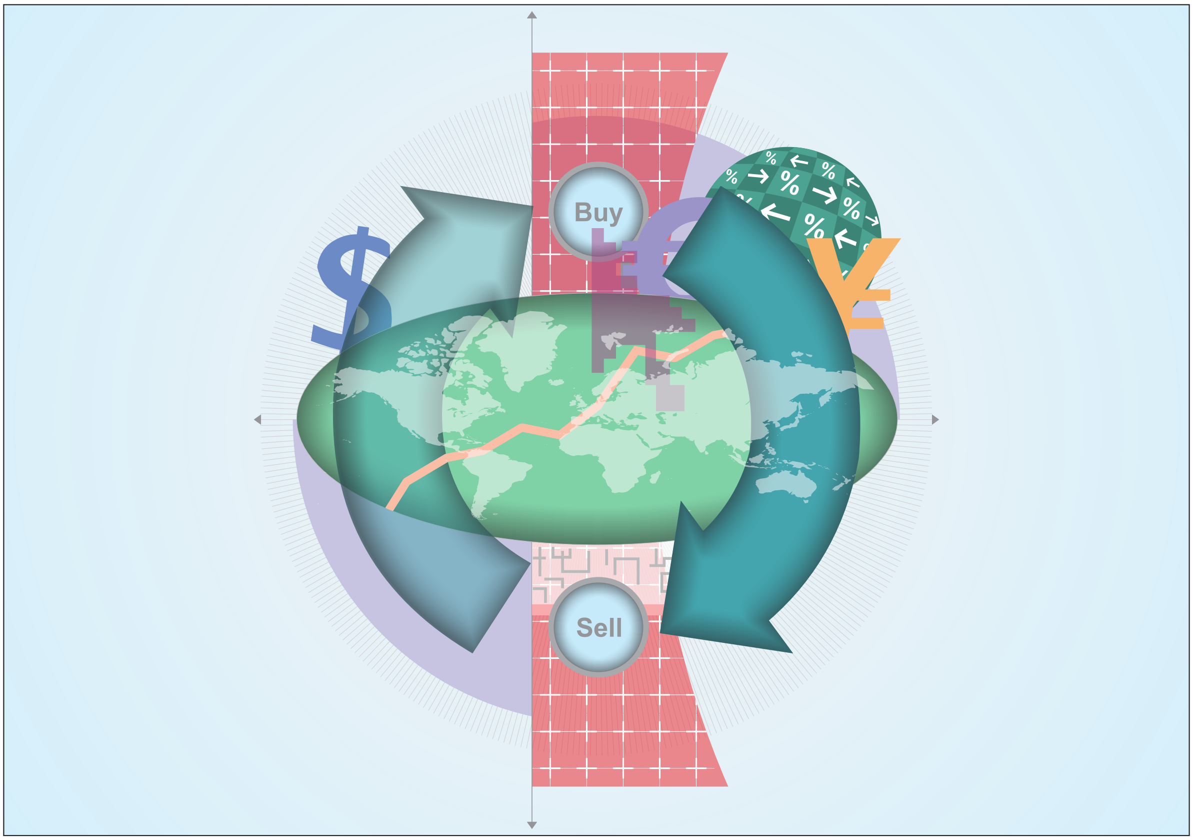

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Improving of Efficiency in Data Envelopment Analysis with Interval Data

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Abstract: Aim of this research is study efficiency of Decision Making Units (DMUs) with interval data using Data Envelopment Analysis (DEA) models. The DEA is a widely applied approach for measuring the relative efficiencies of a set of DMUs which uses multiple inputs to produce multiple outputs. An assumption underlying DEA is that all the data are known exactly. But in reality, many factors cannot be measured in a precise manner. In recent years, in different applications of DEA, inputs and outputs have been observed whose values are indefinite. Such data are called imprecise. Imprecise data can be probabilistic, interval, ordinal, qualitative or fuzzy. In this study, we investigate an interval DEA model, in the case that the inputs and outputs are located within the bounded intervals. The resulting model is non-linear and then we convert it to a linear one. Also minimal variations of input and output intervals are computed to achieve to full efficiency. Indeed, we propose a new method for improving of efficiency classifications of DMUs with interval data in data envelopment analysis.

