Asian Journal of Scientific Research

ISSN 1992-1454

OPEN ACCESS Asian Journal of Scientific Research

ISSN 1992-1454
DOI: 10.3923/ajsr.2017.169.177

Research Article

Exponential Inverse Exponential (EIE) Distribution with Applications to Lifetime Data

[1]Pelumi Emmanuel Oguntunde, [1,2]Adebowale Olusola Adejumo and [1]Enahoro Alfred Owoloko

[1]Department of Mathematics, Covenant University, Ota, Ogun State, Nigeria
[2]Department of Statistics, University of Ilorin, Kwara State, Ilorin, Nigeria

Abstract

Objective: This study aimed to extend and generalize the Inverse Exponential (IE) distribution by using the exponential generalized family of distributions. **Materials and Methods:** The compound distribution was studied while the expressions for its densities and basic statistical properties were derived and established. It was applied to three real life data and a simulation study was also conducted. **Results:** The shape of the Exponential Inverse Exponential (EIE) distribution is unimodal (or inverted bathtub) and its densities are more tractable than that of the Beta Inverse Exponential distribution. It was discovered that the EIE distribution has a better fit than the baseline distribution in two of the three applications but when the data set is over-dispersed, it failed to perform better. **Conclusion:** The EIE is an improvement over the IE distribution except for cases with outliers. The parameters of the EIE are stable and the mean square error reduces as the sample size increases.

Key words: Distributions, inverse exponential, lifetime data, properties, simulation

Received: February 15, 2017 Accepted: May 17, 2017 Published: June 15, 2017

Citation: Pelumi Emmanuel Oguntunde, Adebowale Olusola Adejumo and Enahoro Alfred Owoloko, 2017. Exponential Inverse Exponential (EIE) distribution with applications to lifetime data. Asian J. Sci. Res., 10: 169-177.

Corresponding Author: Pelumi Emmanuel Oguntunde, Department of Mathematics, Covenant University, Ota, Ogun State, Nigeria
Tel: +234 806 036 9637

Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

The exponential distribution is a very simple and popular lifetime model but its inability to properly model real life phenomena whose failure rate are not constant led to several modifications and generalization of the exponential distribution[1].

An inverted version of the exponential distribution called the IE distribution has been introduced in the literature[2]. It has an inverted bathtub hazard rate and can be suitable for modeling real life phenomena with inverted bathtub failure rates. An illustration of its application to breast cancer data has also been discussed[1]. It has also been described as a model that is useful in survival analysis[3].

In recent years, the IE distribution has been generalized and extended to yield the Generalized Inverse Exponential (GIE) distribution[4] and the beta inverted exponential distribution[5].

The GIE distribution[4] introduced only one additional parameter to its baseline distribution. Its statistical properties were studied including the estimation of model parameters and its application was demonstrated using a real life data. It was discovered that the GIE distribution performed better than the IE distribution using the likelihood ratio test and the Kolmogorov Smirnov statistic as selection criteria. It was also noted that the GIE distribution can provide better fit than the Gamma distribution. The GIE distribution is positively skewed and its shape is unimodal while the shape of its hazard function could be decreasing or increasing. The GIE distribution can be used in accelerated life testing, horse racing, queue theory, modeling wind speeds, etc[6].

The Beta Inverted Exponential (BIE) distribution was introduced recently[5] and it was derived using the Beta Generalized family of distributions[7]. The cumulative density function (cdf) of the BIE distribution is given by Eq. 1:

$$F(x)=\frac{1}{B(a,b)}\int_0^{e^{-\theta/x}} w^{a-1}(1-w)^{b-1}dw \quad (1)$$

For x>0, a>0, b>0, θ>0.

Its associated probability density function (pdf) is given by:

$$f(x)=\frac{1}{B(a,b)}\frac{\theta}{x^2}e^{-a\theta/x}\left(1-e^{-\theta/x}\right)^{b-1} \quad (2)$$

For x>0, a>0, b>0, θ>0.

where, a and b are shape parameters and θ is the scale parameter.

