PAPR Reduction in OFDM systems using Quasi Cyclic LDPC Codes

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ABSTRACT
Orthogonal Frequency Division Multiplexing (OFDM) is a multi carrier modulation technique where the revolution of 4G wireless communication is focused towards OFDM systems. The major drawback of OFDM system is high Peak to average power ratio. The proposed study is based on Peak to Average Power Ratio (PAPR) reduction by the implementation of Quasi Cyclic Low Density Parity Check Codes (QCLDPC). Comparison of PAPR is carried out for LDPC and turbo codes. Simulation results show that the QCLDPC codes give a better reduction of PAPR when compared to LDPC and turbo codes.

Key words: Orthogonal frequency division multiplexing, low density parity check code, quasi cyclic LDPC, peak to average power ratio, quadrature amplitude modulation, quadrature phase shift keying

INTRODUCTION
Nowadays the wireless applications are focused towards high data rates. The concept of multi carrier transmission provides high data rates in communication channel. The OFDM is a special kind of multi carrier transmission technique that divides the communication channel into several equally spaced frequency bands. Here the bit streams are divided into many sub streams and send the information over different sub channels. A sub-carrier carrying the user information is transmitted in each band. Each sub carrier is orthogonal with other sub carrier and it is carried out by a modulation scheme. Data's are transmitted simultaneously in super imposed and parallel form. The sub carriers are closely spaced and overlapped to achieve high bandwidth efficiency (Thenmozhi et al., 2012). The main disadvantage of OFDM is high peak to average power ratio. The peak values of some of the transmitted signals are larger than the typical values (Foomooljareon and Fernando, 2002). High PAPR of the OFDM transmitted signals results in bit error rate performance degradation, inter modulation effects on the sub carriers, energy spilling into adjacent channels and also causes non linear distortion in the power amplifiers. The main work of this study was to reduce the high peak powers in OFDM systems. Several PAPR techniques like clipping, selective mapping, partial transmit sequence, tone reservation and tone injection are there to reduce high peak signals (Tarokh and Jafarkhani, 2000). In this study, the concept of coding technique is applied to the OFDM symbols to reduce high peak signals. Coding techniques not only applicable for reducing the PAPR in OFDM systems but also it is well suited for error correcting performances. The literature survey defines the usage of low density parity check codes and turbo codes (Daoud and Alami, 2009) for PAPR reduction. These two codes show that the theoretical limit
values attain closer to the Shannon limit and performs good role in the PAPR reduction of OFDM systems (Sharma and Verma, 2011). The proposed work is based on the utilization of quasi cyclic LDPC codes. Quasi cyclic structure allows parallel encoding and decoding which acts as a tradeoff between encoding complexity and encoding speed. The memory requirement of QCLDPC is very small and it can solve the memory problem due to their linear time encodability (Yahya et al., 2010). The encoding procedure is carried out for LDPC, turbo and QCLDPC in OFDM transmitter section. Peak to average power reduction ratio is calculated and compared with all the three codes (Velmurugan et al., 2010). The power signals of all the above codes are viewed in Complementary Cumulative Distribution Function (CCDF) plot. The results state that the utilization of QCLDPC codes attains a good PAPR reduction and the encoding complexity is reduced when compared to LDPC and turbo codes.

LOW DENSITY PARITY CHECK CODES

LDPC code is a type of linear block codes. The structure of LDPC is entirely expressed by the parity check matrix \( H \) where \( H \) is a sparse (ie) the matrix mostly consists of 0’s and few 1’s. The sparse is a M×N parity check matrix where N>M and M = N-K. The message bits are said to be ‘M’, the parity bits are said to be ‘K’ and ‘N’ defines the total number of bits in the encoded data (ie) (M+K). There are two classes of LDPC. One is regular and another is irregular LDPC. This study deals with regular LDPC, in that the rows and columns of \( H \) have the uniform weights. The size of the parity check matrix is \( P_1\times P_2 \), where \( P_1 \) represents the size of the row and \( P_2 \) defines the size of the column in parity check matrix. The number of 1’s in a row is stated as row weight \( w_r \) and the number of 1’s in a column is represented as column weight \( w_c \), where \( Z \) is a zero matrix, \( I_1 \), and \( I_2 \) are said to be identity matrix, \( g \) is a gap matrix used to change the matrix into upper triangular matrix. Then \( g = \gamma b \) where \( \gamma \) is the total number of blocks in the ‘g’ sub matrix. The row and column weight distribution are \( \{w_1, w_2, ..., w_n\} \) and \( \{w_1, w_2, ..., w_n\} \) where \( w_1 \) and \( w_2 \) represents the weight of the ith block rows and jth columns respectively. Two different set of weight distribution have been generated for the matrix columns and rows (Di et al., 2006).