Key words: DEA, interval data, degree holding true, efficiency classification

INTRODUCTION

Performance measurement and evaluation are fundamental to management planning and control activities, and accordingly, have received considerable attention by both management practitioners and theorists. Data Envelopment Analysis (DEA) is a non-parametric method to evaluate the relative efficiency of Decision-Making Units (DMUs) which are based on multiple inputs and outputs (Charnes *et al*., 1978; Banker *et al*., 1984). DEA has different models which are created according to application. If collected data is imprecise for evaluation, then we must use the models which can deal with imprecise data. DEA models with imprecise data were first proposed by Sengupta (1992). He applied principles of fuzzy set theory to data envelopment analysis.

Cooper *et al*. (1999) investigated an Interval Data Envelopment Analysis (IDEA) model to deal with imprecise data in DEA and measured efficiency as a crisp value. A method for computing efficiency as interval in IDEA was suggested by Despotis and Smirlis (2002). Chiang (2006) used a two-level mathematical programming approach to model the efficiency interval for imprecise data. In this method, the lower bound and upper bound of the efficiency interval are measured by transforming the two-level nonlinear program to the ordinary one-level linear program, based on the concept of productive efficiency and by

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applying a variable substitution technique. Wang *et al*. (2005) developed a different approach of interval DEA model to determine efficiency, using a fixed and unified production frontier.

The purpose of this study is investigation the interval DEA model. The first objective is to propose a simple framework to measure and evaluate the best and the worst relative efficiencies of DMUs. The second objective presents the proposed models for sensitivity of efficiency classification in IDEA, and then calculates variation of data for improving in efficiency and to change classification of DMUs. We use an example to illustrate our method.

EFFICIENCY IN INTERVAL DEA

Assume that there are n units (DMU), each using m different inputs to produce s different outputs. Especially, DMU_i consumes amount $X_j = \{x_{ij}\}$ of inputs (i = 1, 2,…, m) and produces amount $Y_j = \{y_{rj}\}$ of outputs (r = 1, 2,…, s). The efficiency score of a specific DMU_0 can be evaluated by the CCR model of DEA, in the input and output orientation as follows:

$$\begin{aligned} \theta_o^* = \text{minimize} \quad & \theta \\ \text{S.t.} \quad & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} && i=1,2,\ldots,m \\ & \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge \theta y_{ro} && r=1,2,\ldots,s \\ & \lambda_j \ge 0 && j=1,2,\ldots,n \end{aligned}$$

A DMU is a perfect efficient, if and only if the improvement possibility of none of inputs and outputs exist without worsening other inputs and outputs (Pareto-Koopmans Efficiency); therefore, the most efficiency of DMU_o results through the solution of the model which $\theta_o^* \le 1$. If DMU_o is efficiency $\theta_o^* = 1$ and if inefficient, DMU is $\theta_o^* < 1$.

In this study, we assume that all the input and output data x_{ij} and y_{rj} (i = 1,…, m; r = 1,…, s; j = 1,…, n) cannot be exactly obtained due to the existence of uncertainty. They are only known to lie within then lower and upper bounds represented by the intervals.

$$x_{ij} \in [x_{ij}^l, x_{ij}^u]\ ,\quad y_{ij} \in [y_{ij}^l, y_{ij}^u] \text{ where, } x_{ij} > 0 \text{ and } y_{rj} > 0$$

If consider the degrees holding true p and q, for each of the above mentioned intervals, we will have:

$$x_{ij} = x_{ij}^l + p(x_{ij}^u - x_{ij}^l), \quad y_{rj} = y_{rj}^l + q(y_{rj}^u - y_{rj}^l) \qquad 0 \le p,q \le 1$$

By these kind of substitutions, model DEA is finally transformed into the following program:

$$\begin{aligned} \text{S.t.} \quad & \sum_{j=1,\, j\ne o}^{n} \lambda_j [x_{ij}^l + p(x_{ij}^u - x_{ij}^l)] + \lambda_o [x_{io}^l + p_o(x_{io}^u - x_{io}^l)] \le \theta [x_{io}^l + p_o(x_{io}^u - x_{io}^l)], \quad i=1,\ldots,m \\ & \sum_{j=i,\, j\ne o}^{n} \lambda_j [y_{rj}^l + q(y_{rj}^u - y_{rj}^l)] + \lambda_o [y_{ro}^l + q_o(y_{ro}^u - y_{ro}^l)] \ge y_{ro}^l + q_o(y_{ro}^u - y_{ro}^l), \quad r=1,\ldots,s \\ & \lambda_j \ge 0, \qquad j=1,\ldots,n \end{aligned} \tag{1}$$

