A New Assembly Sequences Generation of Three Dimensional Product Based on Polychromatic Sets

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Abstract: This study presents a new systematic approach for automatic assembly sequence plan generation by using an integrated framework of the polychromatic sets and assembly information model. The purpose of this research is to propose a unified and integrated mathematical representation which can be managed automatically by computer and optimal assembly sequences can be generated for various different assembly conditions. Firstly, the position, assembly sequences and conjunction relations of the original product are simplified into the so-called polychromatic sets and the part information, associate relation and mating relation are arranged, simplified and systematically represented by polychromatic matrixes and the assembly information model is constructed. By doing so, the assembly relation model can be obtained easily. Thirdly, with the model presented above, localization relation equations, obstacle disassembling relation equations and conjunction relation equations can be obtained. Thirdly, a new sequences extract algorithm called assembly sequence generation algorithm based on polychromatic sets (PSAPS) is presented to obtain the feasible sequences of product. A product assembly example is presented to illustrate the effectiveness of the proposed method.

Key words: Index terms-polychromatic sets, assembly information model, locating equations, obstructing disassembly equations

INTRODUCTION

Assembly sequence planning is a major portion of process programming in a production system. Good assembly sequence planning has been recognized as a practical method for reduction of operation difficulty, tool quantity and work hours (Lai and Huang, 2004). The traditional generation methods of sequence have been considered in four types: logical precedence relation question-answering method, matching requirements method, code method and hierarchical compression decomposition method. Bourjault (1984) proposed a question-and-answer procedure to obtain the precedence constraint among the components of the final product. However, in this approach the number of questions grows exponentially as the part number in the assembly increases.

De Fazio and Whitney (1987) developed a method based on Bourjault's procedure by reducing question-count to twice the number of liaisons in the assembly. Similarly De Mello and Sanderson (1989) presented a method according to the conjunction mode to obtain sequences; De Mello and Sanderson (1989) used a representation of the directed graph of feasible assembly sequences in which an AND/OR graph is used to represented all possible sets of the assembly sequences of a product. However, this approach become unmanageable as the nodes and the connected paths increase exponentially with number of assembly components needed for a completed representation. Bourjault (Lai and Huang, 2004) proposed an inverted tree procedure based on graphical representation. Using the matrix mode to represent the scheme of assembly planning. Dini and Santochi (1992) proposed a mathematical model of liaison properties for the components of the production. Recently, some studies have been done on integration of planning design, modeling based on features and evaluation sets of assembly (Huijun and Mingmin, 1996). Deng et al. (2000) described the importance of assembly features in assembly modeling and planning and he presented the junction relations between the assembly features and the relation between parts, based on these relations, an algorithm for assembly planning was developed.

These representation schemes described above comprise the major theories of assembly sequence planning, however, when considering the assembly sequence planning of a complex product consisting of a large number of parts, these schemes require a large
amount of memory and long operation times thereby making them unsuitable for computerization and subsequent automation.

In this study, the assembly features are extended according to the idea presented by De Mello and Sanderson (1989), we see the disassembly as the inverse procedure of assembly and an assembly sequence generation method is presented according to the disassembly procedure. Firstly, we described the integration of assembly information in detail, then analyze the relations in assembly and the summarized has been done about assembly information. Finally, a new assembly planning sequences based on polychromatic sets algorithm is presented which firstly construct the assembly relation model, then built the location relation equations, obstructing disassembly equations and junction relation model, by multi class assembly analysis all the feasible sequences can be obtained.

**ASSEMBLY RELATION MODEL**

Assembly relation model is the basis for sequence generation which supplies the all information of parts (Huijun and Mingmin, 1996; Li and Xu, 2003; Zeng, 2001). The assembly relation in a product mainly contains the relations the relations between the structure attributions of parts and parts. In this assembly information model, the parts information is supplied and the model is used to describe the location relations and interference relations between parts.

**Problem statement:** The initialization phase in assembly sequence planning is the assembly information modeling, which consist of all the information that needed in assembly sequences generation. The model is vital to the efficiency of reasoning in assembly sequence generation. ACAD model of a final product could described all the geometrical features and spatial relations, which are suitable for display and some man-machine interactive, however, which cannot supply enough information for assembly sequences planning (Zeng, 2001).