\[
\{a_1, a_2, ..., a_n\}, \text{ where } a_j = w_a, 1 \leq j \leq (n-m+\gamma):
\]

\[
w_a,-1 \leq (n-m+\gamma) \leq j \leq n
\]  

\[
\{b_1, b_2, ..., b_n\}, \text{ where } a_i = w_m, 1 \leq i \leq (n-\gamma).
\]

\[
w_m, (m-\gamma+1 \leq i \leq m)
\]

Here we are considering (3, 6) parity check matrix, where \( P_1 = 6, P_2 = 12 \) and the gap value is \( g = 2 \):

\[
H = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1
\end{bmatrix}
\]
Normally the LDPC codes are represented using bipartite and Tanner graph (Ahmadi et al., 2012). But by using the method of back substitution in this LDPC construction, the encoding complexity is reduced by transforming the parity check matrix into upper triangular form (Fig. 1). The idea is to do as much of the transformation as possible by only row and column permutations to keep the 'H' as sparse. The 'H' matrix is sub divided into six blocks are shown below, using the blocks the message signals are encoded (Richardson and Urbanke, 2001).

$$H = \begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}$$

The different block sizes in 'H' are designed by the following procedure:

- $A = [P_1 \cdot g] \times [P_2 \cdot P_1] = [6-2] \times [12-6] = 4 \times 6$ (4 rows 6 columns)
- $B = [P_1 \cdot g] \times g = [6-2] \times 2 = 4 \times 2$ (4 rows 2 columns)
- $C$ block is an upper triangular matrix where, $C = C^{-1}$
- $C = [P_1 \cdot g] \times [P_2 \cdot P_1] = [6-2] \times [6-2] = 4 \times 4$ (4 rows 4 columns)
- $D = g \times [P_2 \cdot P_1] = 2 \times [12-6] = 2 \times 6$ (2 rows 6 columns)
- $E = g \times g = 2 \times 2$ (2 rows 2 columns)
- $F = g \times [P_1 \cdot g] = 2 \times [6 - 2] = 2 \times 4$ (2 rows 4 columns)

The block structure of 'H' is given as:

$$H = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
\end{bmatrix}$$

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The generated code word is obtained by $[S_i, R_i, R_2]$

- $S_i$ = Information bit
- $R_i$ = Parity bit, calculated using the formula
  \[ R_i = FCAS_i^T \]
- $R_2$ = Parity bit, calculated using the formula
  \[ R_2 = [C (AS_i)^T + BR_i^T] \]
- Let the information bit be $S_i = (1, 0, 0, 0, 0, 0)$
- $R_1 = FCAS_i^T = (0, 1)$
- $R_2 = [C (AS_i)^T + BR_i^T] = (1, 0, 1, 0)$
- Encoded data $= [S_i, R_1, R_2]$
- Encoded data $= [1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0]$
- $S_iR_1R_2$

**PROPOSED TECHNIQUE**

Figure 2 represents the transmitter block diagram of the proposed system. Random bits are given as an input from the serial to parallel converter. The serial data is converted into parallel data and the inputs are sending to the QC-LDPC encoder.

