Analysis of Structure properties of Petri Nets Using Transition Vectors

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Abstract: This study introduces transition vectors based on place transitive matrix derived from graph theory, to study the structure of Petri nets using structure theoretical results that exists in Petri net theory. It has been established that transition vectors provide a simplified and more adequate approach than transitive matrix towards the structural analysis of PN. Some structural classes of Petri nets have been decided and basic concepts about the structure of Petri net have been derived through novel idea of transition vectors. Firstly, new representation of place transitive matrix has been introduced for acyclic Petri nets. Secondly, Petri net structure has been analyzed and an algorithm to find a directed cycle has been presented with a simplified representation, using transition vectors. Thirdly, transition vectors have efficiently been used to identify the particular structures of Petri nets. Finally, useful concepts relevant to the structure of Petri nets have been derived.

Key words: Structure properties, petri net, transition vectors, transitive matrix

INTRODUCTION

Petri Nets (PNs) are promising mathematical and graphical tool for modeling, analysis and design of information processing systems. Petri nets have been frequently used to model and analyze systems that are characterized as being distributed, parallel, concurrent, asynchronous nondeterministic, having discrete events, conflicts, deadlocks or resource sharing (Zurawski and Zhou, 1994; Murata, 1989; Peterson, 1981). Petri nets have a power to analyze the modeled system revealing important information about structure and dynamic behavior of system and same model can be used for performance evaluation. Petri net method for the specification and verification of systems is becoming more and more important as systems increase in size and complexity.

The structural analysis of PN depicts useful information about the behavior of modeled system and the structure of underlying net. Since PN corresponds in a natural way to a directed graph, so the static structure of PN facilitates to study structural i.e., graph theoretic properties depending on the place-transition relationship of underlying net by the flow relation. The study of structure of PNs has the objectives to increase their modeling power and to study the underlying properties of model. The structural analysis of PN has advantage over reachability tree method which exhibit state space explosion problem for larger and concurrent systems. Using structural analysis, important information about the behavior of the modeled system can be achieved. The behavioral properties that are of foremost interest and less easily analyzable in the analysis of systems may be reduced to easier-to-investigate structural properties (Best, 1987). Parallel and cycle structure of PN has been used for classification of solution of matrix equation in order to obtain the firing count vector as a solution of matrix equation (Miyazawa et al., 1996). Up till now, no broad structural classification of PNs has been decided instead of general PNs, ordinary PNs, self-loop free PNs and restricted PNs defined by Peterson (1981). But some important structural sub-classes of PN have been investigated in the literature (Best, 1987; Murata, 1989; Amer et al., 1999). Place and transition invariant method has been used to investigate the structural properties of the underlying unmarked net by Murata (1989). The relationship between PN structure and directed graph has also been discussed and result has been described for a cycle structure of PN (Miyazawa et al., 1998). A method for structural analysis of PN model using transition invariants has been proposed by Cai et al. (1993). Authors identify the cycle or/and parallel structure by the solutions of the homogeneous equations of transition invariants. However, study of some basic properties such as boundedness and liveness etc., through structural analysis of PNs is still to be investigated.

Transitive matrix depicts all the place-transition relationships and entries of transitive matrix describe the transferring relation from one place to another place.
through transitions. The characteristic polynomial equation of transitive matrix has been proposed to identify the cyclic structure in PN (Itoh et al., 1999). Transitive matrix method has also been introduced to study the behavioral properties of PN model (Itoh et al., 1999; Liu et al., 1999). Transitive matrix has been used for decomposition and slicing the basic unit of concurrency in PN model (Lee and Korbaa, 2005; Lee, 2005, 2002).

In this study, new concept of transition vectors to represent and analyze the structure of PN has been proposed. Transition vectors provide a simplified approach to describe the structure of PN and all the information relevant to the structure. New representation of transitive matrix has also been suggested. Some important concepts relevant to structure and classes of PN have been verified using transition vectors.

