Effect of Transmission Power Adjustments on Network Availability

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Abstract: In this study, we have derived and analysed network availability expressions for sleep mode enabled wireless sensor networks under two transmission power models that result into specific data relay models. We conclude that staircase lattice path traversal offers comparative increase in network availability for small to medium sized sensor grids over Delannoy number based lattice path traversal. The later is a better candidate for large networks especially for wide spatial distribution of sensor nodes. Furthermore Delannoy data relay model offers better resilience to unavailable paths in sensor networks that implement sleep mode for energy conservation.

Key words: Sensor battery life, joint PDF, network availability, transmission power

INTRODUCTION

Wireless sensor networks are energy-constrained due to compact form factors resulting into limited lifetime. The study of network longevity is fundamental to have operation critical performance. Many definitions of network availability have been proposed by Sauve and Coelho[1]. They ascertain relationship between transmission power levels and network life time by suggesting two power control algorithms[2]. In this study, our contribution is to formulate explicit expressions of network availability for sleep mode enabled wireless sensor networks. We analyze the effects of inter-node distance variation under transmission power adjustments on network availability.

MODEL AND ASSUMPTIONS

As given in Fig. 1, we consider a reference grid of \( n \times k \) equidistant sensor nodes. Each node has an index as \((i, j)\), where, \( i \) and \( j \) refer to rows and columns of the grid, respectively. Following assumptions are made to formulate the model:

- \( E_i \) is the initial energy of the sensor node \((i, j)\) at the reference time \( t_0 \), distributed across the network as \( \eta_iE_i^{t_0} \) with mean \( 1/\eta_i \).
- Sensing activity by node \((i, j)\) is given by \( \beta_iE_i^{t_i} \) with mean \( 1/\beta_i \) and the relaying activity is given by \( \gamma_iE_i^{t_i} \) with mean \( 1/\gamma_i \).
- Detection is a Poisson process with mean \( \lambda_i \).

Fig. 1: Reference topology of sensor networks

- Sensor node \((i, j)\) consumes energy at a rate of \( a_i \) and \( a_i' \) per second to remain awake and in sleep mode, respectively.

Here, we derive the network availability of a twodimensional topology in the form of Chapman-Kolmogorov equations[3] for two special cases of data relaying models, namely Staircase and Delannoy number based lattice path traversals.

ANALYSIS

We denote \( y_i \) as the total energy consumed by sensor node \((i, j)\) in sensing, relaying, during sleep and in idle mode. Furthermore, \( x_i \) denotes energy consumption just being sensing and relaying data.

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only. \( F(x_i), \forall ij = \{11, 21, 22, 23, 33\} \) is the joint probability density function (pdf) of all nodes at time \( t \). Since Poisson processes are pure birth processes, the joint pdf of all the sensor nodes can be given by the differential-difference equation as:

\[
\begin{align*}
&\frac{dF_t(x_{i_1}, \ldots, x_{i_n})}{dt} = -\sum_{i=1}^{n} \sum_{j=1}^{k} \lambda_i \delta_t(x_{i_1}) + \sum_{i=1}^{n} \sum_{j=1}^{k} \lambda_j \delta_t(x_{i_n}) \prod_{m=1}^{j-1} \gamma_m x_{i_m} \times \\
&\prod_{m=1}^{j-1} \gamma_m x_{i_m} \\
&\exp\left[ -\sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i \delta_t(x_{i_1}, \ldots, x_{i_n}) \prod_{m=1}^{j-1} \gamma_m x_{i_m} \right]
\end{align*}
\]

Equation (1) reflects the overall energy consumption in awake state and in active state, i.e., during sensing and relaying data. An interesting observation is that the second term on the right hand side of (1) implies that due to sensing and relaying activity, the energy consumption reaches from \( y_i \) to \( x_i \). Considering (1) to be an initial value problem, we obtain \( R(s) \) as Laplace transform of \( F(x_i) \):

\[
R(s) = \exp\left[ -\sum_{i=1}^{n} \sum_{j=1}^{k} \lambda_i + \sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i \prod_{m=1}^{j-1} \gamma_m x_{i_m} \right]
\]

(2)

Now if we include the notion that sensor nodes occasionally adopt sleep mode on detecting no activity, the total energy consumption of a sensor node is given by \( y_i = y_i^n + y_i^s + y_i^t, \) where \( t = t^r + t^i \). According to this expression, introduction of sleep mode into sensor nodes suggests reduction in the overall energy consumption of the sensor networks proportionate to the sleep duration of sensor nodes \( \gamma_i \). This is however an implicit phenomenon that occurs simultaneously to energy conservation; when a node sleeps, it does not participate in relaying the data from neighbouring nodes. Thus the overall data relay activity is compromised for individual nodes’ energy conservation. Let \( Z(t) \) be the Laplace transform of \( F(y_i) \):

\[
Z_i(t) = R(s)^n \times \exp\left[ -\sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i s t_i + \sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i s t_j \right]
\]

