Radiation Spectral Integral for Tapered Structure in Optical Communication

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Abstract: The study here is mainly adapted to the implementation of a radiation spectral integral, involved to describe modal field behaviour outside a non uniform structure for optical communication purposes. This is achieved by simply associating with each individual spectral plane wave incident on any boundary of the structure, a refracted wave with appropriate transmission coefficient. This spectral integral describes rigorously and systematically a source-free field behaviour not only before and after a branch point singularity (transition region), but also outside a tapered structure too. In this sense, the implementation of the resulting spectral formulation, for the case of homogenous media, contains all informations pertinent to the modal propagation mechanism, not only outside the structure, but also inside it, before and after the singularity caused by cut off of the propagating mode.

Key words: Integrated Optics, optical communication, thin film waveguide, fiber optics

INTRODUCTION

As the purpose of this study is an original investigation of a structure using a tapered configuration; the impetus was given in order to analyse the field distribution involved in the propagation process outside such a structure using the tapered waveguide as a main body (Arnold, 1985). It is then, necessary to track the motion of any observation point X, not only along and inside the tapered waveguide itself, but also across the cross section of its adjacent bottom medium (n₁). The observation point X is locally positioned at a thickness T, as shown in Fig. 1.

From a physical point of view, rays of the spectrum undergo bottom reflections at Γ₁₂, interface adjoining the wedge angle a and at vicinity of the critical angle θc defined by the relation cos (θc) = (n₂/n₁); when total internal reflections prevail. But, for observation points X located beyond critical transition range, rays reaching X begin to radiate through Γ₁₂. It is the purpose of this article to investigate the modal field distribution in the adjacent bottom medium (n₁). To achieve this, it will be necessary to introduce the concept of a radiation spectral integral which governs the field propagation in medium (n₁) and which will describe the radiation mechanism taking place in the structure. The following treatment could also apply to top adjacent medium (n₁) which is the free-space. But we restrict ourselves to the radiation process occurring at bottom medium (n₁) only; for one has appropriately chosen the refractive indices such as, n₁ > n₂ > n₃.

MATERIALS AND METHODS

Model presentation: The mathematical model implementing the radiation integral in an open region can be presented by a spectral integral representation R(X,θ). This is accomplished by extending the preceding plane wave spectral analysis fully expressed by Arnold (1985) and Belghoraf (1992), hence:

\[ R(X,\theta) = (2\pi)^{-1/2} \int_{\gamma} \exp[jkS(X,\theta)]d\theta \]  \hspace{1cm} (1)

Where the integration contour (\(\gamma\)) is fully developed by Belghoraf (1988). The phase S(X,θ) will explicitly be defined later. The wavenumber k in here refers to bottom medium (k = n₁ k₀). The parameter k₀ is the free-space wavenumber. The expression given by Eq. 1 must satisfy the boundary conditions at Γ₁₂. That is at X = 0 or X = a, respectively (Fig. 1). In this respect, one constructs the radiation integral R(X,θ) by tracking the spectrum of a particular and appropriate species of wave that radiates by refraction into the corresponding boundary and which satisfies that boundary condition. To present the radiation integral referred to bottom boundary Γ₁₂, we consider only one type of wave among the four species introduced by Arnold (1985) and Belghoraf (1992) and which is characterised by the phase Sᵣ(X,θ). This choice is justified by the fact that [jkSᵣ(X,θ)] is a wave which is destined to be refracted at the bottom medium (n₁). Multiplying the appropriate downward propagating plane waves Sᵣ(X,θ) in Eq. 1 by the transmission coefficient at...
\[ R(X, \theta) = \frac{(2a)^{1/2}}{a} \left\{ 1 + \exp[i\phi(\theta)] \right\} \exp\left[-n_1 k_1 r \cos(\theta) - \left(\pi / (2a)\right) - \left(\pi q / a\right)\right] d\theta \]