**BASIC DEFINITIONS AND CONCEPTS**

Here, some basic definitions, notations, structural classification of ordinary PN (for the sake of simplicity) and its matrix representation are described. The related terminology and notations are mostly taken from (Peterson, 1981; Murata, 1989). Directed and transitive graphs are also explained. The concepts and terminology related to graph theory are taken from (West, 2001).

**Definition 1:** (Petri net structure) A Petri net structure $N = (P, T, I, O)$ where $P = \{p_1, p_2, ..., p_m\}$ is a finite set of places, $|P| \geq 0$, $T = \{t_1, t_2, ..., t_n\}$ is a finite set of transitions, $|T| \geq 0$. $I: T \rightarrow P^*$ is the input function, a mapping indicating the directed arcs from places to transitions; $O: T \rightarrow P^*$ is the output function, a mapping indicating the directed arcs from transitions to places. $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$.

Let $I(t_i)$ represents the set of input place(s) of transition $t_i \in T$, then place $p_j \in I(t_i)$ is an input place of a transition $t_i$ if $p_j \in I(t_i)$ and transition $t_j \in I(t_i)$ represents the set of output place(s) and $p_j$ is an output place of $t_i$ if $p_j \in O(t_i)$. Incoming arc to $t_i$ is represented by $(p_i, I(t_i))$ and outgoing arc from $t_i$ be $(p_j, O(t_i))$. The input and output functions can be extended to map set of places $P$ into the set of transitions $T$. Then arc $(t_i, p_j)$ represents the incoming ar to $p_j$ as $t_i \in I(p_j)$ and arc $(t_i, p_j)$ represents outgoing arc from $p_j$ as $t_i \in O(p_j) \forall t_i \in T$, $\forall p_j \in P$.

There exists classification of the topological structure of PN having specific characterizations. The following definitions of structural subclasses of PN are due to (Peterson, 1981; Murata, 1989).

**Definition 2:** (Self-loop-free PN) A PN structure $N = (P, T, I, O)$ is said to be self-loop-free or pure if and only if $\forall t_i \in T$, $I(t_i) \cap O(t_i) = \emptyset$ i.e., no place may be both an input and an output of the same transition.

**Definition 3:** (State-machine) A PN structure $N$ is said to be state-machine if and only if $\forall t_i \in T$, $|I(t_i)| = 1 = |O(t_i)|$ i.e., every transition $t_i \in T$ has exactly one input place and exactly one output place.

**Definition 4:** (Marked graph) A marked graph is a PN $N$ such that $\forall p_i \in P$, $|p_i| = 1 = |O(p_i)|$ i.e., for every place $p_i \in P$, there exist one and only one input transition and one and only one output transition.

**Definition 5:** (Free-choice Petri net) PN $N$ is said to be free-choice Petri net if and only if $\forall t_i \in T$, $\forall p_i \in P$, either $I(t_i) = \{p_i\}$ or $O(p_i) = \{t_i\}$.

**Definition 6:** (Asymmetric choice PN) An asymmetric choice Petri net is a PN $N$ such that $\forall p_i \in P$, $I(p_i) \cap O(p_i) = \emptyset$ i.e., $(p_i) \subset O(p_i) \cup O(p_i) \cup O(p_i)$.

**Definition 7:** (Matrix definition of PN structure) A PN $N = (P, T, B, B')$ where $B$ and $B'$ are $|P| \times |T|$ matrices of input and output functions, respectively, defined by $B'[i,j] = 1$, if there is an arc $(p_i, t_j)$; otherwise, $B'[i,j] = 0$. The incidence matrix $B$ is defined by $B = B' - B^*$. For example, given as $B^* = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, matrix of output function $B^*$ and incidence matrix $B$ of PN in Fig. 1.c are, respectively

\[ B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  

\[ B' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  

**Definition 8:** (Graph and directed graph) A graph $G = (V, E)$ consists of a set $V$ of vertices and a set $E$ of edges. If $E$ is set of directed arcs, $G$ is called a directed graph or digraph denoted by $D$.

