Asian Journal of Scientific Research
ISSN 1992-1454
Solar Radiation Simulation by Using Zero-dimensional Climate Model

1Rahmat Riza, 2Bambang Ari wahjoedi and 1Shaharin Anwar Sulaiman

1Department of Mechanical Engineering, 2Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia

Corresponding Author: Rahmat Riza, Department of Mechanical Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia

ABSTRACT

Solar radiation is the major energy source for entire earth. This causes the variation of solar radiation had strong relation with variation of global temperature. This relation was studied more detail by using simulation of zero-dimensional climate model. On this model, the equilibrium system was used to study the effect of solar radiation to temperature change. Overall, the model consists of two mathematical equations that describe condition for the atmospheric layer and earth surface system. The outputs of the model consisted of the temperature at the atmospheric layer and the earth surface. They were solved by three numerical methods, namely: Newton-Raphson, Quasi Newton-Raphson and Steepest Descent method. Absolute error analysis was used to determine the solution for the Newton-Raphson and Quasi Newton-Raphson methods while alpha value for Steepest Descent method. Relation for the solar variation and global temperature change was obtained by assuming the other variables on the equations to be constant as the average value. The relation graphs describe that variation of the solar radiation could cause significant global temperature change.

Key words: Zero-dimensional climate model, solar variation, system equation, Newton-Raphson, Quasi Newton-Raphson, steepest descent

INTRODUCTION

The increasing of the global temperature in entire world had taken concern for the scientist as reported by Soltani and Soltani (2008). This could be caused by increasing solar radiation entering the world (Elmallah, 2011). Nowadays, the solar activity has been reported by BBC to reach the highest position than previous time. This has been interesting to the scientist since solar radiation is known as the major energy for the life in entire earth. Variation of the activity of the solar radiation supposes to take relation to change of global temperature. This could be caused that the amount of energy that received by the earth.

This news was supported by work of Solanki et al. (2004). On their work, the new activity of the Solar has been reconstructed. Moreover, this work was also quoted on article from Leidig (2004) on The Washington Times.

The increasing of the solar activity has been also supported by some NASA scientist as published by Scientific American.
On this work, the effect of solar variation to the global temperature change is studying by using computer modeling. Zero-dimensional climate model offers some advantages to study relation between solar variation and global temperature change. The model offers the equilibrium state to explain relation between the parameters and simplifying to explain global understanding of climate modeling (McGuffie and Sellers, 2005).

Three iterative numerical methods were used to solve the zero-dimensional climate model, namely: Newton-Raphson, Quasi Newton-Raphson and Steepest Descent method. The results from the iterative solution of these numerical methods were comparing to published result by Boeker and van Grondelle (1999) as the validation part of this work.

Study showed that solar radiation had been varied around 0.25 W m\(^{-2}\) of its average value (Hansen et al., 2012). Analyzing the data on this reference, the effect of solar variation had the minimum value at 1364.85 W m\(^{-2}\) in 2003 and reached the maximum position at 1367.5 W m\(^{-2}\). Indeed, the variation on this work was taken from 1364-1368 W m\(^{-2}\). The result showed that the effect of solar variation influenced both the atmospheric temperature and temperature of the earth surface.

**ZERO-DIMENSIONAL CLIMATE MODELING**

**Climate modeling:** Basic Climate modeling was developed based on Energy Balance Model (EBM). On this model the Earth is assuming as black body that receives the energy radiated by the Solar. This situation is drawing by Eq. 1 (McGuffie and Sellers, 2005):

\[
(1-a)\pi R^2 S = 4\pi R^2 \alpha T^4
\]  

where, \(a\) is the albedos for the solar radiation, with \(a_e\) is defined for atmosphere and \(a_r\) is the Earth’s average albedo. \(R\) is Earth’s radius-approximately 6.371\(\times\)10\(^6\) m. \(S\) is The solar constant-solar radiation per unit area-\(\alpha = \) The Stefan-Boltzmann constant-approximately 5.6\(\times\)10\(^{-8}\) J K\(^{-4}\) m\(^{-2}\) s\(^{-1}\)-T is The radiation temperature of the earth.