We bear in mind that \( \phi(\theta) \) is the phase of the Fresnel reflection coefficient (Tamir, 1979) and \( q \) is the mode number to be exited along the structure. We also recall that the incidence angle \( \theta \) becomes \( \theta' \) corresponding to the direction of propagation of the refracted wave in the adjacent bottom medium \( n_2 \). These angles are both interrelated by Snell’s law, which stipulates that:

\[ n_1 \cos(\theta) = n_2 \cos(\theta') \]  

Combining Eq. 2 and 3, after expanding the cosine term in the integrand of Eq. 2, we obtain after neglecting terms which vanish as \( a \to 0 \) (this is physically compatible with our structure geometry):

\[ kS(X, \theta) = \frac{1}{(2a)} \int \phi(\theta')d\theta' - \left[ \frac{\pi X}{(2a)} \right] (2q - 1) - \left[ (k_2 n_1 r) / n_1 \right] [n_1 \cos(\theta) \cos(X - a) - \sin(X - a)(n_2^2 - n_1^2 \cos^2(\theta))^{1/2}] \]

\[ \frac{dS}{d\theta}(X, \theta_{1}) = 0 \]  

Combining Eq. 5 and 6, yields the new characteristic equation, which is:

\[ \sin(\theta_{1}) \cos(X - a) + [\phi(\theta_{1}) - \pi (2q - 1)] /[2nk_1 n_2] + \sin(X - a) - n_1 \sin(\theta_{1}) \cos(\theta_{1}) = 0 \]

\[ (n_2^2 - n_1^2 \cos^2(\theta_{1}))^{1/2} \]  

One notices the dissimilarity between Eq. 7 and the original eigenvalue equation treated (Arnold, 1985; Belghoraf, 1992). The former exploits the concept on an intrinsic field integral used to analyse field propagation inside the tapered waveguide; whereas the latter makes use of a concept of radiation integral developed to investigate the field behaviour outside the tapered waveguide. In Eq. 7 the additional terms are due to the \( X \)-dependence quantity which mathematically accounts for the dependence on depth in the adjacent bottom medium \( n_2 \); and which appears here to behave as a parameter. At interface \( \Gamma_{12} \), say at \( X = a \), Eq. 7 and that found by Arnold (1985) and Belghoraf (1992) are identical. Hence, they engender the same saddle points \( \theta_{1} \). For at
Fig. 2: Computation of eigenvalue equation Eq. 7 of the radiation integral. Solutions in the complex $\theta$ plane are the incident angle $\theta_1$ (saddle points) for the lowest mode, as the thickness $T$ is arbitrary reduced. Three multiple values of the wedge angle $a$ are considered for the cross variable $X$, the wedge angle $a = 0.027$ rads. The refractive indices in each medium are: $n_1 = 2, n_2 = 1.76, n_3 = 1$. The $V$ sign in figures locates the branch point $\theta_v$.

$X = a$, matching boundary values are necessarily required between the construction of the radiation integral $R(X,\theta)$ whose eigenvalue equation is given by Eq. 7 and that of the intrinsic integral $I(X,\theta)$ developed by Arnold (1985) and Belghoraf (1992) whose eigenvalue equation is also given. One recalls that $I(X,\theta)$ describes the field inside the tapered waveguide and it is analytically expressed in the cited references. A proper substitution of $\phi(\theta)$ in Eq. 7 by the phase of the Fresnel's coefficient leads to the corresponding eigenvalue equations which engenders the saddle points $\theta_v$. The saddle points are then located for each observation point $X$ defined by the polar coordinates $X=(X, r)$. The parametrically computed eigenvalue Eq. 7 is depicted in Fig. 2 for successive values of the transverse variable $X (X=2a)$ and for the lowest mode only. In other words, Fig. 2 show the physical effect on the eigenvalue Eq. 7, of moving the observation point $X$ outside the tapered waveguide and away from bottom boundary $\Gamma_1$. Figure 2a shows a plot corresponding to the parameter $X = a$, where the observation point is located on bottom interface $\Gamma_1$.