**QC-LDPC ENCODER**

The encoding of LDPC is based on parity check matrix ‘H’ where the QC-LDPC code can be constructed by applying circulant matrices (Spagnol and Marnane, 2009). The circulant matrix ‘M’ is shown in Fig. 3. Based on the circulant matrix the parity check matrix ‘H’ is subdivided into two matrix maintaining the equal row and column length (Honary et al., 2005):

![Transmitter block diagram](image)

Fig. 2: Transmitter block diagram of the proposed system, QAM: Quadrature amplitude modulation, IFFT: Inverse fast Fourier transform

![General circulant matrix structure](image)

Fig. 3: General circulant matrix structure
The circulant concept in ‘H’ matrix defines that each row in the above two matrix is one time right cyclic shift of the previous one, since each column and row is a shift of the previous column and row, where the column and row weight is uniform for all the shift in the ‘H’ circulant matrix (Spagnol and Marmane, 2006). The generator matrix is obtained while defining the ‘H’ matrix as $GH^T = 0$. The bits are encoded through the generator matrix and the codeword should obey the property $CH = 0$, where ‘C’ is a codeword. The explanation for the shifting is given below:
The above operation is done for all the rows of matrix 1 and matrix 2. After all the shifting, the obtained weights of the matrix is:

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 \\
\end{bmatrix}
\]

The circulant parity check matrix is sub divided into six block matrix namely A, B, C, D, E and F. The size of each block matrix gets varied. Except the circulant shift procedure in QCLDPC all the encoding procedures (i.e., Block size calculation) are same as in the LDPC. The obtained block structure of QCLDPC is given below:

Let the information data S1 = (0, 1, 1, 0, 1, 0):

- The parity bit R1 and R2 are calculated using the formula
- R1 = [FCAS,7] = [0 0]
- R2 = [C (AS,7) + BR,7] = [0 0 1 0]
- The encoded data = [S1 R1 R2] = [0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0]

The encoded data from QCLDPC encoder is mapped using 16QAM for each sub carrier at a low symbol rate maintaining better data rate. The purpose of using QAM is to increase the efficiency of transmission in both amplitude and phase variations of OFDM signals (Latif and Gohar, 2008). The output from the mapping is sent to the N point IFFT where the frequency domain data is converted to time domain data. Here the ‘N’ defines the number of sub carriers in
the system. The signal from N point IDFT is converted into serial data using parallel to serial converter. Finally the transmitted OFDM signal is generated.

PEAK-TO-AVERAGE POWER RATIO

In presence of large number of independently modulated sub-carriers in OFDM systems, the peak value of the some signals can be very high as compared to the average of the whole system. The complex envelope of an OFDM signal is an overlap of N complex oscillations with different frequencies, phases and amplitudes. As a result, we get a time domain signal with high Peak to Average Power Ratio. These peaks may cause signal clipping at high levels and may force the amplifier in the transmitter side to work in the non linear region, thereby producing frequency components in addition to the original and results in out of band radiation. The major concept of this paper is to reduce the high peak value before the transmission is carried out (Tarokh and Jafarkhani, 2000). The ratio of the peak to average power value is termed as Peak-to-Average Power Ratio. Mathematically PAPR can be given as:

$$\text{PAPR} = \frac{\max |x(t)|^2}{E[|x(t)|^2]}$$

where, \(\max |x(t)|^2\) is the peak signal power and \(E[|x(t)|^2]\) is the average signal power.

The average power is calculated using the formula:

$$\text{Average power} = \frac{\text{Sum of magnitude of all the symbols}}{\text{No. of symbols}}$$

The Complementary Cumulative Distribution Function (CCDF) of the PAPR is one of the most frequently used method to check how often the PAPR exceed the threshold values (Vimal and Kumar, 2011). Graph is plotted among threshold and CCDF values. The CCDF can be calculated by the relation \(P(\text{PAPR}>X)=1-P(\text{PAPR}<X)\). The fixations of threshold value range from zero to maximum value. The formula for calculating the threshold value is:

$$\text{Threshold} = \frac{0.1(\text{Maximum PAPR} - \text{Minimum PAPR})}{\text{Maximum PAPR} - \text{Minimum PAPR}}$$

Let the maximum value be 10, minimum value be 5. Therefore:

$$\text{Threshold} = 0: (10-5)/10:10 = 0: 0.5:10$$

Then the threshold values are 0, 0.5, 1, 1.5 ... 10.

