Asian Journal of Scientific Research

ISSN 1992-1454

Asian Journal of Scientific Research 6 (3): 411-422, 2013
ISSN 1992-1454 / DOI: 10.3923/ajsr.2013.411.422

Development of an Optical Strain Measurement Method Using Digital Image Correlation

Khoo Sze Wei, Saravanan Karuppanan and Muhamad Ridzuan Bin Abdul Latif
Department of Mechanical Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia

Corresponding Author: Khoo Sze Wei, Department of Mechanical Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia

ABSTRACT

Strain measurement is important in mechanical testing. There are many strain measurement methods; namely electrical resistance strain gauge, extensometer, Geometric Moiré technique and optical strain measurement method among others. Each method has its own advantages and disadvantages. There is always a need to develop a precise and yet simple strain measurement method in mechanical testing. The objective of this study is to develop a two-dimensional deformation measurement algorithm that calculates the strain in a loaded structural component. This can be achieved by using the digital image correlation technique which compares the displacement of the random speckles pattern in the reference and the deformed images. The strain measurement algorithm was coded into MATLAB program utilizing the MATLAB's Image Processing toolbox. Next, tensile test experiments for two different materials were conducted and videos were recorded simultaneously by using a consumer version of high-definition video camera. The recorded videos (images) were then analyzed and the strain values were determined by using the optical strain measurement method. The modulus of elasticity of the respective materials was then determined and the results obtained by the extensometer and the optical strain measurement method were compared to each other. The modulus of elasticity of the mild steel and the polypropylene specimens were found to be 5.1 and 2.4% off the benchmark values, respectively. A good agreement was achieved upon comparison between the results determined by the two methods mentioned above. In conclusion, the two-dimensional deformation measurement algorithm had been successfully developed.

Key words: Digital image correlation, optical strain measurement method, random speckles pattern, two-dimensional deformation measurement algorithm

INTRODUCTION

The knowledge of strain is vital to the engineers as it plays an important role in most of the engineering designs and experimental works. For example, almost all the engineering designs are adopting small strain analysis. It means only small deformations are allowed and most of the structural members are designed to be rigid or the deformations are barely noticeable. As a result, the strain measurement becomes ultimately important in many engineering applications since the manmade structures and machines are getting more complex than before. A precise strain measurement method is always desirable as a misleading result might cause a catastrophic incident and also put human lives in jeopardy.

In order to overcome this situation, different types of strain measurement methods are invented by the engineers, i.e., electric resistance strain gauge, extensometer, brittle coating method, geometric moire technique and optical strain measurement method. However, each of the strain measurement methods mentioned previously has its own advantages and disadvantages. For example, the electrical resistance strain gauges provide precise results in strain measurement but their high price tag and handling difficulty make them an imperfect strain measurement method (Khan and Wang, 2001). By using the extensometer, the handling difficulty is solved as the extensometer is easily mounted on the specimen (http://www.epsilontech.com/3542.htm). In contrast, the contact points of the extensometer's arms and the specimen surface create unwanted stress concentration to the specimens. As for the brittle coating method, the preparation works before the experiment is easy but the coating is not commonly acceptable as it exhibits both flammability and toxicity (http://stresscoat.com/SCManual.pdf). Therefore, safety precautions against these dangers must be taken. Geometric Moiré technique is another method used in strain analysis where interference fringes are produced by superimposing two or more sets of gratings (SPOTS Standard, 2005). However, this technique is not suitable for high temperature strain measurement and is not sensitive enough to measure small strains accurately. The holographic interferometry method has received substantial attention since it provides full-field deformation measurement with high degree of accuracy (Khan and Wang, 2001). Nevertheless, the drawback of this method is that the entire equipment must be completely isolated from the vibrations. Otherwise, the vibrations would alter the optical path and the recorded hologram becomes blur.

In order to improve the strain measurement method, a vision system known as Digital Image Correlation (DIC) had been introduced for experimental stress analysis (Peter and Ranson, 1982). Sutton *et al.* (1983) improved the DIC technique to obtain the full-field in-plane deformations of a cantilever beam. Basically, this vision metrology is ideally suitable for non-contact deformation and strain measurement in a remote system. The DIC provides full-field deformation measurement that mathematically compares the two images that are acquired at two different states, one before deformation and the other one after deformation. The acquired images are stored in the digital form and the measurement algorithm is applied to perform the image analysis. During the image analysis, DIC or image matching has been executed by choosing two subsets (small aperture for pattern matching) from the reference and deformed images for correlation. As a result, the two subsets must have the same level of light intensity or contrast, so that the image matching can be performed accurately (Sutton, 2008). Finally, the measurement of deformation of a structural component under loading can be achieved by tracking the displacement of random speckle pattern which is deposited on the surface of the sample. Normally, the random speckle patterns used in DIC technique are generated by spray painting or etching (Jin *et al.*, 2008).