One notes that such a parametrical case has already been treated via the eigenvalue Eq. 7, whose numerical plotting is reported (Arnold, 1985, Belghoraf, 1992); a case dealing with the analysis inside the tapered optical waveguide. Locating an observation point $X$ at $X = 2a$ or at $X = 3a$, as shows by Fig. 2b and c, respectively, has the effect of shifting up the solutions of Eq. 7 (saddle points $\theta_v$), particularly those lying beyond the transition region, to a region near the branch point $\theta$, which is denoted by $V$ sign in figures. For those solutions which are situated in the guided wave region (region delimited by $\theta < \theta_v$), they exhibit a slight positive imaginary component. In the case ($X = 2a$ or $X = 3a$), all solutions which are supposed to belong to the leaky wave region (region delimited by $\theta > \theta_v$), seem to be coinciding with the branch point $\theta$. In other words, choosing a thickness $T$ beyond transition region, will have the effect of bringing all complex saddle points to coalesce with $\theta$. Such an effect, exhibited by Fig. 2b and c, can geometrically be explained by reference to Fig. 3. As a matter of fact, in the guided wave region, as in Fig. 3a, the saddle points $\theta_v$ are within the interval $0 < \theta < \theta_v$. Consequently, all rays inside the tapered waveguide undergo total multiple reflections at $\Gamma_1$ and $\Gamma_3$ interfaces. For any observation point $X$ located outside the tapered waveguide and within the guided wave region, there are evanescent waves accommodate by the complex $\theta_v$. As $X$ moves away from $\Gamma_1$ (as $X$ increases), the transverse propagation constant $\tau$ of that evanescent wave becomes 'more complex' and entails a strongly decaying evanescent field. The constant $\tau$ is defined as follows:

$$\tau = (n_2^2 - n_3^2 \cos^2(\theta_q))^{1/2}$$

(8)

Hence, one expects the imaginary part of the saddle point to increase. However, in the leaky wave region in contrast, Fig. 3b shows how the rays are refracted into medium ($n_3$), once the branch point $\theta$ is exceeded by $\theta_v$. In this case, any observation point $X$ outside the tapered waveguide receives the contribution of 2 distinct rays impinging at $\Gamma_1$ interface, from 2 angles of incidence, $\theta_\alpha$ and $\theta_\beta$. Therefore, 2 saddle points ($\theta_\alpha$ and $\theta_\beta$) are required at a given thickness $T$ and at a given transverse variable $X (X>a)$. Because, one of these is at incidence.
angle near to $\theta_c$, the corresponding ray refracts from the tapered waveguide to the open medium ($n_i$), with a near grazing refraction angle. As the saddle point is condensed near the singularity $\theta_c$, it is very difficult (if not impossible) to numerically assess absolutely the branch cut contribution. Such an effect, which manifests itself strongly when the observation point $X$ is located outside the tapered waveguide ($X>a$) and beyond transition region, complicates the evaluation of the integral $R(X,\theta)$ in Eq. 4 by the saddle point method.

RESULTS AND DISCUSSION

Integrand variation of the radiation integral along the real axis: To circumvent those above difficulties and to achieve integration of Eq. 4 systematically, we suggest another method, already used (Belghoraf, 1992); which is to keep the undeformed original contour (C) and to abandon the saddle point method. The original contour (C) has a path coinciding with the real axis. We omit however the two lower tails because computational results have shown that their evanescent contributions are negligible compared to the real axis integration. Hence, we shall restrict the integration contour (C) to the interval $0<\theta<\pi/2$. For justification, let us plot the variation of the phase $S(X,\theta)$ of the integrand in Eq. 4 along the real axis at a given observation point $X$ situated on bottom boundary $\Gamma_{12}$ ($X = a$). Only the lowest mode is concerned.
Fig. 4: a) Computation of the phase in the radiation integral Eq. 4, versus the incident angle \( \theta \) varying along the real axis. b) Computation of the integrand in the radiation integral Eq. 4 versus the incidence angle \( \theta \) varying along the real axis. The saddle point \( \theta_s \) is located in the guided wave region at \( k_x T = 5 \). Mode 1, normalised critical thickness = 1.75. The x sign locates a saddle point \( \theta_s \); the \( \nabla \) sign locates the branch point \( \theta_b X = a = 0.027 \) rds.