In this study, we classified the assembly information into three kinds of relations: localization relation, constrain relations between parts which are involved in sequence planning and conjugat relation. The assembly information has two kinds of feature: part level features and associated features. The part level feature contains the geometry feature and structure attribute such as ‘with screw’ or non screw and so on; the associate level feature contains the connect relation, mating relation, position relation and motion of part. Recently, a large amount of methods are presented for describing product assembly information, such as connected graph mode, contact model, construction graph model, associated model, hierarchy model and recently assembly model based on geometry information, all these methods mainly emphasis on the geometry constrain information. Most of these models cannot be used in computer recognition and contain uncompleted information.

In this study, we built the associated diagram (Bourjault, 1984) of assembly body based on concurrent engineering method and construct concurrent model which integrated the part level features and associate level features. These two features are all modeled with polychromatic matrix which is very easy for product modeling accurately and recognition for computer. Polychromatic sets is a novel modeling tools which is easily organized for complex model and can be used in information modeling for assembly information modeling and it is described by Bourjault (1984). The produce of modeling is consisting of two phases as follows:

- **Extract the parts level features, associate level features and mating relations information in assembly body according to the assembly requirements, then construct the assembly information tables.** In technical system modeling, it is not needed to study all property of system and its elements and only by using the attributes of a concrete problem we can construct the assembly information tables.

- **Construct the assembly information model by using polychromatic sets.** Each elements and the entity are pigment with various color to represent the attributes of object. The traditional sets are an example of polychromatic sets in which all objects were pigmented with one kind color. The color set $F(a)$ correspond the attribute of a element $a \in A$ and the color set $F(A)$ represents the attribute of the entire set $A$. The individual color of all elements can be described with $|c_{i}| = [A \times F(a)]$, where $F(a) = F(a) \cup \ldots \cup F(a)$.

In this study, we consider that the product $A$ consist of basic parts $a$, and each parts has its individual color and we use the individual color $F(a)$ to describe the part level features and associate level features of a part. In polychromatic model, the parts are the basic elements and the part level features and associate level features are the attributes, then we can construct the polychromatic sets. If $a \in A$ and there are $F(a) \in F(a)$, the $c_{i} = 1$, we represent this relation with '○' in product assembly information matrix; if $a \in A$ and there not the relation $F(a) \in F(a)$, then $c_{i} = 0$. 

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Fig. 1: Parts in reducer output shaft

Fig. 2: The associated diagram of output shaft and the assembly information table

Here, we will describe the method to construct product assembly information matrix with an example of reducer output shaft. In Fig. 1 the parts of reducer output shaft are shown, with the method described above, the assembly information matrix can be construct as Fig. 3 the procedure as follows.

- Construct the assembly information tables of reducer output shaft. We can construct the part associated diagram as shown in Fig. 2 according parts in reducer output shaft, from Fig. 2a, all the assembly relations between parts can be obtained, then the information between parts and reducer output shaft can be obtained, mainly are the structure information and the mating relation between parts. In this phase, only the information related with assembly are considered, which decreased the scale of the matrix and increase the efficiency of assembly. Figure 2b shows all parts level information and associated level information related with output shaft.

Fig. 3: Assembly information model of output shaft based on polychromatic

- Construct the assembly information matrix of output shaft. The assembly information of output shaft can be classified as two kinds, one is part structure information: general part $F_1$, part with screw $F_2$, connector $F_3$, another is mating information of part: shrink fit $F_4$, transition fit $F_5$, gap fit $F_6$. By using polychromatic sets, the assembly information table can be converted into Boolean matrix. Set the output shaft as the assembly entity $A = \{a_0, a_1, \ldots, a_n\}$, in which $a_i$ represents the part contained by output shaft, the structure information and mating information of parts can be see as individual $F(a_i)$ and the entity color can be represented as $F(a_i) = F(a_0) \cup F(a_1) \cup \ldots \cup F(a_n)$. If $a_i \in A$ and $F(a_i) \subseteq F(a_0)$, then $c_0 = 1$ which is showed in Fig. 3 as $\circ$, if not, $c_0 = 0$.

So, the assembly information of output shaft can be represented with a Boolean relation graph, which easily for extraction and computing in next phase.