The theoretical simplicity is attractive but the iteration procedure for the strain determinations are lengthy and always cause the errors (Wang *et al.*, 2002). For example, the determination of the correlation coefficient by Sutton *et al.* (1983) and Chu *et al.* (1985) are very time consuming, especially during the estimation of the initial range of the displacement values. Sutton *et al.* (1986) modified the iterative Digital Image Correlation algorithm and they applied the Newton-Raphson method with differential corrections to speed up the searching time. Bruck *et al.* (1989) optimized the DIC algorithms which converged faster and hence fewer calculations were needed. Besides, they developed a complete model of Newton-Raphson method of partial differential corrections which determines the six deformation parameters by using less computation time if compared to the previously used coarse-fine search method. In 1998, much refinement and optimization of the basic algorithms to increase the robustness of convergence had been carried out by Vendroux and Knauss (1998). Nonetheless, the equations involved are still too complex and required long running time in order to determine the results.

However, Hovis (1989) developed an algorithm known as Centroidal Tracking algorithm which performed the two-dimensional full-field deformation measurements by tracking the displacement of the speckles' centroid. By using this method, the interpolation procedures in smoothing the intensity values of the subset in the reference and the deformed images were avoided. Besides, the initial estimations of the displacement values, u, v and the correlation coefficient were no longer be necessary as the coordinates of the speckles in the deformed images were determined using the centroidal tracking algorithm. In his work, the Geometric Approach equation (Glover and Jones, 1994) was applied in order to calculate the strain components and is expressed in the form:

$$\frac{1}{2}\varepsilon^2 + \varepsilon = \varepsilon_{xx}\cos^2\theta_x + \varepsilon_{yy}\cos^2\theta_y + 2\varepsilon_{xy}\cos\theta_x\cos\theta_y \quad (1)$$

Hovis *et al.* (1999) adopted the simplicity of the geometric approach concept and developed a compact instrument for strain measurement in a variety of environment. In their work, Microsoft Visual C++ software was used to load pairs of images manually and the strain values were determined successfully. In contrast, the presented algorithm was limited by manual method of measuring the two points. Besides, the video camera used to capture images during the strain inducing event is expensive. Therefore, it is always desirable if the DIC technique can be simplified further without compromising its accuracy. The objective of this study is to develop a measurement algorithm that calculates the strain in a loaded structural component. This can be achieved by comparing the displacement of the selected speckles on the deformed and undeformed images with the aid of MATLAB program where the Geometric Approach equation was embedded. In order to improve the robustness of this technique, a consumer version of high-definition video camera was used to capture images during the experiment. The scope of this study consists of two main parts:

- Development of a measurement algorithm by using image processing toolbox in MATLAB software
- Validation of the optical strain measurement method with the experimental results

MATERIALS AND METHODS

Development of algorithm: Detailed algorithms were developed by using image processing toolbox in MATLAB software and these algorithms were divided into a total of eight main parts. First, it starts with the input of the two images that are acquired at two different states, one before deformation and the other one after deformation. The dimensions of the images in x and y directions are entered in the unit of pixel/mm. Next, the red, green and blue (RGB) images (Abbas *et al.*, 2001) captured earlier are converted into the binary format and the images' features are enhanced according to the light intensity. By selecting the four speckles within the two images respectively, the program will extract the speckles from the random speckle pattern. After that, the centroids of the speckles in term of x and y pixel-coordinate are obtained and the selected speckles are labelled according to the number in sequence. The pixel-coordinate is further converted into the unit of millimeter. Lastly, the embedded Geometric Approach equation in the program will perform the calculation and display the strain values in the MATLAB command windows. Since the surface deformation is studied in this work, only the normal strains, ε_{xx} and ε_{yy} and the shear strain, ε_{xy} are calculated by the MATLAB program.