Since PN corresponds to a directed graph, so the following concepts are taken directly from graph theory and explained in the context of PN. A directed path is a set of $k$ nodes $x_i \in V$ such that $x_i \in V$, $i = 1,...,k$ and set of $k-1$ arcs $E \subset (P \times T) \cup (T \times P)$, the path connects the $i$-th node to the $(i+1)$-th node. The directed cycle is a directed path from one node back to itself. PN structure has two important types of connectivity, intensively studied to explain and verify many basic properties of system modeled by PN i.e., connected PN and strongly connected PN. A PN is simply connected if, for any pair of nodes $x_i, x_j \in V \cup T$, there is a path either from $x_i$ to $x_j$ or from $x_j$ to $x_i$, i.e., one of
Fig. 1: Different petri net models; (a) without source and sink transition (b) with sink transition and (c) source transition

Fig. 2: (a) The place transitive graph of Fig. 1c (b) The transition transitive graph of Fig. 1c

them is reachable from other. A PN is said to be strongly connected if any node $x_i \in P_uT$ is reachable from any other node $x_j \in P_uT$. A connected PN is strongly connected if and only if for each directed arc $(x_k, x_j)$ there is a directed path leading from $x_k$ to $x_j$.

**Definition 9:** (Transitive graph) A directed graph $D_p = (V_p, E_p)$ (or $D_r = (V_r, E_r)$) for PN is called a place (or transition) transitive graph, where $V_p = P$ (or $V_r = T$) is the set of vertices and $E_p = T$ (or $E_r = P$) is the set of directed arcs and $1_{t_i} \in E_p$ (or $1_{t_i} \in E_r$) is input transition (or place) of $p_i$ (or $t_i$) and the output transition (or place) of $p_i$ (or $t_i$), respectively. For example, Fig. 2a shows the place transitive graph and Fig. 2b is a transition transitive graph for PN model shown in Fig. 1c.

**TRANSITION VECTORS**

Here, definitions of transitive matrix and labeled place transitive matrix are presented which are based on (Itoh et al., 1999; Liu et al., 1999). New representation of place transitive matrix and the idea of transition vectors are also being introduced in order to analyze the structure of PN.

**Definition 10:** (Transitive matrix) Let $V = P \cup T$ be the set of vertices and $E \subseteq (P \times T) \cup (T \times P)$ the set of directed arcs. $D = (V, E)$ is a directed graph of the PN $N$. The adjacent matrix $A$ and transitive matrix $S$ are $A = \begin{bmatrix} 0 & (B^*)^T \\ B^* & 0 \end{bmatrix}$ and $S = AA = \begin{bmatrix} (B^*)^T B^* & 0 \\ 0 & B^*(B^*)^T \end{bmatrix}$. Then $B_i = B^* (B_i)^T$ and $B_i = (B_i)^T B_i$ are the place transitive matrix and transition transitive matrix, respectively.

**Definition 11:** (Labeled place transitive matrix) Let $L_{p_i}$ be the labeled place transitive matrix, defined by $L_{p_i} = B \text{ diag}(1, t_1, \ldots, t_n)(B^*)^T$, where $t_i$ ($i = 1, 2, \ldots, n$) represent the labels. The entry in the $i$th row and $j$th column as $L_{p_i}[i, j] = t_i$ means the transitive relation from input place $p_i$ (row $i$) to the output place $p_j$ (column $j$) through firing of the transition $t_i$.