**Zero-dimensional climate model:** In this study, variation of the solar radiation is simulated to study the effect to the global temperature change. This could be described by using zero-dimensional climate model. This model is part of EBM model that assumes the Earth has one-mean temperature. Boeker and van Grondelle (1999) had been developed zero-dimensional climate model as the Eq. 2 and 3 that consisted of two simultaneous equation. The first equation is to model the condition on the surface of the earth. This is given following Equation:

\[
-(1-a_1-a_2)\frac{S}{4}+\varepsilon(T_s-T_e)+\sigma T_s^4(1-a_2)-\sigma T_e^4=0
\]  

\[
(1-a_1-a_2)\frac{S}{4}+\varepsilon(T_s-T_e)+\sigma T_s^4(1-a_2)+2\sigma T_e^4=0
\]  

The second Equation is modeling process occurred on atmosphere layer. This is given on Eq. 3:

This model is the improved EBM to accommodate process that occurred between the atmosphere layer and earth surface.
MATERIALS AND METHODS

Data: Zero-dimensional climate model as the Eq. 2 and 3 describe the earth surface as one single point. On this situation, the parameters on the both equations describe general condition or assumed as average value of different position on the surface of the earth.

Data that used on this paper namely: Albedo of the atmosphere (a) is 0.3, albedo of the atmosphere for long-wavelength radiation (a_l) is 0.53, albedo of the earth (a_s) is 0.11, transmission of the atmosphere (τ) for short-wavelength radiation, transmission of the atmosphere for long-wavelength radiation (τ_l) is 0.06 and the last data is coefficient for the interaction between the atmospheric layer and the earth surface (ε) that equal to 2.7 W m⁻²·K⁻¹ (Boeker and van Grondelle, 1999).

Numerical methods: Climate model as on Eq. 2 and 3 is consisted of two simultaneous equations with two variables name temperature of the earth surface (T_s) and temperature of the atmosphere layer (T_a). This system equation is solved by using iterative numerical methods. Three numerical methods were used to solve the model equations on this work, namely: Newton-Raphson, Quasi Newton-Raphson and Steepest Descent methods.

Newton-Raphson: This method is described by following steps (Chapra and Canale, 2006):

Step 1: Determine the initial guessing for both the variable T_s and T_a
Step 2: Determine matrix Jacobian of the model for the initial guessing
Step 3: Calculate determine of matrix Jacobian
Step 4: Calculate the value of the function of the both equation
Step 5: Compute the Newton-Raphson solution
Step 6: Analyze the error

Quasi Newton-Raphson: This method is described by following steps (Faires and Burden, 2002):

Step 1: Determine the initial guessing for both the variable T_s and T_a
Step 2: Define matrix Jacobian and obtain its determinant
Step 3: Determine the inverse of matrix Jacobian, assumed as inverse of matrix A_0
Step 4: Compute the value of both equations for the initial guessing
Step 5: Compute the new guessing namely T_{s1} and T_{a1}
Step 6: Determine the value of both equations for the second guessing as step ‘4’
Step 7: Define the inverse of new matrix A_1 based on matrix A_0
Step 8: Compute the second iteration step by using matrix A_1
Step 9: Analyze the error to decide the iteration process

Steepest descent: This method is described by following steps (Faires and Burden, 2002):

Step 1: Determine the initial guessing for both the variable T_s and T_a
Step 2: Determine the value of the function for both equations for the initial guessing
Step 3: Compute function of “g” for the initial guessing
Step 4: Analyze the gradient of the equation at the initial guessing
Step 5: Analyze alpha value, for α>0, near the initial guessing to obtain best path to the solution
Step 6: Define the new value that the closest one to the solution
Step 7: Analyze function "g" by using new value guessing for the using alpha
Step 8: Re-iterate the methods when step '7' was not satisfied the boundary

Initial guessing: Theoretically, initial guessing is random number selected to begin the iteration process but some methods have limitation on the using any random number. They can result divergence solution when the initial guessing is placed too far from the exact solution.

However, the zero point is the most affordable to start the iteration point. Based on this point, there are two possibility of zero point on this work can be selected, namely: zero Kelvin and zero degree Celsius. Zero Kelvin is looked quite far from the expected solution. This brought that zero Celsius to be selected on this work.