Several statistical properties of the BIE distribution have been derived and studied[5]. The shape of the hazard function for the BIE distribution could be non-monotonic and inverted bathtub. It was noted that the distribution can be used to model lifetime data. In their study, a simulation study was conducted using R-software by assuming the values of the parameters to be; θ = 3, a = 0.3, b = 2 and at θ = 5, a = 0.5, b = 4. Hence, two sets of data were obtained, the sample size considered are n = 30, 50, 80 and 100. The purpose was to compare the performances of the Maximum Likelihood Estimates (MLE) with that of the Bayes estimates. The BIE distribution was in turn applied to two real data sets and its performance was compared with that of GIE distribution, IE distribution, Inverse Rayleigh distribution, Beta Weibull distribution and Beta Exponential distribution using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) values posed by each of the distribution. Meanwhile, it was discovered that the BIE distribution failed to perform better than any of the competing distributions.

It can be observed in Eq. 1 that the cdf of the beta inverted exponential distribution involves a special function; the incomplete beta function. This would make the distribution to be difficult to handle. As a result of this, the IE distribution shall be generalized in this study using the exponential generalized family of distributions which is obtained as a special case of the Weibull generalized family of distributions[8]. In this case, only one additional parameter would be introduced and the resulting generalized distribution is expected to be more tractable than the BIE distribution.

It is worthy of note that some other generalized versions of the IE distribution also exist in the literature; examples include the transmuted inverse exponential distribution[9]; Kumaraswamy IE distribution[10] and exponentiated GIE distribution[11]. Meanwhile, these distributions have not yet been applied to real life data sets.

In this article, the densities of the EIE distribution are derived; its statistical properties are also established including estimation of parameters. The flexibility of the proposed distribution was assessed using three real life data sets and a simulation study was conducted in order to investigate the performance of the model parameters.

MATERIALS AND METHODS

The methods used in developing the proposed EIE were systematically explained.

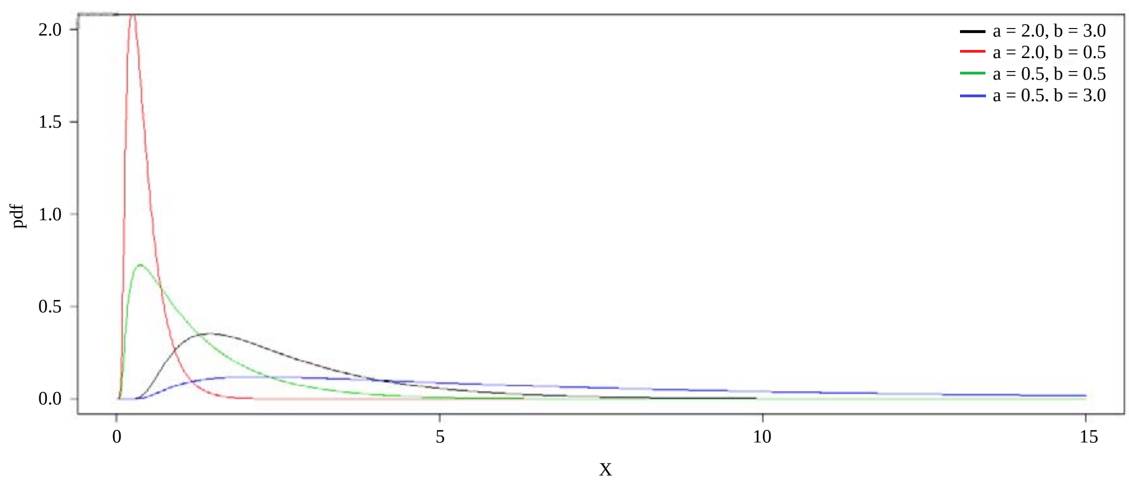

Fig. 1: PDF of the exponential inverse exponential distribution (a = α, b = θ)

EIE distribution: Following the concept of Bourguignon *et al.*[8], the exponential-G family of distribution can be obtained using the relation:

$$F(x) = \int_0^{\frac{G(x)}{1-G(x)}} \alpha e^{-\alpha t} dt \tag{3}$$

where, G(x) is an arbitrary cdf.

It then follows that:

$$F(x) = 1 - \exp\left\{-\alpha\left[\frac{G(x)}{1-G(x)}\right]\right\} \tag{4}$$

To derive the corresponding pdf; the expression in Eq. 4 is differentiated with respect to x to give:

$$f(x) = \alpha \frac{g(x)}{\left[1-G(x)\right]^2} \exp\left\{-\alpha\left[\frac{G(x)}{1-G(x)}\right]\right\} \tag{5}$$

where, $g(x) = \frac{dG(x)}{dx}$ is an arbitrary pdf.