The adaptive set of points to system (1) is called feasible region S. The following special cases are being considered from the system (1). If in the above mentioned model $p_o = 0$,

$q_o = 1$, $p = 1$, $q = 1$, ideal model is resulted and feasible region in this case is called S_I (DMU_o has lower inputs and outputs in upper bound (in the best state) and other DMUs produce minimal outputs by maximal inputs).

$$\begin{aligned} \theta_I^* = \text{Minimize} \quad & \theta \\ \text{S.t.} \quad & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^u + \lambda_o x_{io}^l \leq \theta x_{io}^l, & i = 1,...,m \\ & \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}^l + \lambda_o y_{ro}^u \geq y_{ro}^u, & r = 1,....,s \\ & \lambda_j \geq 0, & j = 1,...,n \end{aligned} \tag{1-1}$$

Similarly, if $p_o = 1$, $q_o = 0$, $p = 0$, $q = 1$ feasible region, S_A (Anti-ideal space), is resulted.

$$\begin{aligned} \theta_A^* = \text{Minimize} \quad & \theta \\ \text{S.t.} \quad & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij}^l + \lambda_o x_{io}^u \leq \theta x_{io}^u, & i = 1,...,m \\ & \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj}^u + \lambda_o y_{ro}^l \geq y_{ro}^l, & r = 1,....,s \\ & \lambda_j \geq 0, & j = 1,...,n \end{aligned} \tag{1-2}$$

Based on these mentioned models, the following theorem is proved easily.

Theorem 1

$$S \subseteq S \subseteq S_A,\ \theta_A^* \leq \theta^* \leq \theta_I^*$$

Corollary 2

If θ_A^* then $\theta = \theta_I^*$ and DMU_o is always efficient.

Corollary 3

If $\theta_I^* < 1$ then $\theta_A^* \leq \theta^* < 1$ and DMU_o is always inefficient.

Corollary 4

If θ_I^* and $\theta_A^* < 1$ then DMU_o will be efficient in some intervals and sometimes inefficient in the others.

When DMUs are evaluated with interval data; therefore, they can classify as follows: E^{++} is as all the DMUs which are efficient with any combination of their inputs and outputs which are called as fully efficient. E^+ is the DMUs which are efficient in their maximal status but for some data levels they lose their efficiency and E^- is the DMUs that are inefficient in each case.

We investigate the state of the efficiency scores for the efficient units in E^+, and will determine part of intervals of data inputs and outputs in which DMU is fully efficient.

IMPROVING OF EFFICIENCY IN THE INTERVAL DEA

If DMU_o is a member of E^+, thus DMU_o in state $p = 1$, $q = 0$ is efficient and θ_I^*. In this state, model (1) is finally transformed into the following program:

$$\begin{aligned}
&\theta^* = \text{Minimize} \quad \theta \\
&\text{S.t.} \quad \sum_{j=1,\, j\neq o}^{n} \lambda_j x^u_{ij} + \lambda_o [x^l_{io} + p_o (x^u_{io} - x^l_{io})] \leq \theta [x^l_{io} + p_o (x^u_{io} - x^l_{io})], \qquad i = 1,\dots,m \\
&\quad \sum_{j=1,\, j\neq o}^{n} \lambda_j y^l_{rj} + \lambda_o [y^l_{ro} + q_o (y^u_{ro} - y^l_{ro})] \geq y^l_{ro} + q_o (y^u_{ro} - y^l_{ro}), \qquad r = 1,\dots,s \\
&\quad \lambda_j \geq 0, \qquad j = 1,\dots,n
\end{aligned} \tag{2}$$

Theorem 5

In model (2), if $p_{o1} \leq p_{o2}$ and $q_{o1} \leq q_{o2}$ with feasible solution, respectively, S_{o1} and S_{o2} then $S_{o1} \subseteq S_{o2} \Rightarrow \theta^*_{o2} \leq \theta^*_{o1}$.

