(a-d): Transverse displacements (A/D) at internal flow velocity (a) 0, (b) 10, (c) 20 and (d) 30 m sec\(^{-1}\).

achieve the highest maximum amplitude due to resonance state. The maximum amplitude is decreased significantly after the frequency ratio is further increased.

**Effect of internal flow velocity**: To investigate the effect of internal flow velocity to vortex-induced responses, the variation of the resonant responses with different internal flow velocities are displayed in Fig. 5. The results exhibit that the amplitude A/D of resonant responses decreases as the internal flow velocity increases. Since only fluid damping is considered in this mathematical model, without the internal flow, the amplitude of A/D in Fig. 5a is higher.
than others due to zero damping. If the responses are extended to 300 sec, it is observed that when the internal flow increases, the time duration for the response to converge onto steady state decreases. In Fig. 5d, after 40 sec, the resonant response starts to be in steady state compares to the others which take longer time to converge onto steady state. It is found that the internal flow velocity plays a significant role in reducing the maximum amplitude of the riser responses and time duration of the response to converge onto steady state.

CONCLUSION

As a conclusion, the dynamic characteristics of the GRP riser pipeline due to vortex induced vibration are discussed in this paper. The mathematical model to predict the dynamic response of the GRP marine riser is presented in this paper. The finite element model is solved and simulated by using the MATLAB software. Two parameters which are frequency ratio and internal flow velocity are varied to study the effects on the riser response.

From the results, those parameters have certainly affected its vibration and their characteristics. Two phenomenon of VIV which are beating effect and resonance state were captured successfully. During the resonance state, vortex shedding frequency is equal to the natural frequency. The response is in lock-in with the natural frequency and the amplitude becomes larger. The beat phenomena appeared when the vortex shedding frequency is close with natural frequency. The resonant responses decrease and as the internal flow velocity increases. The presence of internal flow and vortex shedding frequency show some effects on the dynamic response of the riser.

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