α>0 is a shape parameter whose role is to induce skewness and vary tail weights.

These results would be equivalent to the result of Bourguignon *et al.*[8], when β = 1.

Since the cdf and pdf of the IE distribution were given by:

$$G(x) = \exp\left(-\frac{\theta}{x}\right) \tag{6}$$

and:

$$g(x) = \frac{\theta}{x^2} \exp\left(-\frac{\theta}{x}\right) \tag{7}$$

respectively.

For x>0, θ>0.

where, θ is a scale parameter.

Then, the cdf and pdf of the EIE distribution were given by:

$$F(x) = 1 - \exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x}\right)}{1-\exp\left(-\frac{\theta}{x}\right)}\right]\right\} \tag{8}$$

and:

$$f(x) = \alpha \frac{\theta}{x^2} \exp\left(-\frac{\theta}{x}\right) \frac{1}{\left[1-\exp\left(-\frac{\theta}{x}\right)\right]^2} \exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x}\right)}{1-\exp\left(-\frac{\theta}{x}\right)}\right]\right\} \tag{9}$$

respectively.

For x>0, α>0, θ>0.

where, α is the shape parameter and θ is the scale parameter.

The plots for the pdf of the EIE distribution at various selected parameter values shown in Fig. 1 and Appendix.

Reliability analysis for the EIE distribution: Here, the expression for the survival function, hazard function, reversed hazard function and the odds function of the EIE distribution were provided.

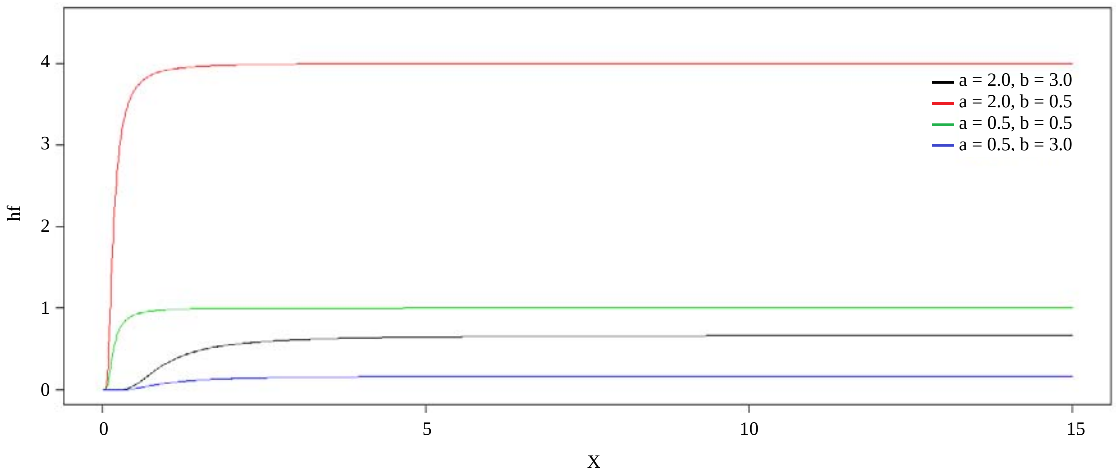

Fig. 2: Hazard function of the exponential inverse exponential distribution (a = α, b = θ)

Survival function: The general formula for the survival function is:

$$S(x) = 1\text{-}F(x)$$

Therefore, the survival function for the EIE distribution was given by:

$$S(x)=\exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x}\right)}{1-\exp\left(-\frac{\theta}{x}\right)}\right]\right\} \quad (10)$$

For x>0, α>0, θ>0.

Hazard function: The hazard function (failure rate) was derived from:

$$h(x)=\frac{f(x)}{1-F(x)}$$

Hence, the expression for the failure rate of the EIE distribution becomes:

$$h(x)=\alpha\frac{\theta}{x^2}\exp\left(\frac{\theta}{x}\right)\frac{1}{\left[1-\exp\left(\frac{\theta}{x}\right)\right]^2} \quad (11)$$

For x>0, α>0, θ>0.

The plots for the hazard function of the EIE distribution at various selected parameter values shown in Fig. 2.