Samples preparation and the experimental set-up: Two types of samples made of mild steel and polypropylene were fabricated in accordance to the ASTM E8M and the ISO 527-1, respectively. In order to create the random speckle pattern on the mild steel and the polypropylene samples, black paint was deposited on the surface of the specimens. The entire experimental set-up such as the position of the video camera, light sources and the specimens were carefully arranged as shown in Fig. 1. For example, a series of alignments checking were carried out to ensure that the video camera was installed correctly as the tripod can be rotated in yaw, pitch and roll-axis. The right angle between the video camera and the specimen was checked by installing a mirror in the middle of the Universal Testing Machine (UTM) grippers. By referring to the reflected image captured by the video camera as shown in Fig. 2, the right angle between the video camera and the specimen was achieved if the center of the video camera's lens coincided with the cross mark generated in the video camera screen. This method was found to be extremely sensitive in detecting out-of-plane displacement. Besides, the alignment of the specimen in the vertical axis was verified by placing a spirit level alongside the specimen during the installation of the specimen into the middle of the UTM grippers.

Tensile test experiment: According to ASTM E8M and ISO 527-1, at least five specimens each for mild steel and polypropylene were tested using the Universal Testing Machine (Zwick/Roell Amsler HA 100). The extensometer was mounted on the specimen and the results obtained served as a benchmark for values determined from the optical strain measurement method. The cross-head

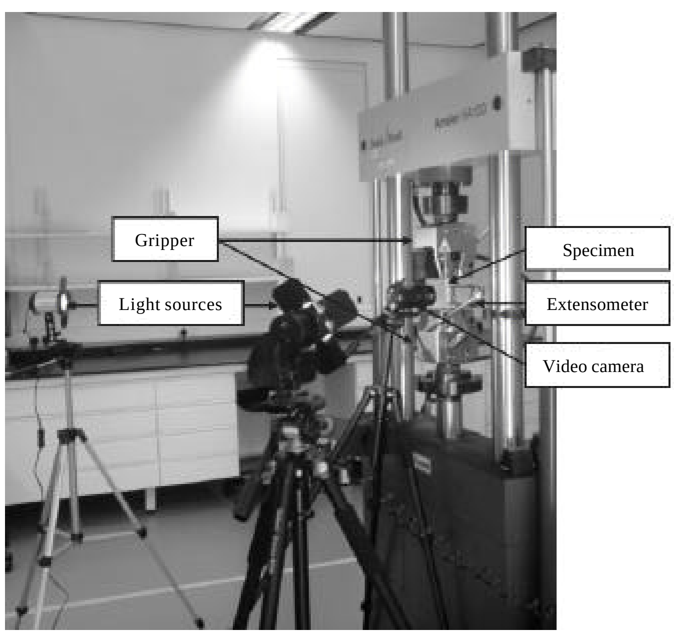

Fig. 1: Experimental set-up for the tensile test

Fig. 2: Mirror image used to detect out-of-plane displacement

speed was set to the rate of 0.02 mm sec^{-1} (Savic *et al*., 2010). At the same time, images were captured during the strain-inducing event by using a consumer version of high-definition video camera. By using the consumer version of high-definition video camera, more than 650 pairs of images that were acquired at the undeformed and the deformed states were extracted from the recorded videos. These extracted images were then processed by using the MATLAB program and the normal strains, ε_{xx} and ε_{yy} and the shear strain, ε_{xy} were determined. From the determined results, only the normal strain, ε_{yy} was selected and compared with the benchmark values as the extensometer only provides the normal strain values in y direction (ε_{yy}).

RESULTS AND DISCUSSION

Validation of the optical strain measurement method with the experimental results

Mild steel: A stress-strain curve for the mild steel specimen was plotted as in Fig. 3. Based on Fig. 3, the strain values determined from the extensometer readings (solid line) served as the benchmark for the strain values determined by the optical strain measurement method (dotted points). The results for the strain values determined by the extensometer were only up to 0.18 mm mm^{-1}. This is because the extensometer was uninstalled from the specimen as the elongation was about to exceed the extensometer's measuring range.

By referring to Fig. 3, the strain values determined by the extensometer and the optical strain measurement method were close to each other. The yield point phenomenon was observed in the transition from the elastic to the plastic deformation. As shown in Fig. 3, the stress increased linearly in the elastic region, followed by a sudden drop at the upper yield point. Next, the stress fluctuated around a constant value (lower yield point) and then rose when the strain was approaching 0.024 mm mm^{-1}. This phenomenon occurred due to the dislocation motion in the