Assembly constrain relation model: The generation of constrain relation is very important in assembly sequences planning. There are some constrain relations between the parts of assembly body in assembling and the localization method of parts the other part which obstacle the part assembly should be took into account. In general, the assembly procedure is reversible, so, we defined the assembly constrain relations as three kinds: localization relation, obstacle disassembling relation and junction relations. There are constrain relations between the mating parts and there maybe some constrain relations between those not mating parts, such as obstacle disassembling relation. In these section, an assembly constrain relation model is construct based on the method described above which consist of localization relation equation, obstacle disassembling relation equation and conjugation relation equation and the interference relation between localization and assembly motion.

Localization relation equation: Localization of parts must be taken into account in assembly planning. Each part are
consist of simple plane or curved face, in assembly part must contact each other with localization face and hold the status, so, the localization relation contains two: plane mating and curved face mating. If a part localized do not depend on other part in the assembly entity instead of a factor \((a_i)\) out of assembly entity, the part can be named as self-localization part. The localization relation equation is used to describe the localization relation between parts (Li and Xu, 2003; Zeng, 2001). To a part, if there are a set of parts which localized it, then the logic equation construct by the logic relation of the set of parts is called localization relation equation.

\[
B(a_i) = b(a_i) \land (\lor) b(a_j) \land (\lor) \cdots (\lor) b(a_n)
\]  

(1)

Where, \(b(a)\) represents the logic value that \(a_i\) in \(a_i\)'s localization, if \(a_i\) was localized early than that of \(a_i\), then \(b(a_i) = 1\), if not, \(b(a_i) = 0\). According the relation between parts, the \(b(a_i)\) can be calculated. Accordingly (1), a part can be known localized or not can be confirmed, for example, the localization status of \(a_i\) part is decided by its localization relation equation \(B(a_i) = 1\), if \(a_i\) is localized, then \(B(a_i) = 1\), if not, \(B(a_i) = 0\).

**Obstacle disassembling relation equation:** Disassembling procedure is more easily understood for human, in general the obstacle disassembling and assembly produce are a pair of inverse procedures. So, here we used the obstacle disassembling equation instead of obstacle assembling equation. The obstacle disassembling equation is used to describe the situation that it is reversed by other parts. The obstacle disassembling equation can be constructed according the logic relation of parts that obstacle the part:

\[
W(a_i) = w(a_i) \land (\lor) w(a_j) \land (\lor) \cdots (\lor) w(a_n)
\]  

(2)

Where, \(w(a)\) represents the logic value that \(a_i\) in \(a_i\)'s localization, if \(a_i\) was assembly early than that of \(a_i\), then \(w(a_i) = 1\), if not, \(w(a_i) = 0\). According the relation between parts, the \(w(a_i)\) can be calculated.

According to Eq. 2, a part can be know disassemblies or not can be confirmed, for example, the obstacle interference status of \(a_i\) part was decided by its obstacle disassembly relation equation \(B(a_i)\), if \(a_i\) was disassemblies, then \(W(a_i) = 1\), if not, \(W(a_i) = 0\).

**Conjunction relation equation:** Because the connector and fastening piece are all standard part, it do not participate the generation sequences problem of assembly, so, many studies ignore its effective in assembly planning. However, the conjunction is very important for the stability of assembly entity in sequence generation planning. Conjunction connects consist of bolt and screw structure, welding connect, rivet bond, cementation, shimming. Most of product contains of bolt and screw structure, here, we used the bolt and screw structure as an example to construct the conjunction relation equation:

\[
D(a_i) = d(a_i) \land (\lor) d(a_i) \land (\lor) \cdots (\lor) d(a_n)
\]  

(3)

Where, \(d(a)\) represents the logic value that \(a_i\) in \(a_i\)'s connection, if \(a_i\) fulfill the Eq. 1 and 2, the \(d(a_i) = 1\), if not, \(d(a_i) = 0\). According the connect relation to decided the and/or relation of \(d(a_i)\), ..., \(d(a_n)\), to obtain \(D(a_i)\). If \(D(a_i) = 1\), it represented that the parts connected with \(a_i\) are fulfilled the connection requirements, otherwise, it represented that the parts connected with \(a_i\) are not fulfilled the connection requirements.