Reversed hazard function: The reversed hazard function was obtained from:

$$r(x)=\frac{f(x)}{F(x)}$$

Hence, the reversed hazard function of the EIE distribution was:

$$r(x)=\frac{\alpha\frac{\theta}{x^2}\exp\left(-\frac{\theta}{x}\right)\frac{1}{\left[1-\exp\left(-\frac{\theta}{x}\right)\right]^2}\exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x}\right)}{1-\exp\left(-\frac{\theta}{x}\right)}\right]\right\}}{1-\exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x}\right)}{1-\exp\left(-\frac{\theta}{x}\right)}\right]\right\}} \quad (12)$$

For x>0, α>0, θ>0.

Odds function: Odds function were obtained from:

$$O(x)=\frac{F(x)}{S(x)}$$

Therefore, the odds function for the EIE distribution was given by:

$$O(x)=\frac{\exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x}\right)}{1-\exp\left(-\frac{\theta}{x}\right)}\right]\right\}}{1-\exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x}\right)}{1-\exp\left(-\frac{\theta}{x}\right)}\right]\right\}} \quad (13)$$

For $x>0$, $\alpha>0$, $\theta>0$.

Quantile function and median: Quantile function was obtained from:

$$Q(u) = F^{-1}(u)$$

Therefore, the quantile function of the EIE distribution was obtained as:

$$Q(u)=\frac{\theta}{\log\left\{\left[-\alpha^{-1}\log(1-u)\right]^{-1}+1\right\}} \tag{14}$$

Hence, random samples from the EIE distribution can be generated from:

$$x=\frac{\theta}{\log\left\{\left[-\alpha^{-1}\log(1-u)\right]^{-1}+1\right\}} \tag{15}$$

where, u is uniform (0, 1).

Also, when u = 0.5 in Eq. 14, the median of the EIE distribution was obtained as:

$$Q(u)=\frac{\theta}{\log\left\{\left[-\alpha^{-1}\log(0.5)\right]^{-1}+1\right\}} \tag{16}$$

where, u is uniform (0, 1).

Parameter estimation: The unknown parameters of the EIE distribution can be estimated using the method of maximum likelihood. Let $x_1, x_2, ..., x_n$ be a random sample each having the pdf of the EIE distribution, then the likelihood function was given by:

$$f(x_1,x_2,.....,x_n;\alpha,\theta)=\prod_{i=1}^{n}\left[\begin{array}{c}\alpha\frac{\theta}{x_i^2}\exp\left(-\frac{\theta}{x_i}\right)\frac{1}{\left[1-\exp\left(-\frac{\theta}{x_i}\right)\right]^2}\\ \exp\left\{-\alpha\left[\frac{\exp\left(-\frac{\theta}{x_i}\right)}{1-\exp\left(-\frac{\theta}{x_i}\right)}\right]\right\}\end{array}\right] \tag{17}$$

Then its log-likelihood function was given by:

$$l=n\log(\theta)+n\log(\alpha)-2\sum_{i=1}^{n}\log(x_i)-\theta\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)-2\sum_{i=1}^{n}\log\left[1-\exp\left(-\frac{\theta}{x_i}\right)\right]-\alpha\sum_{i=1}^{n}\left[\frac{\exp\left(-\frac{\theta}{x_i}\right)}{1-\exp\left(-\frac{\theta}{x_i}\right)}\right] \tag{18}$$

$$\frac{dl}{d\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\left[\frac{\exp\left(-\frac{\theta}{x_i}\right)}{1-\exp\left(-\frac{\theta}{x_i}\right)}\right] \tag{19}$$

$$\frac{dl}{d\theta}=\frac{n}{\theta}-\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)-2\sum_{i=1}^{n}\frac{\frac{1}{x_i}\exp\left(-\frac{\theta}{x_i}\right)}{\left[1-\exp\left(-\frac{\theta}{x_i}\right)\right]}-\alpha W(x;\theta) \tag{20}$$

where:

$$W(x;\theta)=\frac{d}{d\theta}\left[\frac{\exp\left(-\frac{\theta}{x_i}\right)}{1-\exp\left(-\frac{\theta}{x_i}\right)}\right]$$

The solution of $\frac{dl}{d\alpha}=0$ and $\frac{dl}{d\theta}=0$ gives the maximum likelihood estimates of parameters a and θ. Meanwhile, the solution cannot be obtained analytically but it can be solved numerically with the aid of suitable software like R, SAS and so on when data sets are available.

RESULTS

As an illustration, the EIE distribution was fitted to three real life data sets and its performance over the inverse exponential distribution is compared. The best distribution wa selected using the log-likelihood and Akaike Information Criteria (AIC) values posed by each of the competing distributions.