**Saddle point \( \theta_s \) located in the guided wave region:** Figure 4a shows the complex variation of \( S(X,\theta)-S(X,\theta_s) \) versus the incidence angle \( \theta \) varying on the real axis. The real saddle point \( \theta_s \) is located in the guided wave region at \( k_x T = 5 \). Real \( [S(X,\theta)-S(X,\theta_s)] \) has an extremum at the chosen saddle point \( \theta_s \) which is denoted by the x sign in figures. Its imaginary part is zero for all real \( \theta \)'s lying in the region \( \theta < \theta_s \); where \( \theta_s \) is the branch point for the phase \( S(X,\theta) \) and it is denoted by \( \nabla \) sign in figures.

This means that \( \text{Imag} [S(X,\theta)] \) is constant in the guided wave region. However when \( \theta > \theta_s \), the imaginary part increases in magnitude, then leakage occurs. It is that very imaginary part of \( S(X,\theta) \) that accounts for the amplitude decay of the field in the leaky wave region. In contrast, Real \( [S(X,\theta)] \) decreases towards the constant Real \( [S(X,\theta_s)] \) as \( \theta \) tends to \( \theta_s \) and increases as \( \theta \) tends to \( \theta_b \) and beyond. Figure 4b shows the complex variation of the integrand in Eq. 4, when the incidence angle \( \theta \) varies on the real axis. Both components exhibit extrema at location of the saddle point. They oscillate initially, then decay exponentially as the incidence angle \( \theta \) approaches \( \pi/2 \). This signifies that for \( \theta > \pi/2 \), the radiation integral of Eq. 4 engenders a vanishing field. In other words, there is no contribution of rays whose incidence angle is higher than \( \pi/2 \). This important statement justifies and accounts for the neglect of all types of subsequent rays in the establishment of Eq. 4.

**Saddle point \( \theta_s \) located in the leaky wave region:** Figure 5a shows the same phenomena for a saddle point \( \theta_s \) located in the leaky wave region at \( k_x T = 1.2 \). We notice that \( S(X,\theta)-S(X,\theta_s) \), accommodates a shift due to the fact that the saddle point \( \theta_s \) is complex in this leaky wave region. Therefore, any variation of \( \theta \) on the real axis, will never coincide with \( \theta_s \). Henceforth, \( S(X,\theta)-S(X,\theta_s) \) will never fall to zero. This explains why \( S(X,\theta)-S(X,\theta_s) \) exhibits a shift in its real part. This shift is more accentuated at \( \theta_s \), becomes strongly complex, that is to say, as the observation point \( X \) tends towards the apex (or as \( T \) diminishes). As for the variation of the integrand of Eq. 4, Fig. 5b shows that initially it exhibits more rapid oscillation and then decays exponentially and faster than in Fig. 4b.

Hence, here too, the field engendered by the radiation integral Eq. 4 vanishes as \( \theta \) tends towards \( \pi/2 \). For any location of the saddle point \( \theta_s \) with respect to \( \theta_b \), it is safe to neglect the field contribution outside the range \( 0 < \theta < \pi/2 \). Returning to the radiation integral in Eq. 4, we shall then perform it along the real axis and in the interval \( 0 < \theta < \pi/2 \). In this case, the presence of any branch point will automatically be taken care of (Belghonif, 2001). The convergence of Eq. 4 is, however, guaranteed by taking an integration step much smaller than the oscillating periods of Fig. 4 and 5. It is also necessary to maintain the same branch conventions for \( (\theta-\theta_s)i0 \) and \( (\theta-\theta_b)i0 \).
defined earlier. In as far as we are not using the saddle point method to calculate Eq. 4, knowledge of the saddle point \( \theta_e \) is not necessary. In spite of this, we still represent each observation point \( X = (X, r) \) by its equivalent notation \( X = (X, \theta_e) \). Note that Fig. 4 and 5 have all been numerically carried out for an observation point \( X = a \) and for the first mode \( q = 1 \) only. Similar qualitative results could have been obtained for any other parameter.