**ASSEMBLY SEQUENCE GENERATION ALGORITHM BASED ON POLYCHROMATIC SETS (PSAPS)**

PSAPS algorithm select the first level feasible assembly parts according the self-localization conditions and according the localization equation and obstacle disassembling equation, the next level assembly feasible parts can be selected, until the last level part, all the feasible assembly sequence can be obtained. Different with existing algorithm which emphasis on the geometry structure and assembly directivity, the PSAPS is based on the reversibility of disassembling and assembly, it emphasis on the assembly ability to clarify the algorithm.

**Definition 1:** \(a_n\) represents self-localization conditions, which contains the external factor that made the part can self-localization, \(a_i\) \(\{i = 1, ..., n\}\) represent these parts.

**Definition 2:** \(s_i, \{i = 1, ..., n\}\) represents the \(i\)th level feasible sets of assembly part, if there \(n\) parts in assembly entity, then the assembly procedure can be classified \(n\) level, \(S =\{s_1, s_2, ..., s_n\}\), feasible assembly sequence is those feasible sequence which fulfill localization equation and obstacle disassembling equation.

**Definition 3:** \(B(a_i) = b(a_i) \land (\lor) b(a_i) \land (\lor) \cdots (\lor) b(a_n)\), \(i, j, l, m, c, \{i, j, l, m\} \in \{1, ..., n\}\), represent the localization relation between parts in assembly entity. \(W(a_i) = w(a_i) \land (\lor) w(a_i) \land (\lor) \cdots (\lor) w(a_n)\), \(i, j, l, m, c, \{i, j, l, m\} \in \{1, ..., n\}\), represent the disassembly path constrains of parts in assembly body.

D(a) = \{d(a)i ∨ d(a)j ∨ d(a)k | i, j, k ∈ \{1, ..., n\}, i ≠ j ≠ k\}, represents the conjunction relations in conjunction body.

PSAPS Algorithm

Input: a0, ai, i = 1, ..., n, all parts in assembly body, the localization equation of part is B(ai), the obstacle disassembly equation of part is W(ai), the conjunction relation equation is D(ai).

Output: All feasible assembly sequence.

Initialization: If there N parts, M connector, according Eq. 1-3, the b(ai), w(ai), d(ai), i = 1, ..., N can be obtained and the equation B(ai), W(ai), D(ai) i = 1, ..., N, can be obtained.

Step 1: According the self-localization a0, the first level assembly parts set s0 can be obtain which fulfilled the follows equation: s0 = \{ai | i = 1, ..., n\}, where, a fulfisfied B(ai) = b(ai)i \vee b(ai)j \vee b(ai)k ..., and, i ≠ j ≠ k, that is to say, B(ai) = 1, in this step the first level assembly parts sequence s0 can be obtained;

Step 2: According s0 to calculate s1:

Begin Calculate s1
1: for j = 1,1:N and i ≠ j
2: if (b(ai) = 1 and W(ai) = 0)
3: s1 = s0 + a1
4: end if
5: for temp = 0:1:M
6: if (a0D(a0) \cap D(a0) = 1)
7: s1 = s1 + a1
8: endif
9: endfor
End

Step 3: Calculate next level until all N level assembly feasible parts set can be obtained.

Begin CalculateNextLevel
1: for k = 1,1:N and k ≠ j and k ≠ j
2: if (B(ai) = 1 and W(ai) = 0)
3: s1 = s1 + a1
4: end if
5: for temp = 0:1:M
6: if (a0D(a0) \cap D(a0) = 1)
7: s1 = s1 + a1
8: endif
9: endfor
End

Step 4: In step 2, the feasible sequences sets S = \{s1, s2, ..., sn\} can be obtained, in which, s2i = 1, ..., n is the part sequence set, it can be represented as:

S = \{s2, s2, ..., sn\} \{\{a1\}2, \{a2\}2, ..., \{an\}2\}, i ≠ j ≠ k.

Obviously, matrix S is a m × n matrix, m is the number of feasible sequence, which was decided by minimum feasible sets from Step 2 and each row elements in S represent a set of feasible assembly sequence. And all feasible sequence can be obtained.

CASE

In reference (Zeng, 2001), a method for assembly sequence generation of reducer output shaft was presented, it suitable for 2D assembly. The PSAPS presented in this study suitable for complex product by using Boolean matrix, as an example, the algorithm for assembly of reducer output shaft shows in Fig. 4.