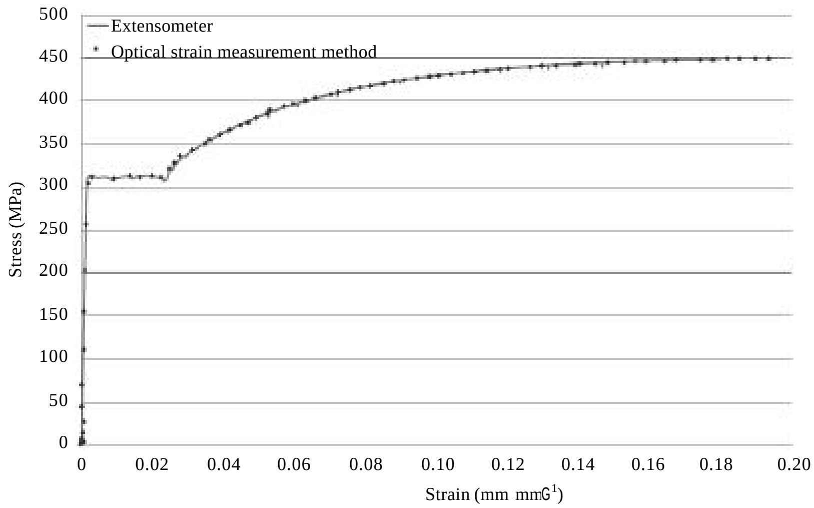

Fig. 3: Stress-strain curve for mild steel specimen

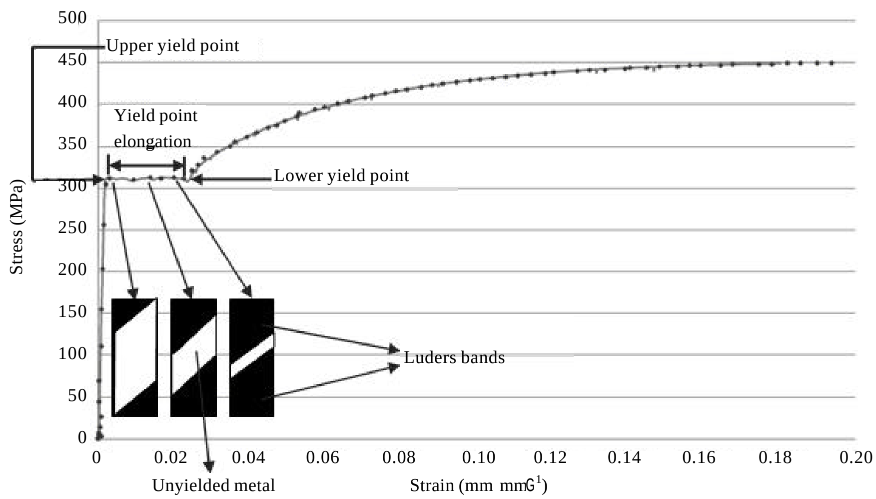

Fig. 4: Yield point phenomenon of mild steel specimen

low-carbon steel such as the one used in this study. As the small interstitial atoms in the material interfered with the dislocation slip plane, the stress was increased to the upper yield point. Once the dislocations plane started to slip, the stress was lowered to the lower yield point (Askeland and Phule, 2003). Besides, the Luders Bands were observed throughout the yield point elongation and approximately 45° to the tensile axis (Davis, 2004). In this study, the complete yield point phenomenon was recorded by the video camera and the Luders Bands observed were shown in Fig. 4. In general, the stress value started to increase once the Luders Bands have propagated over the entire gauged section.

Since the deformations that occurred throughout the yield point elongation were reported to be heterogeneous or irregular (Parker and Sikorsky, 2002), the selection of the speckles during the execution of the MATLAB program was different than before. The four speckles have to be selected within the Luders Bands rather than from the random speckles pattern. This is because no deformations occurred within the unyielded metal section. Despite of the yield point phenomenon, the optical strain measurement method was still able to determine the deformations that occurred in the specimen. Based on the results determined by the optical strain measurement method, the deviations were found to be in the range of minimum 4.8% to the maximum of 10.8% off the benchmark values.

When the strain value was about to exceed the lower yield point or to be exactly 0.024 mm mm^{-1}, the deviations between the results determined by the extensometer and the optical strain measurement method were found to be very small. For example, when the strain values determined by the extensometer were in between 0.03 and 0.07 mm mm^{-1}, the results determined by the optical strain measurement method were in the range of minimum 0.2% to the maximum of 2.8% off the benchmark values. Besides, the deviations of the results determined by the optical strain measurement method were less than 5.0% off the benchmarks values when the strain values determined by the extensometer were below 0.12 mm mm^{-1}. However, the deviations for the results determined by the extensometer and the optical strain measurement method were found to be increasing when the strain values were further increased. Based on Figure 3, the results determined by the optical strain measurement method were in the range of minimum 5.0% to the maximum of 7.9% off the benchmark values when the strain values determined by the extensometer were in between 0.12 and 0.18 mm mm^{-1}.