Stopping technique and error analysis: Newton-Raphson and Quasi Newton-Raphson have same technique to determine that the iteration steps need to find new solution or to stop the process. They use absolute error as the technique. Absolute error could be written as Eq. 4 (Heath, 2002):

\[
\text{Error} = |T - T_i|
\]  

(4)

where, \(T\) is the latest solution and \(T_i\) is the previous solution. This error is used for both \(T_a\) and \(T_s\). However, the process of the iteration path for all the methods could be defined by the absolute error whether the methods go straight convergence or zigzag procedure to obtain the solution.

While the Steepest Descent uses the \(\alpha\) value as the point to make decision. This was shown as on the step iteration given.

RESULTS AND DISCUSSION

Results for numerical solution of the system equations: As the discussion on ‘Initial Guessing’ Section, the results of the iteration process that obtained by all methods were given based on initial guessing \(T_a = 273.15\) K and \(T_s = 273.15\) K, the results of the system equation solution for the Newton-Raphson and Quasi Newton-Raphson are consecutively given as in Table 1 and 2. While the results for the Steepest Descent method are given in Table 3.

<table>
<thead>
<tr>
<th>Step</th>
<th>(T_a) (K)</th>
<th>(T_s) (K)</th>
<th>(T_a)</th>
<th>(T_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>290.40</td>
<td>251.59</td>
<td>17.25</td>
<td>21.55</td>
</tr>
<tr>
<td>2</td>
<td>288.35</td>
<td>248.56</td>
<td>2.05</td>
<td>3.04</td>
</tr>
<tr>
<td>3</td>
<td>288.31</td>
<td>248.51</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>288.31</td>
<td>248.52</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1: Results of the iteration steps of the Newton-Raphson method

<table>
<thead>
<tr>
<th>Step</th>
<th>(T_a) (K)</th>
<th>(T_s) (K)</th>
<th>(T_a)</th>
<th>(T_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>290.40</td>
<td>251.59</td>
<td>17.25</td>
<td>21.55</td>
</tr>
<tr>
<td>2</td>
<td>288.35</td>
<td>249.14</td>
<td>2.00</td>
<td>2.46</td>
</tr>
<tr>
<td>3</td>
<td>288.34</td>
<td>248.62</td>
<td>0.0039</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table 2: Results of the iteration steps of the quasi Newton-Raphson method

356
Table 3: Results of the iteration steps of the steepest descent method

<table>
<thead>
<tr>
<th>Step</th>
<th>( x )</th>
<th>( T_s (K) )</th>
<th>( T_a (K) )</th>
<th>( T_s )</th>
<th>( T_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.02</td>
<td>248.06</td>
<td>248.32</td>
<td>0.0151</td>
<td>0.0132</td>
</tr>
<tr>
<td>8</td>
<td>0.02</td>
<td>248.98</td>
<td>248.32</td>
<td>0.0199</td>
<td>0.0122</td>
</tr>
<tr>
<td>9</td>
<td>0.01</td>
<td>248.06</td>
<td>248.33</td>
<td>0.0016</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

In Table 1 and 2, column 2 and 3 give the results of each step iteration calculation for \( T_s \) and \( T_a \), respectively. The values of both temperatures are given in Kelvin unit measurement. The errors calculation for \( T_s \) and \( T_a \) are given in column 4 and 5 for both Table 1 and 2.

Table 3 has extra column, namely column 2 for selected alpha value for each iteration procedure. Table 3 only shows last three step of the iteration process.

**Result for solar variation:** The results for variation solar radiation from 1364-1368 W m\(^{-2}\) are given in Fig. 1-2. Figure 1 describes the relation between solar variation and the values of \( T_s \) while Fig. 2 gives relation between solar variation and \( T_a \) that resulted from the climate model as given in Eq. 2-3.

The x-axis uses the solar variation and y-axis is as the temperature change of the earth surface for Fig. 1 and Atmospheric layer for Fig. 2.

By using \( T_s \) and \( T_a \) equal to 273.15 K (°C), the results for all the methods as in Table 1-3 gave that solution of the model for \( T_s \) is around 288 K. Newton-Raphson method needs four steps iteration to get the expected solution. Results for this method have showed that it had straightforward convergence process in order to obtain the solution. Both variable \( T_s \) and \( T_a \) had near error value for the every step of iteration.