If in model (2) $p_o = 0$, will determine minimal value of q_o so that DMU_o stays efficient. In other words, we are going to determine q_o^* in such $q_o^* \leq q_o \leq 1$ for DMU_o to be efficient. For this purpose, we make use of dual model (coefficient model):

$$\begin{aligned}
&q_o^* = \text{Min} \quad q_o \\
&\text{S.t.} \quad \sum_{r=1}^{s} u_r y^l_{rj} - \sum_{i=1}^{m} v_i x^u_{ij} \leq 0, \qquad j = 1,\dots,n \;,\; j \neq o, \\
&\quad \sum_{r=1}^{s} u_r [y^l_{ro} + q_o (y^u_{ro} - y^l_{ro})] = 1 \\
&\quad \sum_{i=1}^{m} v_i x^l_{io} = 1 \\
&\quad u_r, v_i \geq 0, \qquad r = 1,\dots,s, \quad i = 1,\dots,m
\end{aligned} \tag{3}$$

Model (3) is always feasible and $0 \leq q_o^*$. Nonlinear model (3) convert to linear model (4) by transformations:

$$T = \sum_{r=1}^{s} u_r (y^u_{ro} - y^l_{ro})$$

and

$$t = \frac{1}{T}\;,\; v'_i = \frac{v_i}{T}\;,\; u'_r = \frac{u_r}{T}$$

$$\begin{aligned}
&q_o^* = \text{Min} \quad q_o \\
&\text{s.t.} \quad \sum_{r=1}^{s} u'_r y^l_{rj} - \sum_{i=1}^{m} v'_i x^u_{ij} \leq 0, \qquad j = 1,\dots,n \;,\; j \neq o \\
&\quad \sum_{r=1}^{s} u'_r y^l_{ro} + q_o = t, \\
&\quad \sum_{r=1}^{s} u'_r (y^u_{ro} - y^l_{ro}) = 1, \\
&\quad \sum_{i=1}^{m} v'_i x^l_{io} = t, \\
&\quad u'_r, v'_i \geq 0, \quad r = 1,\dots,s, \quad i = 1,\dots,m
\end{aligned} \tag{4}$$

Theorem 6

Optimal solution of models (3) and (4) are equivalent.

Proof

Let $(u_1^*, u_2^*,.., v_1^*, v_2^*,..., v_m^* q_o^*)$ be an optimal solution to (3), then by setting

$$u'_r = \frac{u_r^*}{\sum_{r=1}^{s} u_r^*(y_{ro}^u - y_{ro}^l)}, \quad v'_i = \frac{v_i^*}{\sum_{r=1}^{s} u_r^*(y_{ro}^u - y_{ro}^l)}$$

and

$$t = \frac{1}{\sum_{r=1}^{s} u_r^*(y_{ro}^u - y_{ro}^l)}$$

Then, we have:

$$\sum_{r=1}^{s} u'_r (y_{ro}^u - y_{ro}^l) = 1$$

and

$$\sum_{r=1}^{s} u_r^*[y_{ro}^l + q_o(y_{ro}^u - y_{ro}^l)] = 1 \Rightarrow \sum_{r=1}^{s} u'_r\, y_{ro}^l + q_o = t$$

and

$$\sum_{r=1}^{s} u_r y_{rj}^l - \sum_{i=1}^{m} v_i x_{ij}^u \le 0 \quad j = 1,...,n \quad j \ne o \Rightarrow \sum_{r=1}^{s} u'_r\, y_{rj}^l - \sum_{i=1}^{m} v'_i\, x_{ij}^u \le 0 \quad j = 1,...,n \quad j \ne o$$

and

$$\sum_{i=1}^{m} v_i x_{io}^l = 1 \Rightarrow \sum_{i=1}^{m} v'_i\, x_{io}^l = t,$$

So,

$$t = \frac{1}{\sum_{r=1}^{s} u_r^*(y_{ro}^u - y_{ro}^l)}, \; u'_r = \frac{u_r^*}{t}, \; v'_i = \frac{v_i^*}{t}$$

is a feasible solution to (4).