But $L_{p_i}$ of PN having source or/sink transition(s) can not include all the transitions appearing in the structure of PN. If $t_i \in T$ is a source transition of given Petri net such that $I(t_i) = 0$ and $p_i \in O(t_i)$ is the output place of source transition $t_i$, then all the entries of $i$th row of $L_{p_i}$ are zero. Similarly if $t_i$ be a sink transition in PN as $O(t_i) = 0$ and $p_i \in I(t_i)$, then all the entries of $i$th column of $L_{p_i}$ are zero. So place transitive matrix representation of PN having source or/sink transition(s) does not give the complete portrait of PN model e.g., PNs in Fig. 1a-c all have the same transitive matrix i.e., $L_{p_i} = \begin{bmatrix} 0 & 0 & t_i \\ 0 & 0 & t_i \\ 0 & 0 & 0 \end{bmatrix}$.

In order to overcome the vague representation of place transitive matrix of Petri net having source or/sink transition(s), new representation of $L_{p_i}$ is suggested. This advantageous representation admits all the transitions of PN model to appear in $L_{p_i}$, provides the complete place-transition relationship for PN having source or/sink transition(s) and accommodating the synthesis to Petri net model.
Property 1: If Petri net $N$ has source (sink) transition $t_i$ such that $I(t_i) = \emptyset \cap p_i \in O(t_i)$ and $O(t_i) = \emptyset \cap p_i \in I(t_i)$, then the first entry of $i$th zero column (row) of transitive matrix will be labeled as $L_{t_i} = \begin{bmatrix} 1 & 0/t_i & 0 & 0 \end{bmatrix}$ and $L_{p_i} = \begin{bmatrix} 0 & 0/t_i & 0 & 0 \end{bmatrix}$, respectively. The example of new representation of place transitive matrix of PNs in Fig. 1b and c can be shown as $L_{p_i}$.

Property 2: Let $N$ be a PN having $[P \times |P|]$ labeled place transitive matrix $L_{p_i}$, then $T_q = \sum_{i=1}^{p_{i}} L_{p_i} [i,j]$ is a row $|P|$-vector of transitions and $T_q = \sum_{i=1}^{p_{i}} L_{p_i} [i,j]$ is a column $|P|$-vector of transitions. Transition vector $T_q$ is mapping as $T_q : P \rightarrow P$ where $I : P \rightarrow I$ is input function, $T_q$ is a mapping where $T_q : P \rightarrow O(p_i)$ where $O : P \rightarrow T$ is output function.

Sth component of $T_q$ which is $T_q(p)$ gives the set of input transitions of place $p_i$ and ith component $T_q(p)$ is a set of output transitions of place $p_i$.

The components of $T_q$ and $T_q$ are finite linear combinations of transitions with positive integer coefficients. Coefficients of transitions appearing in transition vector $T_q(T_q)$ represents number of incoming arcs i.e., indegree or number of input places (number of outgoing arcs called outdegree or number of output places) of transitions. The ith zero component of vector transition $T_q(T_q)$ can be interpreted as source place $p_i$ (sink place $p_i$) such as $I(p_i) = \emptyset$ and $O(p_i) = \emptyset$. Labeled zero component of $T_q(T_q)$ appears in transition vector $T_q(T_q)$, indicates the zero incoming arc to/outgoing arc from $p_i$ and hence $t_i$ stands for source (sink) transition. As the first labeled zero entry/ies (of zero column(s) (zero row(s)) belong only to zero column(s) (zero row(s)) of transitive matrix $L_{p_i}$, therefore they can only be added to $t_i$ (T_q) e.g., transitions vectors $T_q$ and $T_q$ of PN in Fig. 1c can be written as $T_q = \begin{bmatrix} 0/t_i & 0/t_i & 0/t_i \end{bmatrix}$ and $T_q = \begin{bmatrix} 0/t_i & 0/t_i & 0/t_i \end{bmatrix}$, where first and second component of $T_q$ is $0/t_i$ and zero label of transition $t_i$ indicating the zero indegree of transition $t_i$ i.e., source transition while third zero component of $T_q$ communicates $p_i$ as sink place.