There are 32 parts in reducer output shaft and 12 bolts, 24 screws. Vent cap, oil leveler, choke plug and eye bolt as the attach structure of reducer output shaft, have little effect to the assembly sequence, so, in this study, we ignore them in study of assembly sequence generation.

Most mechanical products consist of parts with hierarchy structure and in assembling, parts are assembly to components and components assembly to product. So, according the hierarchy relations, product can be disassembling into several sub-assembly body which consist of one or more parts. In assembly body P = \{p1, p2, ..., pn\} if there are m = 2m = N-1 parts, built which the set, S = \{p1, p2, ..., pn\} fulfilled the conditions as follows:

- If S is an un-self disjoins structure according the relations of parts, then S is a stabilization structure;
- After assembling in S, it does no effect to assembling of other parts in assembly body P.

If S satisfies conditions above, it was called sub-assembly. The recognition of sub-assembly can be carrying out according the associated diagram. In associated diagram, this kind parts have an feature that it lie in center location of local region and it have most number of connected parts.

Here, we constructed the associated diagram as Fig. 5.

By using the method described above, the reducer can be simplified into 13 components: shell (S1), reducer input shaft (S2), reducer output shaft (S3), reducer cover (S4), blind output shaft cover (S5), 2 input
shaft cover (S₃, S₄), output shaft cover (S₇), the conjunction S₅ which connect S₁ and S₆, sems S₁₀ which connect S₆, two sems S₁₁, S₁₂ which connect input cover S₆, S₇, respectively, sems S₁₃ which connect S₉.

- Construct assembly information model of reducer. The shell of reducer was connected with reducer cover by bolt and the output shaft cover and the input shaft cover was fasten on shell and reducer cover by bolts. The bolt and screw are standard parts; it can be ignored in assembly information tables. By using polychromatic sets, the assembly information can be transform into simple Boolean matrix, as shows in Fig. 6.

- Construct the localization equation, obstacle disassembly equation and conjunction relation equation of reducer.

According to the information shown in Fig. 6, the localization equation, obstacle disassembly equation and conjunction relation equation of reducer can be extracted as follows.

- Localization equations

\[ B(S₁) = S₃; B(S₂) = S₃; \]
\[ B(S₃) = S₇ B(S₄) = S₇; \]
\[ B(S₄) = (S₅ \lor S₆) \land S₉; \]
\[ B(S₅) = (S₆ \lor S₇) \land S₉; \]
\[ B(S₆) = (S₇ \lor S₈) \land S₉; \]

- Obstacle disassembly equations

\[ W(S₁) = S₅ \lor S₆ \lor S₇ \lor S₈ \lor S₉ \]
\[ W(S₂) = (S₅ \land S₆) \lor S₇ \lor S₈ \]
\[ W(S₃) = (S₆ \land S₇) \lor S₈ \lor S₉ \]
\[ W(S₄) = S₉ \lor S₇ \lor S₈ \]
\[ W(S₅) = 0; W(S₆) = 0 \]
\[ W(S₇) = 0; W(S₈) = 0 \]
Conjunction relation equations

\[ \begin{align*}
D(S_1) &= S_1 \land S_4 \\
D(S_2) &= S_2 \land S_5 \\
D(S_3) &= S_3 \land S_6 \\
D(S_4) &= S_4 \land S_7 \\
D(S_5) &= S_5 \land S_8 \\
D(S_6) &= S_6 \land S_9 \\
\end{align*} \]

By using the algorithm described above and according to the localization equation, obstacle disassembly equation and conjunction relation equation of reducer as criterions, all the feasible assembly sequences can be obtained, there are 48 sequences, 16 of them are shows in Fig. 7.

CONCLUSION

The generation of assembly sequence is one of the most important techniques in computer aided assembly techniques, many studies have been done, but most of them are emphasis on the description of geometry and assembly directivity. In this study, by using the assembly information model based on polychromatic sets theory the assembly features are transform into simple polychromatic sets matrix, which easily to recognize and extract by computer. And the localization equation, obstacle disassembly equation and conjunction relation equation of product are built based on which a new sequence generation method is presented. This method is suitable for complex product because of its low computation complexity. The case demonstrated the method is correct and effective and suitable in industry.

REFERENCES