On the other hand, if $S_2 \subseteq S_1$ is an optimal solution to (4), then:

$$u_r = \frac{u'^*_r}{t^*}, \; v_i = \frac{v'^*_i}{t^*}, \; q_o = q'^*_o$$

is a feasible solution to (3), because:

$$\sum_{r=1}^{s} u'_r (y_{ro}^u - y_{ro}^l) + q_o = t \Rightarrow \sum_{r=1}^{s} \frac{u'_r}{t^*} (y_{ro}^u - y_{ro}^l) + \frac{q_o}{t^*} = \frac{1}{t^*}$$

$$\Rightarrow \sum_{r=1}^{s} \frac{u'_r}{t^*} (y_{ro}^u - y_{ro}^l) + q_o \sum_{r=1}^{s} \frac{u'_r}{t^*} (y_{ro}^u - y_{ro}^l) = 1 \Rightarrow \sum_{r=1}^{s} u_r (y_{ro}^u - y_{ro}^l) + q_o \sum_{r=1}^{s} u_r (y_{ro}^u - y_{ro}^l) = 1$$

and the constraint imply that:

$$u'_r = \frac{u_r^*}{t^*},\ v'_i = \frac{v_i^*}{t^*},\ q_o = q_o^*$$

is a feasible solution to (3), and completes the proof.

By replacing $q = q_o^*$ in model (2), and definition $y'_{ro} = y^l_{ro}+q_o^*\ (y^u_{ro}-y^l_{ro})$, we compute maximal value of p_o, such that model stays efficient.

$$\begin{aligned} p'^*_o = \text{Max} \quad & p_o \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y^l_{rj} - \sum_{i=1}^{m} v_i x^u_{ij} \le 0, \qquad j \ne o \\ & \sum_{r=1}^{s} u_r y^u_{ro} = 1, \\ & \sum_{i=1}^{m} v_i [x^l_{io} + p_o (x^u_{io} - x^l_{io})] = 1, \\ & p_o \le 1, \quad u_r, v_i \ge 0, \qquad \forall_{r,i} \end{aligned} \tag{5}$$

Similarity, set:

$$T = \sum_{i=1}^{m} v_i (x^u_{io} - x^l_{io})$$

and

$$t = \frac{1}{T},\ v'_i = \frac{v_i}{T},\ u'_r = \frac{u_r}{T}$$

for each r and i. Then substitute in model (5) to obtain:

$$\begin{aligned} p'^*_o = \text{Max} \quad & p_o \\ \text{s.t.} \quad & \sum_{r=1}^{s} u'_r y^l_{rj} - \sum_{i=1}^{m} v'_i x^u_{ij} \le 0, \qquad j = 1,\dots,n \quad j \ne o \\ & \sum_{r=1}^{s} u_r y'_{ro} = t, \\ & \sum_{i=1}^{m} v'_i x^l_{io} + p_o = t, \\ & \sum_{i=1}^{m} v'_i (x^u_{io} - x^l_{io}) = 1, \\ & 0 \le p_o \le 1, \quad u'_r, v'_i \ge 0, \qquad i = 1,\dots,m, \quad r = 1,\dots,s \end{aligned} \tag{6}$$

At first, the interval of outputs and then the interval of inputs were determined to achieve to full efficiency.

If adjustment data is based on inputs data, the interval of data will be determined in the following way.

By putting $q_o = 1$ in model (2), maximal value of p'^*_o is computed by the similar model with model (5), then by selecting $p_o = p'^*_o$ in model (2) minimal value of q_o is determined in a way that system can be efficient. This value is called q'^*_o.

Theorem 7

$$p'^*_o \geq p^*_o \text{ and } q'^*_o \geq q^*_o$$

Based on above theorems, it is trivial that:

- $\forall p_o, q_o \ni q_o \geq q^*_o \wedge p_o \leq p^*_o \Rightarrow$ DMU_o is fully efficient and $DMU_o \in E^{++}$
- $\forall p_o, q_o \ni q_o \geq q'^*_o \wedge p_o \leq p'^*_o \Rightarrow$ DMUo is fully efficient and $DMU_o \in E^{++}$

It is proved for model (2):

- If $p_o > p'^*_o$ then DMU_o is inefficiency.
- If $q_o < p'^*_o$ then DMU_o is inefficiency.

DMU_o can be efficient or inefficient in region $p_o^* < p_o < p'^*_o$, $q_o^* < q_o < q'^*_o$.

Corollary 8

If model (2) for given value of p_o and q_o and p = 0, q = 1 is efficient then model (2) is efficient for all values of p and q.

Corollary 9

If model (2) for given value of p_o and q_o and p = 1, q = 0 is inefficient then model (2) is inefficient for all values of p and q.

Theorem 10

The objective optimal value of model (1) is always greater than or equal to the objective optimal value of model (2).