In fact, the deviations were expected as the Geometric Approach equation embedded in the MATLAB program contributed to the deviations. This is because the term $\varepsilon^2/2$ in the Geometric Approach equation was found to be contributing a small value even though it was treated as negligible when the relative change in distance between the two nodes, ε is small (Glover and Jones, 1994). From the comparison of the results determined by the extensometer and the optical strain measurement method, the small deviations proved that the optical strain measurement method is capable to determine the strain values correctly. Besides, a good agreement was achieved upon comparison with the benchmark values especially when the strain values were above 0.024 mm mm^{-1}.

Polypropylene: Similarly, a stress-strain curve for the polypropylene specimen was plotted as in Fig. 5. By referring to Fig. 5, the strain values determined from the extensometer readings (solid line) served as the benchmark for the strain values determined by the optical strain measurement method (dotted points). The results for the strain values determined by the extensometer were only up to 0.16 mm mm^{-1}. This is because the extensometer was uninstalled from the specimen once the necking occurred.

By referring to Fig. 5, the strain values determined by the extensometer and the optical strain measurement method were extremely close to each other. When the strain values were in between the elastic limit and the ultimate tensile strength, the deviations between the results determined by the extensometer and the optical strain measurement method were found to be small. The results determined by the optical strain measurement method were in the range of minimum 0.2% to the maximum of 3.6% off the benchmark values. Besides, the deviations of the results determined by the optical strain measurement method were found to be less than 5.0% off the benchmark values when the strain values determined by the extensometer were below 0.14 mm mm^{-1}.

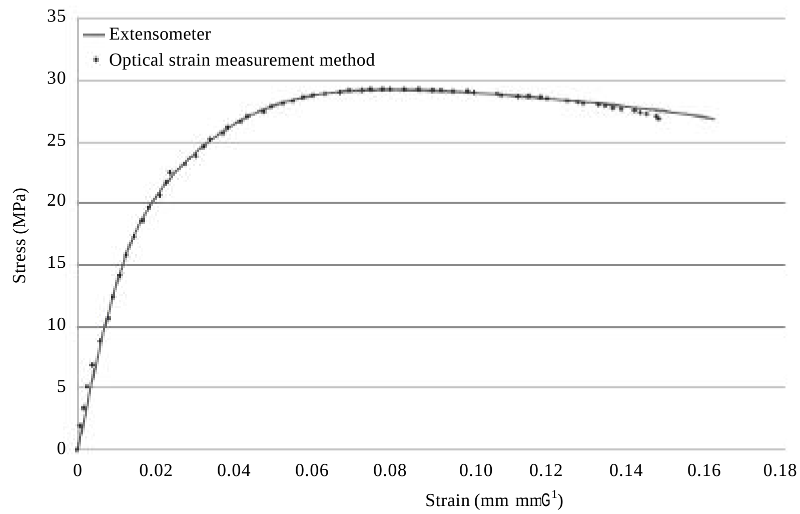

Fig. 5: Stress-strain curve for polypropylene specimen

However, the deviations of the results determined by the two methods mentioned above were higher when the strain values were increased. By referring to Fig. 5, the results determined by the optical strain measurement method were in the range of minimum 5.0% to the maximum of 8.6% off the benchmark values when the strain values were in between 0.14 and 0.16 mm mm^{-1}. As described earlier in the mild steel section, the deviation of 8.0% off the benchmark value was expected to occur when the strain value was approaching 0.16 mm mm^{-1}. However, by referring to Fig. 5 once again, the deviations that occurred were different if compared with the stress-strain curves for the mild steel specimen as shown in Fig. 3. As shown in Fig. 3, the results determined by the optical strain measurement method deviated as much as 8.0% more than the benchmarks values. But according to Fig. 5, the results determined by the optical strain measurement method were found to be 8.6% less than the benchmark values. The deviations that occurred in the polypropylene specimens were different as necking was found to be the factor which contributed to the errors. By referring to Fig. 6, the necking occurred at the mounting points between the extensometer and the specimen.