First data: The first data represents the remission times (in months) of a random sample of 128 bladder cancer patients[12,13]. The observations are as follows:

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46 , 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66,

Table 1: Summary of the data on bladder cancer patients

No.	Minimum	Maximum	Median	Mean	Variance	Skewness	Kurtosis
128	0.080	79.050	6.395	9.366	110.425	3.286569	18.48308

Table 2: Performance of the EIE distribution using data on bladder cancer patients

Distributions	$\hat{\theta}$	$\hat{\alpha}$	Log-likelihood	AIC	Rank
EIE	0.21092	0.02277	-413.6499	831.2998	1
IE	2.4847	-	-460.3823	922.7646	2

AIC: Akaike information criteria

Table 3: Summary of data on survival times of patients with breast cancer

No.	Minimum	Maximum	Median	Mean	Variance	Skewness	Kurtosis
121	0.30	154.00	40.00	46.33	1244.464	1.04318	3.402139

Table 4: Performance of the EIE distribution using the data on survival times of breast cancer patients

Distributions	$\hat{\theta}$	$\hat{\alpha}$	Log-likelihood	AIC	Rank
EIE	0.350733	0.007599	-584.9013	1173.803	1
IE	10.3215	-	-677.2791	1356.558	2

AIC: Akaike information criteria

1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69

The summary of the data is as given in Table 1. The performance of the EIE distribution when applied to the data on bladder cancer patients is as given in Table 2.

Remark: The lower the AIC (or the higher the log-likelihood) value, the better the model. It can be seen in Table 2 that the exponential Inverse exponential distribution has the lowest AIC value compared to that of the IE distribution. As a consequence, the EIE distribution was an improvement over the IE distribution based on the data considered.

Second data: This second data represents the survival times of 121 patients with breast cancer obtained from a large hospital in a period from 1929-1938[14,15]. The observations are as follows:

0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3, 11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1, 30.0, 31.0, 31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0, 38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0, 41.0, 42.0, 43.0, 43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0, 47.0, 48.0, 49.0, 51.0, 51.0, 51.0, 52.0, 54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0, 111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0, 154.0

The summary of the data given in Table 3. The performance of the EIE distribution when applied to the data on breast cancer is given in Table 4.

Remark: The EIE distribution also has the lowest AIC value and the highest log-likelihood value. Hence, the Exponential Inverse exponential distribution can be considered better than the Inverse exponential distribution.

Third data: This data represents the survival times of a group of patients suffering from head and neck cancer diseases and treated using a combination of radiotherapy and chemotherapy (RT+CT)[16,17]. The observations are as follows:

12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776

The summary of the data given in Table 5. The performance of the EIE distribution when applied to the data on survival times of patients suffering from head and neck cancer given in Table 6.

Remark: Unlike in the two previous examples, the IE distribution has the lowest AIC value and can be considered the best based on the data considered. However, it was observed that this could be due to the fact that, the variance of the data set used is very large (93,286.41).

Table 5: Summary of data on survival times of patients suffering from head and neck cancer

No.	Minimum	Maximum	Median	Mean	Variance	Skewness	Kurtosis
44	12.20	1776.00	128.50	223.50	93,286.41	3.2691	16.5596

Table 6: Performance of the EIE distribution using data on head and neck cancer

Distributions	$\hat{\theta}$	$\hat{\alpha}$	Log-likelihood	AIC	Rank
EIE	33.44690	0.16085	-280.4043	564.8086	2
IE	76.7000	-	-279.5773	561.1546	1

AIC: Akaike information criteria

Table 7: Result from the simulation study

No.	Parameters	Estimate	Standard error	Mean square error	Bias	Abs. bias
50	$\alpha = 2$	1.5592	0.6067	0.012134	0.4408	0.4408
	$\theta = 3$	2.4548	0.6036	0.012072	0.5452	0.5452
100	$\alpha = 2$	1.0018	0.2855	0.002855	0.9982	0.9982
	$\theta = 3$	1.7954	0.3573	0.003573	1.2046	1.2046
150	$\alpha = 2$	2.4363	0.5033	0.003355	-0.4363	0.4363
	$\theta = 3$	3.4207	0.3987	0.002658	-0.4207	0.4207
200	$\alpha = 2$	2.0845	0.3905	0.001953	-0.0845	0.0845
	$\theta = 3$	3.2450	0.3572	0.001786	-0.2450	0.2450
500	$\alpha = 2$	1.4384	0.1812	0.000362	0.5616	0.5616
	$\theta = 3$	2.4072	0.1952	0.000390	0.5928	0.5928
800	$\alpha = 2$	1.6760	0.1616	0.000202	0.3240	0.3240
	$\theta = 3$	2.6778	0.1603	0.000200	0.3222	0.3222