Proof

The feasible solution of model (1) and model (2) are called S_1 and S_2, respectively. At first, we will prove $S_2 \subseteq S_1$.

Since:

$$x^l_{ij} \leq x^u_{ij}$$

we have:

$$x^l_{ij} + p(x^u_{ij} - x^l_{ij}) \leq x^u_{ij}$$

For

$$\forall(\lambda_1, \lambda_2, ,..\lambda_n) \in S_2$$

by multiplying the above relation in λ_j and summing all the inequalities over j and adding

$$\lambda_o[x^l_{ij} + p_o(x^u_{ij} - x^l_{ij})]$$

to both sides of the relation, we will have:

Table 1: Efficiency of DMU_o by according to p_o and q_o

q_o							
p_o	0	…	q_o^*	…	q'^*_o	…	1
0	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$
⋮	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$
p_o^*	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$
⋮	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	Undefined	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$
p'^*_o	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*=1$	$\theta_o^*=1$	$\theta_o^*=1$
⋮	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$
1	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$	$\theta_o^*<1$

$$\sum_{j=1}^{n}\lambda_j[x_{ij}^l + p(x_{ij}^u - x_{ij}^l)] + \lambda_o[x_{io}^l + p_o(x_{io}^u - x_{io}^l)] \le \sum_{j=1}^{n} x_{ij}^u + \lambda_j[x_{ij}^l + p_o(x_{ij}^u - x_{ij}^l)]$$
$$\le \theta[x_{io}^l + p_o(x_{io}^u - x_{io}^l)]$$

also, we have:

$$y_{rj}^l + q(y_{rj}^u - y_{rj}^l) \ge y_{rj}^l$$

By multiplying in λ_j and summing all the inequalities over j and adding

$$\lambda_o[y_{rj}^l + q_o(y_{rj}^u - y_{rj}^l)]$$

to both sides of the relation, we will have:

$$\sum_{j=1}^{n}\lambda_j[y_{rj}^l + q(y_{rj}^u - y_{rj}^l)] + \lambda_o[y_{ro}^l + q_o(y_{ro}^u - y_{ro}^l)] \ge \sum_{j=1}^{n}\lambda_j y_{rj}^l + \lambda_o[y_{ro}^l + q_o(y_{ro}^u - y_{ro}^l)]$$
$$\ge y_{ro}^l + q_o(y_{ro}^u - y_{ro}^l)$$

Thus, $(\lambda_1, \lambda_2,..., \lambda_n)$. Minimum over smaller region is greater of minimum over larger region, and completes the proof.

Corollary 11

In the all region that DMU_o in model (2) is inefficient then in model (1) is inefficient.

Also, for given value of p_o, q_o in model (1), if $p = p_i$ and $q = q_i$ with feasible solution S_{1i} $i = 1,2$, which $p_1 \le p_2$ and $q_1 \le q_2$ then and $\theta_1^* \le \theta_2^*$.

Corollary 12

If model (1) for given values of p_o, q_o and p, q is efficient then model is efficient for all values of $p'^*_o \le p_o$ and $q'^*_o \ge q_o$.

Let DMU_o be a member of E^+ (on the other hand, the DMU_o is efficient in their maximal status but for some data levels it isn't efficient) corresponding to results of theorem 7, the efficiency regions of DMU_o by according to p_o and q_o are given in Table 1.

As noted earlier, the inputs and outputs intervals of DMUo are equivalent of following statements:

$$x_{io} \in [x_{io}^l, x_{io}^u] \Rightarrow x_{io} = x_{io}^l + p_o\,(x_{io}^u - x_{io}^l),$$
$$y_{ro} \in [y_{ro}^l, y_{ro}^u] \Rightarrow y_{ro} = y_{ro}^l + q_o\,(y_{ro}^u - y_{ro}^l)$$

Table 2: Efficiency and classification of DMUs

DMU	Labor x_1	Capital x_2	Customer satisfaction y_1	Income level y_2	Efficiency in S_A	Efficiency in S_I	DMUs classes
1	[12,15]	[0.21,0.48]	[138,144]	[21,22]	0.217	0.80	E^-
2	[10,17]	[0,1.7]	[143,159]	[28,35]	0.227	1.00	E^+
3	[4,12]	[0.16,0.35]	[157,198]	[21,29]	0.581	1.00	E^+
4	[19,22]	[0.12,0.19]	[158,181]	[21,25]	0.361	0.88	E^-
5	[14,15]	[0.06,0.09]	[157,161]	[28,40]	0.775	1.00	E^+
6	[8,10]	[0.08,0.1]	[150,180]	[36,39]	1.000	1.00	E^{++}