Throughout the tensile tests for the five polypropylene specimens, the necking was observed at the same position as shown in Fig. 6 when the strain value determined by the extensometer was approaching 0.14 mm mm^{-1}. From this observation, it proved that the extensometer has introduced the stress concentrations on the mounting points and created premature necking to the specimens. Since the necking occurred in one of the extensometer's clips, the extensometer was still able to measure the elongation and hence compute the strain values successfully. However, the strain values determined by the optical strain measurement method started to decrease when the benchmark value was approaching 0.14 mm mm^{-1}. This is because the video camera was set to capture the middle of the gauged section of the specimen during the tensile test. Since the elongation occurred mostly in the extensometer mounting point and not in the middle of the gauged section, the captured images were unable to record the elongation correctly. As a result, the strain values determined by the optical strain measurement method were smaller and 8.6% off the benchmark values when the strain value was approaching 0.16 mm mm^{-1}.

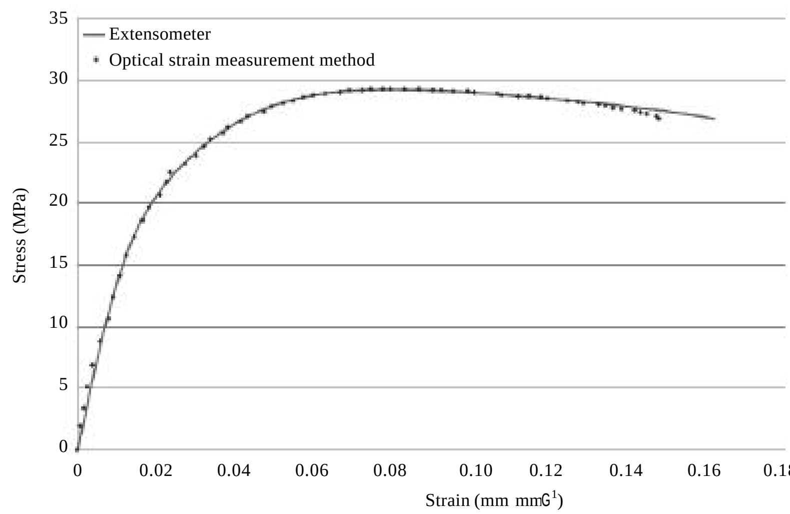

Fig. 6: Necking observed in polypropylene specimen

From the comparison of the results determined by the extensometer and the optical strain measurement method, the small deviations proved that the optical strain measurement method is capable to determine the strain values correctly. Besides, a good agreement was achieved upon comparison with the benchmark values especially when the strain values were above 0.008 mm mm^{-1}. Although some deviations were still observed during the comparison of the results determined by the two methods mentioned above, they were within the acceptable limit. Through the comparison between the results determined by the extensometer and the optical strain measurement method, the deviations will start to increase once the necking was found to occur in the specimens.

Comparison of the specimens' material properties

Mild steel: In order to find the modulus of elasticity of the mild steel specimens, the gradient of the stress-strain curve was plotted as in Fig. 7. The strain values determined from the extensometer readings (triangle dotted points) served as the benchmark for the strain values determined by the optical strain measurement method (diamond dotted points). Meanwhile, two straight lines which best fitted the dotted points were plotted and the modulus of elasticity of the mild steel specimen was successfully determined.

Table 1 shows the comparison of the modulus of elasticity determined by the extensometer and the optical strain measurement method for all five specimens that have been tested in the tensile test. The average value of the modulus of elasticity determined by the extensometer was 204 GPa. While for the average value of the modulus of elasticity determined by the optical strain measurement method was found to be 193 GPa. As a result, the average deviation was found to be 5.1% off the benchmark value.

Polypropylene: The gradient of the stress-strain curve for the polypropylene specimen as in Fig. 8 was plotted in order to determine the modulus of elasticity of the tested specimen. The strain values determined from the extensometer readings (triangle dotted points) served as the benchmark

Table 1: Comparison of the modulus of elasticity determined by the extensometer and the optical strain measurement method

Specimens	Extensometer (GPa)	Optical strain measurement method (GPa)	Deviation (%)
1	206	189	8.3
2	218	201	7.8
3	211	207	1.9
4	190	185	2.6
5	195	185	5.1
Average	204	193	5.1