In this study, availability \( A_i \) is adopted to be a measure of network lifetime and is defined as the probability that all the nodes along the paths are alive. Inserting (2) into (3) and manipulating the variables, the network availability is given by:

\[
A_i = \exp\left[ \frac{\gamma_i x_{i_1} \prod_{m=1}^{j-1} \gamma_m x_{i_m}}{\sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i \prod_{m=1}^{j-1} \gamma_m x_{i_m}} \times t_i \right]
\]

(4)

At this stage, we investigate the effect of regulating the transmission power on network availability of spatial distributions of sensor nodes by considering two unicast data relaying models, i.e., Delannoy number based and staircase lattice path traversals. If the transmission range is adjusted to \( R_i \), as shown in Fig. 1, staircase lattice paths are used, i.e., only leftwards or downwards (\( \leftarrow \downarrow \) or \( \leftarrow \uparrow \)) links are formed on relay data from the sensing node to the gateway and assuming a square topology, (4) can be transformed as:

\[
A_i = \exp\left[ \frac{\gamma_i x_{i_1} \prod_{m=1}^{j-1} \gamma_m x_{i_m}}{\sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i \prod_{m=1}^{j-1} \gamma_m x_{i_m}} \times t_i \right]
\]

(5)

Similarly, adjusting the power level such that the transmission range changes to \( R_i \), the data relaying activity turns out to be a different lattice path traversal, i.e., paths from sensing node to the gateway are formed by leftwards, downwards or diagonal-downwards (\( \leftarrow \downarrow \) or \( \leftarrow \uparrow \)) links as given by Delannoy numbers \( n \). The network availability of (4) is now given as:

\[
A_i = \exp\left[ \frac{\gamma_i x_{i_1} \prod_{m=1}^{j-1} \gamma_m x_{i_m}}{\sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i \prod_{m=1}^{j-1} \gamma_m x_{i_m}} \times t_i \right]
\]

(6)

The parameters in (5) and (6) are all assumed to be independent of \( i, j, k, m \) and \( n \) i.e., \( a_i = a, a_j = a, \lambda_i = \lambda, \lambda_j = \lambda, \gamma_{i-1} = \gamma, \gamma_{j-1} = \gamma, \) and \( y_{i-1} = y_{j-1} \). The numeric are as adopted from [7]:

\[
\begin{align*}
&\nu_a = 15 \, \mu s, \quad \nu_y = 1.5 \, \mu s, \quad \lambda = 0.083 \, \text{packets per second,} \\
&\nu_{\gamma} = 12960 \, J, \quad \nu_{\eta} = 42.61 \, J, \quad \nu_{\gamma_{\gamma}} = 140.87 \, J \, \text{for staircase} \\
&\nu_{\gamma_{\gamma}} = 280 \, J \, \text{for Delannoy.} \, \text{The number of nodes} \, n \, \text{varies} \\
&\text{from 4 (or } n = 2 \text{) to 1600 (or } n = 40 \text{).}
\end{align*}
\]

Figure 2 plots the difference between (6) and (5) for two scenarios, i.e., fixed distance and varied distances. According to Fig. 2a, when the distance is fixed to 1000 m and the numerical plots are obtained for \( t = 15552000 \, \text{sec (six months), staircase} \, \text{lattice path} \)}.
as compared to Staircase, avoiding sleeping nodes effectively. Sensor networks that incorporate sleep mode conserve energy at one hand but waste the relaying energy on the other. Delannoy number based traversal can reduce such an adverse effect of sleeping nodes.

Figure 2b is the effect of distance variation onto network availability at t=100000 sec (11.57 days). The distance was varied from 100 m to 5000 m with a step size of 150 m. For very large networks, e.g., for a network size of 1600 nodes or more, when the inter-node distance is increased for a fixed number of sensor nodes, Delannoy number based lattice path traversal starts to outperform staircase based data relay. This suggests an added advantage of increasing the transmission power on the network availability for wide spatial distributions of sensor nodes.

CONCLUSIONS

In this study, we study the spatio-temporal effects of transmission power adjustment onto network availability of sensor nodes deployed across a two-dimensional space that implement sleep mode. We observe that doubling the transmission power of sensor nodes can help incorporate diagonal neighbours into the data relay activity from sensing nodes to the gateway. This results into better network availability due to a decrease in effective number of hops for very large deployment of sensor nodes. It is clearly against the apparent notion that lifetime reduces by increasing the transmission power. It is also noticeable that increasing the transmission power also increases the probability of finding alternate paths for two cases; first, when the sensor nodes are distributed in a wide area; second, when sensor nodes sleep to conserve energy and make the intermediate paths unavailable.

REFERENCES