Table 3: The values of q and p of intervals for fully efficient

DMUo	q_o^*	p_o^*	$p'_o{}^*$	$q'_o{}^*$
DMU_2	0.4375	0.0000	0.0857	0.9980
DMU_3	0.0000	0.8083	1.0000	0.5609
DMU_5	0.0000	1.0000	1.0000	0.0000

For instance, if $p_o \le p_o^*$ and $q_o \le q_o^*$ then DMU_o is always efficient. On the other hand, the DMU_o, with setting of the inputs and outputs intervals, can be member of E^{++}.

NUMERICAL EXAMPLE

Consider a performance measurement problem of manufacturing industry, in which there are six manufacturing industries from different cities (DMUs) participating in the evaluation, each consuming two inputs (Labor and Capital) and producing two outputs (Customer satisfaction and Income level). The data are all estimated and are thus imprecise and only known within the prescribed bounds, which are listed in Table 2. The relative efficiencies of each DMU are calculated under feasible regions S_I and S_A by applying models (1-1) and (1-2), respectively, and classification of DMUs are obtained. Models are implemented in an MS-Excel worksheet and are solved by using the Excel Solver.

In above Table, the unit 6 is fully efficient and units 2, 3 and 5 are efficient in their best status, and for some data levels they lose their efficiency. In Table 3, we give adjusted data levels to achieve fully efficiency in these units. The Table 3 represents the results in accordance with models (4) and (6) on DMUs 2, 3 and 5. For example, DMU_2 is always efficient if its inputs lie on lower bounded ($p_o^* = 0$) and its outputs intervals to be adjusted as $y_{r2} \ge y^l_{r2}+0.4375\ (y^u_{r2}-y^l_{r2})$, r = 1, 2 ($q_o^* \ge 0.4375$), also if inputs of DMU_2 are as $x_{i2} \le x^l_{i2}+0.0857$ $(x^u_{i2}-x^l_{i2})$, i = 1, 2, then outputs of this DMU must be, $y_{r2} \ge y^l_{r2}+0.998\ (y^u_{r2}-y^l_{r2})$ r = 1, 2.

The DMUs will be fully efficient by variation data of inputs or outputs according to the Table 2.

CONCLUSION

Data envelopment analysis (DEA) is a mathematical programming approach to evaluate the relative efficiency of DMUs. In some applications, however, the data may be imprecise. For instance, some of the data are known only within specified bounds. To deal with imprecise data in DEA, imprecise data envelopment analysis (IDEA) models and methods have been developed previously. In this study by a variable as a degree holding true, the interval model is converted to decisive model. The criterion degree holding true can represent indicant view point's managers or experts, which increase the validity of evaluation. Also we proposed an approach to determine the amount of exchanges of inputs and outputs data in a way to convert the efficient units to the fully efficient ones. This method can also be generalized to the inefficient units to convert them to the efficient ones or fully efficient ones.

REFERENCES

Banker, R.D., A. Charnes and W.W. Cooper, 1984. Some models for estimating technical and scale inefficiency in data envelopment analysis. Manage. Sci., 30: 1078-1092.
Charnes, A., W.W. Cooper and E. Rhodes, 1978. Measuring the efficiency of decision making units. Eur. J. Operat. Res., 2: 429-444.
Chiang, K., 2006. Interval efficiency measures in data envelopment analysis with imprecise data. Eur. J. Operational Res., 174: 1087-1099.
Cooper, W.W., K.S. Park and G. Yu, 1999. IDEA and AR-IDEA: Models for dealing with imprecise data in DEA. Manag. Sci., 45: 597-607.
Despotis, D. and Y. Smirlis, 2002. Data envelopment analysis with imprecise data. Eur. J. Operational Res., 140: 24-36.
Sengupta J.K., 1992. A fuzzy systems approach in data envelopment analysis. Comput. Math. Appl., 24: 259-266.
Wang Y., R. Greatbanks and J. Yang, 2005. Interval efficiency assessment using data envelopment analysis. Fuzzy Set Syst., 153: 347-370.