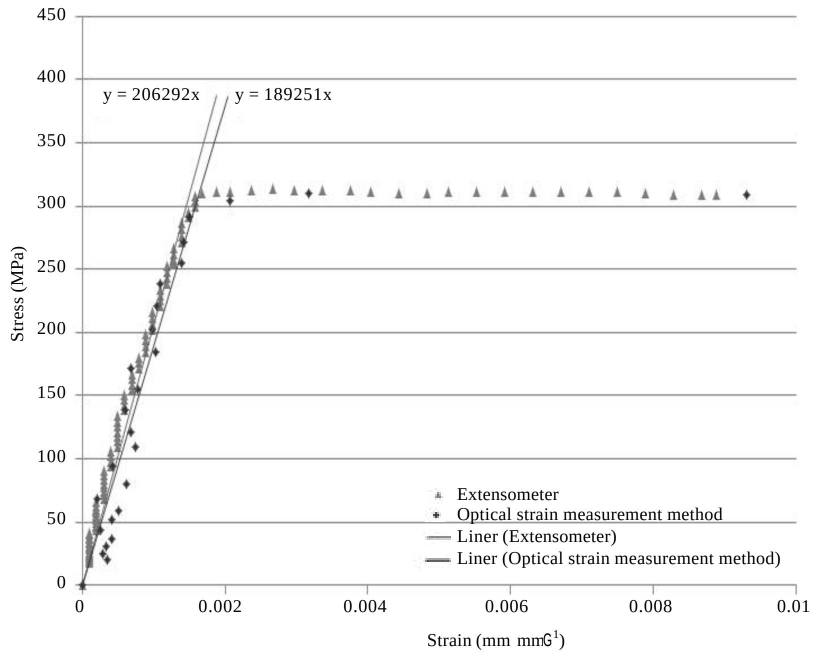

Fig. 7: Determination of the modulus of elasticity for the first mild steel specimen

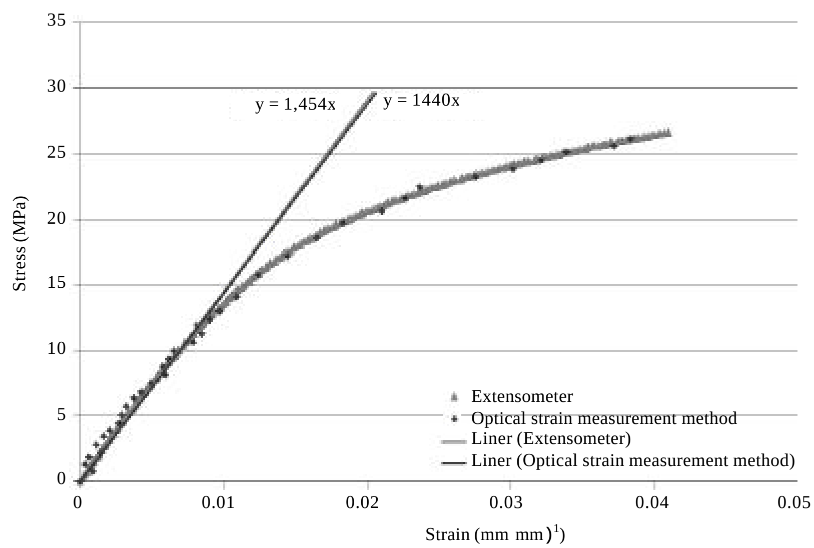

Fig. 8: Determination of the modulus of elasticity for the first polypropylene specimen

Table 2: Comparison of the modulus of elasticity determined by the extensometer and the optical strain measurement method

Specimens	Extensometer (MPa)	Optical strain measurement method (MPa)	Deviation (%)
1	1454	1440	1.0
2	1628	1509	7.3
3	1561	1488	4.7
4	1544	1542	0.1
5	1476	1504	1.9
Average	1533	1497	2.4

for the strain values determined by the optical strain measurement method (diamond dotted points). Meanwhile, all the dotted points were best fitted with the straight lines. As a result, the modulus of elasticity of the polypropylene specimen was determined.

Table 2 shows the comparison of the modulus of elasticity determined by the extensometer and the optical strain measurement method for all five specimens that have been tested in the tensile test. Based on Table 2, the average value of the modulus of elasticity determined by the extensometer was 1533 MPa. While the average value of the modulus of elasticity determined by the optical strain measurement method was found to be 1497 MPa. As a result, the average deviation was found to be 2.4% off the benchmark value. Besides, the straight lines plotted nicely fitted the results determined by the optical strain measurement method (diamond dotted points). By referring to the comparison of the modulus of elasticity determined by the two methods mentioned above, the optical strain measurement method had been verified and a good agreement was achieved as the deviations were found to be 2.4% off the benchmark value.