Simulation study: A data set of size m = 1,000 was simulated from the Weibull distribution; a well-known lifetime distribution with parameters $\alpha = 2$, $\theta = 3$. Six different data sets with sizes n = 50, 100, 150, 200, 500 and 800 were extracted from the main data (m) using the method of simple random sampling with the aid of R software. Then, the EIE distribution was fitted to the data. The idea is to know how the parameters of the EIE distribution would behave as the sample size increases. The maxLik package in R software was used to estimate the maximum likelihood estimates of parameters α and θ. The result of the simulation study as shown in Table 7.

Remark: It can be seen that the error posed by the parameters α and θ decreases as the sample size (n) increases but there was an exception for parameter α when n = 150. It can also be seen that the biasedness of the parameters were not large. The biasedness in this case cannot be zero in because the data was simulated from Weibull distribution and EIE distribution was fitted on it. It is expected that there will be a level of biasedness.

DISCUSSION

Figure 1 shows that the shape of the EIE distribution at varying parameter values were unimodal (or inverted bathtub). The aim of generalizing or extending a standard distribution was usually to make it more flexible; as displayed in Table 2 and 4, the EIE distribution was confirmed to be more flexible than the Inverse exponential distribution because it has the lowest AIC value and the highest log-likelihood value. These results are in accordance with that of previous studies[8,13,15,18] and many more where the generalized distribution performed better than the baseline distribution.

On the contrary, in Table 6, the EIE distribution failed to perform better than its baseline distribution based on the AIC and log-likelihood values posed by these distributions. Now, looking at Table 5, the variance of the data used was very large (93,286.41) and the data can be considered to be over-dispersed. This particular result has been able to uncover that a compound distribution may be tractable and flexible like the proposed EIE distribution but some limitations can exist especially when there are outliers in the data set.

In addition, several studies in distribution theory that involve simulation were performed using random samples generated from the newly developed compound or generalized distribution[19-21] but this present study did not follow that route, though similar conclusions were obtained; the error posed by the parameter estimates decreases as the sample size increases. Here, the two-parameter Weibull distribution was used to generate random samples instead of using the newly developed EIE distribution; as expected, the parameter estimates were close to the true parameter values, thus the bias generated was small.

Unlike other studies where the shape of the hazard rate function shows unimodality[3,4], increasing[22], decreasing or constant shapes, the shape of the EIE distribution as shown in Fig. 2 resembles that of the cumulative density function.

CONCLUSION

The IE distribution has been successfully extended using the Exponential family of distribution. Various statistical properties of the EIE distribution EIE have been explicitly derived including estimation of model parameters. In general, the shape of the EIE distribution is unimodal. The EIE distribution can be seen as an improvement over the BIE distribution as the former does not involve any special function like the incomplete beta function. The EIE distribution was applied to three different data sets and it performed better than the IE distribution except for a case where the data was over-dispersed.

SIGNIFICANCE STATEMENTS

This current study discovers that the exponential family of distribution could be used as an alternative generator of compound distributions as it is mathematically tractable. Also, it uncovers the possible limitation of the newly proposed model. The results in this study will help researchers in distribution theory to develop new compound distributions in modeling lifetime data; an illustration using the EIE distribution has been successfully made in this study.

APPENDIX

R code for plotting the pdf of the EIE distribution

```
a=2
b=3
x=seq(0,15,0.01)
eie.pdf=function(x,a,b)
f=a*(b/x^2)*exp(-b/x)*(1/(1-exp(-b/x))^2)*exp(-a*(exp(-b/x))/(1-exp(-b/x)))
plot(x,eie.pdf(x,2,3),type='l',ylim=c(0,2.0),xlab="x",ylab="pdf")
lines(x,eie.pdf(x,2,0.5),col=2)
lines(x,eie.pdf(x,0.5,0.5),col=3)
lines(x,eie.pdf(x,0.5,3),col=4)
```

ACKNOWLEDGMENT

The authors sincerely appreciate Covenant University, Nigeria for providing an enabling environment for this study.