CONCLUSIONS

Numerical plotting of the radiation field in the guided wave region: Let us now concentrate first on Fig. 6. They shows the variation of the normalised field modulus in medium \( n_3 \), versus the variation \( X \), for mode 1.

Three locations of the observation point \( X \) are considered in the guided wave region, \( \theta_s > \theta_e \). They correspond to the three distinct normalised thicknesses in (i), (ii) and (iii) in figures 6. Thereby, it is clear that, as \( \theta_s \) approaches \( \theta_e \) (that is to say, as \( k \lambda \) approaches the critical thickness of the corresponding mode 1), the evanescent field decays less rapidly in the medium \( (n_3) \). The decay is more strongly evanescent, when \( X \) is located far from \( \theta_e \), as in (i), than when it is near, as in (iii). This is mainly because, in such a region, the waves inside the tapered waveguide are totally guided. When \( \theta_e \) approaches \( \theta_s \), as in (iii), energy starts leaking out from the inside of the tapered waveguide to medium \( (n_3) \). As a matter of fact, this leakage near the transition region, makes the amplitude of the cross section field inside the tapered waveguide decrease in the guided wave region of course, the above remarks hold for any higher mode, characterised by its own corresponding critical thickness.

Numerical plotting of radiation field in the leaky wave region: In this case, all observation points \( X \) are located in the leaky wave region, such that \( \theta_s > \theta_e \). In this region, energy is leaked out from the tapered waveguide to medium \( (n_3) \). Such a leakage characterises the radiation process taking place at bottom interface \( \Gamma_{13} \). The light rays, then, are no longer totally reflected back into the tapered waveguide, but are partially transmitted into medium \( (n_3) \) as refracted waves. Figure 7 show the variation of the normalised modulus of the radiated field, versus the angular variable \( X \), for mode 1.

Each position of \( X \) is located in the leaky wave region. It is seen in each plotting that the field oscillates to a maximum, then decays exponentially because of the continuous refraction taking place in medium \( (n_3) \). Also, in each diagram, the locus engendered by each maximum of the radiated field describes a caustic (Oliver, 1974) whose gradient with respect to the bottom interface \( \Gamma_{13} \) corresponding to \( X = a \), represents the
Fig. 6: Computation of the normalised cross section field in medium \((n_i)\) versus the cross angle \(X\). The saddle points in (i), (ii) and (iii) are located in the guided wave region. Mode 1, normalised critical thickness \(= 1.75\), \(a = 0.027\) rds, \(n_1 = 2\), \(n_2 = 1.76\), \(n_3 = 1\).

Fig. 7: Computation of the normalised cross section field in medium \((n_i)\) versus the cross angle \(X\). The saddle points in (i), (ii) and (iii) are located in the leaky wave region. Mode 1, normalised critical thickness \(= 1.75\). The top of each figure represents the caustic.

directionality of the beam of the radiation pattern in medium \((n_i)\) (Belghoraf, 2001 a,b). It is the existence of this caustic that causes the field (in each diagram of Fig. 7) to oscillate in one part of the cross section pattern and decay exponentially in the other. We also notice that the amplitude of the field maximum becomes smaller as the thickness \(k_0 T\) diminishes from one diagram of Fig. 7 to another. This emphasises the fact that as \(\theta_0\) moves in the leaky wave region, away from \(\theta_0\) and towards \(\pi/2\), the field tends to vanish. The same remarks apply to any higher mode.

REFERENCES