CONCLUSION

The Geometric Approach equation has been successfully embedded in the MATLAB program and used to calculate the strain in a loaded structural component. The optical strain measurement method in this work has successfully determined the strain values in mechanical testing of the mild steel and the polypropylene specimens. The obtained strain values were further compared with the benchmark values set by the extensometer. The optical strain measurement method was found to be capable to determine the strain correctly compared to extensometer readings with very small deviation. A good agreement was achieved upon comparison with the benchmark values especially when the strain value was above 0.008 mm mm^{-1}. In determining the modulus of elasticity, the optical strain measurement method has been verified to give good agreement with the results obtained from extensometer readings. In conclusion, the two-dimensional measurement algorithm that calculates the strain in a loaded structural component had been developed successfully in the study. In addition, the consumer version of high-definition video camera has been proven to be capable to capture the images of deformed specimens. Future work will include the automation of the image processing technique.

ACKNOWLEDGMENT

The authors would like to acknowledge the financial support from Universiti Teknologi PETRONAS for this work.

REFERENCES

Abbas, Y., K. Alsultanny and N. Shilbayeh, 2001. Applying popularity quantization algorithms on color satellite images. J. Applied Sci., 1: 530-533.

Askeland, D.R. and P.P. Phule, 2003. Mechanical Properties and Behaviour, The Science and Engineering of Materials. 4th Edn., Thomson Learning, California.

Bruck, H.A., S.R. McNeill, M.A. Sutton and W.H. Peters, 1989. Digital image correlation using Newton-Raphson method of partial differential correction. Exp. Mech., 29: 261-267.

Chu, T.C., W.F. Ranson and M.A. Sutton, 1985. Applications of digital image correlation techniques to experimental mechanics. Exp. Mech., 25: 232-244.

Davis, J.R., 2004. Mechanical Behaviour of Materials Under Tensile Loads, Tensile Testing. 2nd Edn., ASM International, Ohio.

Glover, C. and H. Jones, 1994. Conservation Principles of Continuous Media. 3rd Edn., McGraw Hill, New York.

Hovis, G.L., 1989. Centroidal tracking algorithm for deformation measurements using gray scale digital images. Ph.D. Thesis, Department of Mechanical Engineering University of South Carolina.

Hovis, G.L., W.F. Ranson, H.J. Reed and C.T. Etheredge, 1999. Vision system for remote strain or deformation measurement. Proceedings of the American Nuclear Society 8th Topical Meeting on Robotics and Remote Systems, April 25-29, 1999, Pittsburgh, PA., USA.

Jin, H., W.Y. Lu and J. Korellis, 2008. Micro-scale deformation measurement using the digital image correlation technique and scanning electron microscope imaging. J. Strain Anal., 43: 719-728.

Khan, A.S. and X.W. Wang, 2001. Strain Measurements and Stress Analysis. Prentice Hall, New Jersey.

Parker, C.A. and P.J. Sikorsky, 2002. Atlas of Stress-Strain Curves. 2nd Edn., ASM International, Ohio.

Peter, W.H. and W.F. Ranson, 1982. Digital imaging technique in experimental stress analysis. Opt. Eng., 21: 427-431.

SPOTS Standard, 2005. Good practice guide to geometric moire for in plane displacement or strain analysis. Technical Report, G6RD-CT-2002-00856.

Savic, V., L.G. Hector Jr. and J.R. Fekete, 2010. Digital image correlation study of plastic deformation and fracture in fully martensitic steels. Exp. Mech., 50: 99-110.

Sutton, M.A., 2008. Digital Image Correlation for Shape and Deformation Measurements. In: Handbook of Experimental Solid Mechanics, Sutton, M.A. (Ed.). Springer, USA., ISBN: 978-0-387-26883-5, pp: 565-600.

Sutton, M.A., W.J. Wolters, W.H. Peters, W.F. Ranson and S.R. McNeill, 1983. Determination of displacements using an improved digital image correlation method. Image Vision Comput., 1: 133-139.

Sutton, M.A., C. Mingqi, W.H. Peters, Y.J. Chao and S.R. McNeill, 1986. Application of an optimized digital correlation method to planar deformation analysis. Image Vision Comput., 4: 143-150.

Vendroux, G. and W.G. Knauss, 1998. Submicron deformation field measurements: Part II. Improved digital image correlation. Exp. Mech., 38: 86-92.

Wang, C.C., J.M. Deng, G.A. Ateshian and C.T. Hung, 2002. An automated approach for direct measurement of two-dimensional strain distributions within articular cartilage under unconfined compression. J. Biomech. Eng., 124: 557-567.