ALGORITHM FOR THE PROPOSED WORK

This algorithm has following steps:

Step 1: Start the program
Step 2: Generate the input bits randomly
Step 3: Convert the serial data into parallel data
Step 4: Construct the sparse H matrix for different coding rate
Step 5: Shift the sparse H matrix (Circulant Procedure)
Step 6: Encode the input bits
Step 7: Modulate the input signals using QAM 16 and QPSK modulation
Step 8: Compute IFFT for the mapped sequence
Step 9: Convert the parallel data into serial bits
Step 10: Calculate the PAPR value
Step 11: Determine the threshold value
Step 12: Check whether PAPR > threshold value
Step 13: Draw the CCDF plot i.e. threshold (vs) probability of PAPR
Step 14: Compare the result with no coding, QCLDPC, LDPC and turbo codes for different Coding rates and spreading rates
Step 15: Stop the program

SIMULATION AND RESULTS

In this study the simulation was carried out by using MATLAB 7.6 software. Randomly generated input data sequence is uniformly distributed. The study is carried out for 1/2 and 1/3 coding rates with two different spreading rates (I = 2 and 3) with a generator polynomial $g = [1 1 1; 1 0 1]$ and the modulation technique used in this work is QAM 16 and QPSK. The simulation includes for the following values $n = 128$, $m = 64$, $\gamma = 6$ and $d_{min} = 8$. Hundred signals are considered for calculating the average power and PAPR. Simulation result shows the CCDF plot for PAPR reduction using LDPC, QCLDPC, turbo codes and PAPR with no compensation (no coding).

Figure 4 shows the CCDF plot for 1/2 coding rate with spreading rate 2, there is a 3.29 dB reduction for QCLDPC. There is a difference of 0.13 dB when compared to LDPC and 0.55 dB for turbo codes. Figure 5 states that the complexity of the OFDM system decreases as the PAPR values get closer to the LDPC results for the coding rate 1/2 with spreading rate 3. Figure 6 shows the simulation results for 1/3 coding rate with spreading rate 2. In that there is a 3.59 dB reduction for QCLDPC and 4.25 dB reductions for LDPC and 3.9 dB for Turbo codes. The simulation results for

![CCDF Plot of PAPR reduction with QAM 16 modulation (Coding rate '1/2', Spreading rate '2')](image)

Fig. 4: CCDF Plot of PAPR reduction with QAM 16 modulation (Coding rate ‘1/2’, Spreading rate ‘2’)

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Fig. 5: CCDF Plot of PAPR reduction with QAM 16 modulation (Coding rate ‘1/2’, Spreading rate ‘3’)

Fig. 6: CCDF Plot of PAPR reduction with QAM 16 modulation (Coding rate ‘1/3’, Spreading rate ‘2’)

<table>
<thead>
<tr>
<th>Table 1: Comparison of PAPR reduction values by using LDPC, QC-LDPC and turbo with PAPR (No coding)</th>
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<tr>
<td>Mapping</td>
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<tr>
<td>16 QAM</td>
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<td>QPSK</td>
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1/3 coding rate with spreading rate 3 in Fig. 7 demonstrates that nearly 1 dB PAPR difference attains from turbo to QC-LDPC and 0.63 dB from LDPC to QC-LDPC codes. Comparison for obtained PAPR reduction values for the above codes is given in Table 1.
Fig. 7: CCDF Plot of PAPR reduction with QAM 16 modulation (Coding rate ‘1/3’, Spreading rate ‘3’)

The comparison Table 1 states the PAPR reduction by applying Quadrature phase shift keying (QPSK) and 16 QAM modulation techniques in OFDM systems. The utilization of 16 QAM modulation techniques attains a good PAPR reduction when compared to QPSK. The simulation results and comparison table defines that LDPC codes shows good PAPR reduction when compared to turbo codes and QCLDPC codes gives better reduction when compared to LDPC and turbo codes.

CONCLUSION

The QCLDPC codes have been used to reduce the PAPR effectively. The required memory size for storing the parity check matrices in QCLDPC codes can be reduced by the utilization of circulant matrix. The advantages of QCLDPC code in OFDM systems is that there is no need to store the full \(H\) matrix since tail bits are not required for coding scheme where it provides additional bits for data transmission. The above work can be improved by using different LDPC, QCLDPC decoding algorithms in the receiver side to calculate the bit error rate of the OFDM systems. Further the work can be extended by increasing the coding and spreading rates with different modulating schemes.

REFERENCES