In other case, Quasi Newton-Raphson has been successful to reduce number of iteration step to solve the system equation of the model comparing to the Newton-Raphson. Moreover, this method got quite different procedure to obtain the solution. While the Newton-Raphson produce the constant error between \( T_s \) and \( T_a \), this method got different thing on the last step iteration as given in Table 2 and the fourth row. The error for \( T_s \) is significantly different with the error for \( T_a \). The error for \( T_s \) in this step dramatically reduces comparing to the error for \( T_a \).

Steepest Descent results as shown in Table 3 need nine steps iteration to produce acceptable solution as produced by Newton-Raphson and Quasi Newton-Raphson. The error of this method show that this method has zigzag path in order to find the acceptable solution as other methods obtained. The step iteration of the Steepest Descent ran for small increment to go to the solution point.

Overall, the results as produced by all the methods are same as the expected solution as the suggestion by Boeker and van Grondelle (1999) that solution of \( T_s \) from the system equation that using the average value should be around 288 K. This proves that these methods are capable to solve the model as in Eq. 2 and 3 with acceptable solution.

Thus, all the methods could be used to simulate variation of the solar radiation in order to study its effect to temperature of the earth surface and temperature of the atmospheric layer.

Based on data reference from NASA, as discussed in Section Introduction, the simulation of the solar radiation was taken to vary from 1364-1368 W m\(^{-2}\) while other parameters were kept constant. The outputs of this simulation were different value for \( T_s \) and \( T_a \).
Fig. 1: The relation for variation of solar radiation value and earth surface temperature ($T_s$)

The results of solar variation from zero-dimensional climate model by using three numerical methods are given as in Fig. 1 and 2. Figure 1 simulates the relation for variation of solar radiation and earth surface temperature. Legend of RTs represents the relation obtained by Newton-Raphson method while the QTs for Quasi Newton-Raphson and STs for Steepest Descent method.

The results of Newton-Raphson and Quasi Newton-Raphson produced constant relation with the variation of solar radiation. In other hand, the results from Steepest Descent give fluctuation relation. This can conclude that the results from both Newton-Raphson and Quasi Newton-Raphson methods are able to be accepted as the simulation solution for the relation between the variation of solar radiation and $T_s$ and $T_a$. Moreover, Steepest Descent method cannot use for simulation this equation. So, this method can be proposed as the way to obtain initial value for the iterative procedure since it gives the result close to solution compared to the initial value used in the iterative step.

The acceptable simulation result showed that the increasing of solar radiation as the main input of the model can cause significant increment of the both temperature of the earth surface and atmospheric layer. The relation these parameters walk with non linear pattern. Moreover, Fig. 1 shows that the plot of temperature of the earth surface had gradient higher than plot of atmospheric layer temperature as in Fig. 2. This is describing that the effect of increasing solar
radiation more pronounced to the earth surface than the atmospheric layer. However, the different
effect to both temperature of the earth surface and atmospheric layer is not too much happened.

CONCLUSION

Three numerical methods, namely: Newton-Raphson, Quasi Newton-Raphson and Steepest
Descent methods are capable to solve zero-dimensional climate model developed by Boeker and
van Grondelle (1999). The results produced from all the methods proved as the acceptable solution
as suggested by Boeker and van Grondelle (1999). However, only Newton-Raphson and Quasi
Newton-Raphson methods can be accepted to do the simulation to vary the solar radiation.
Variation of solar radiation has influenced both temperature of the earth surface and temperature
of the atmospheric layer.

ACKNOWLEDGMENT

The authors are thankful to Universiti Teknologi PETRONAS for providing grant and facilities
for the work.

REFERENCES

New York, USA.

New York, USA.

Elmallah, E.S., 2011. Regional climate interaction with the solar and geomagnetic activities.


Leidig, M., 2004. Hotter-Burning Sun Warming the Planet. The Washington Times Publisher,
Washington, DC, USA.

New York, USA.

during recent decades compared to the previous 11,000 years. Nature, 431: 1084-1087.

Soltani, E. and A. Soltani, 2008. Climate changes of Khorasan, North-East of Iran, during 1950-