REFERENCES

1. Singh, S.K., U. Singh and M. Kumar, 2013. Estimation of parameters of generalized inverted exponential distribution for progressive type-II censored sample with binomial removals. J. Probab. Stat. 10.1155/2013/183652.
2. Keller, A.Z., A.R.R. Kamath and U.D. Perera, 1982. Reliability analysis of CNC machine tools. Reliabil. Eng., 3: 449-473.
3. Lin, C.T., B.S. Duran and T.O. Lewis, 1989. Inverted gamma as a life distribution. Microelectron. Reliab., 29: 619-626.
4. Abouammoh, A.M. and A.M. Alshingiti, 2009. Reliability estimation of generalized inverted exponential distribution. J. Stat. Comput. Simul., 79: 1301-1315.
5. Singh, B. and R. Goel, 2015. The beta inverted exponential distribution: Properties and applications. Int. J. Applied Sci. Math., 2: 132-141.
6. Dey, S. and B. Pradhan, 2014. Generalized inverted exponential distribution under hybrid censoring. Stat. Methodol., 18: 101-114.
7. Eugene, N., C. Lee and F. Famoye, 2002. The Beta-normal distribution and its applications. Commun. Stat. Theory Methods, 31: 497-512.
8. Bourguignon, M., R.B. Silva and G.M. Cordeiro, 2014. The weibull-G family of probability distributions. J. Data Sci., 12: 53-68.
9. Oguntunde, P.E. and O. Adejumo, 2015. The transmuted inverse exponential distribution. Int. J. Adv. Stat. Probabil., 3: 1-7.
10. Oguntunde, P.E., O.S. Babatunde and A.O. Ogunmola, 2014. Theoretical analysis of the Kumaraswamy-inverse exponential distribution. Int. J. Stat. Applic., 4: 113-116.
11. Oguntunde, P.E., A. Adejumo and O.S. Balogun, 2014. Statistical properties of the exponentiated generalized inverted exponential distribution. Applied Math., 4: 47-55.
12. Lee, E.T. and J.W. Wang, 2003. Statistical Methods for Survival Data Analysis. 3rd Edn., John Wiley and Sons Ltd., New York, USA., ISBN: 9780471458555, Pages: 534.
13. Oguntunde, P.E., A.O. Adejumo and K.A. Adepoju, 2016. Assessing the flexibility of the exponentiated generalized exponential distribution. Pac. J. Sci. Technol., 17: 49-57.
14. Lee, E.T., 1992. Statistical Methods for Survival Data Analysis. 2nd Edn., John Wiley and Sons Ltd., New York, USA., ISBN-13: 9780471615927, Pages: 496.
15. Ramos, M.W.A., G.M. Cordeiro, P.R.D. Marinho, C.R.B. Dias and G.G. Hamedani, 2013. The Zografos-Balakrishnan log-logistic distribution: Properties and applications. J. Stat. Theory Applic., 12: 225-244.
16. Efron, B., 1988. Logistic regression, survival analysis and the Kaplan-Meier curve. J. Am. Stat. Assoc., 83: 414-425.

17. Shanker, R., F. Hagos and S. Sujatha, 2015. On modeling of lifetimes data using exponential and Lindley distributions. Biometrics Biostat. Int. J., Vol. 2, No. 5.
18. Khaleel, M.A., N.A. Ibrahim, M. Shitan and F. Merovci, 2016. Some properties of Gamma Burr type X distribution with application. AIP Conf. Proc., Vol. 1739, No. 1. 10.1063/1.4952567.
19. Jafari, A.A., S. Tahmasebi and M. Alizadeh, 2014. The beta-Gompertz distribution. Revista Colombiana de Estadlstica, 37: 141-158.
20. Cordeiro, G.M., E.M.M. Ortega, G.G. Hamedani and D.A. Garcia, 2016. The McBurr XII and Log-McBurr XII models with applications to lifetime data. Int. J. Stat. Probab., 5: 1-18.
21. Hussian, M.A., 2013. A weighted inverted exponential distribution. Int. J. Adv. Stat. Probab., 1: 142-150.
22. Oguntunde, P.E., A.O. Adejumo, H.I. Okagbue and M.K. Rastogi, 2016. Statistical properties and applications of a new lindley exponential distribution. Gazi Univ. J. Sci., 29: 831-838.