ANALYSIS OF STRUCTURE PROPERTIES OF PN USING TRANSITIONS VECTORS

Here, some structural classes and fundamental properties about the structure of PN are presented in terms of transition vectors in order to analyze the structure of PN. An algorithm to find a directed cycle is also presented.

Property 3: A PN $N$ is acyclic if its transition vectors $T_q$ or $T_q$ has at least one zero component (labeled or unlabeled). As explained for property 2, zero component(s) of $T_q$ or $T_q$ correspond to source or sink place(s), respectively, labeled zero component(s) of $T_q$ or $T_q$ correspond to source or sink transition(s), respectively. So PN structure is said to be acyclic if it has at least one source or sink place (or/and transition). An acyclic PN does not imply that it can never contain a cycle.

Property 4: A PN $N$ is cyclic if and only if all the components of both the transition vectors $T_q$ and $T_q$ are non-zero. This condition refers to the situation that there is neither any source and sink place nor any transition in the PN structure. In words, the structure of PN will be cyclic if it doesn’t have any source and sink place (and transition).

Property 5: A PN $N$ is self-loop free or pure if and only if the corresponding same components $T_q(p)$ and $T_q(p)$ don’t have identical transition $\forall i = 1, 2, \ldots, |P|$ i.e., there does not exist $t_i \in T$ such that $t_i \in T_q(p)$ and $t_i \in T_q(p)$ $\forall i = 1, 2, \ldots, |P|$. In words, if different transition(s) are appearing in each corresponding same components of both $T_q$ and $T_q$ then PN structure is said to be self-loop free.

Property 6: Every acyclic PN has at least one sink place (transition) and at least one source place (transition).

Proof: Since $N$ is acyclic, so by property 3 its transition vectors $T_q$ or $T_q$ must have at least one zero (labeled or/and unlabeled) component. Choose any node $x_i \in P \cup T$, if $x_i$ is a source place e.g., $p_i \in P$ (transition e.g., $t_i \in T$), then ith component of $T_q$ will be zero (labeled zero), otherwise there will be a transition $t_i \in T_q(p)$ such that $t_i \in I(p)$ ($p \in O(t_i)$). Similarly, by scanning other entries of $T_q$, there must be source place (transition). Now, choose any other node $x_i \in P \cup T$, if $x_i$ is sink place e.g., $p_i \in P$ (transition $t_i \in T$), then jth component of $T_q$ will be zero (labeled zero), otherwise there will be a transition $t_j \in T_q(p)$, such that $t_j \in O(p_j)$ ($p \in I(t_j)$). Similarly, scanning other entries in $T_q$, there must be sink place (transition).

Property 7: Every self-loop free cyclic PN is strongly connected.

Proof: By property 4 Cyclic PN, every entry of transition vectors $T_q$ and $T_q$ must have at least one transition. As no transition and place of PN is on a self-loop. So it is possible to leave any place $p \in P$ through
an arc \((p_i, t_j)\) such as \(t_j \in T_{c}(p_i)\) and to enter any place \(p_j \in P\) through an arc \((p_j, O(t_j))\) such as \(t_j \in T_{s}(p_j)\). So there exist directed path from any node \(x_i \in P \cup T\) to any other node \(x_i \in P \cup T\). These arguments establish the statement of the property.

**Property 8:** A cyclic PN having place \(p_i \in P\) on self-loop is strongly connected iff ith entry of transition vector \(T_{c}\) has at least two transitions or and the transition on self-loop with \(p_i\) has coefficient at least 2.

**Proof:** Suppose that place \(p_i \in P\) is on self-loop with transition \(t_i \in T\) in the strongly connected cyclic PN \(N\). So by property 5, \(\exists t_i \in T_{s}(p_i)\) and \(t_i \in T_{c}(p_i)\). By definition of strongly connectedness, there must be a directed path from any node \(x_i \in P \cup T\) to any other node \(x_i \in P \cup T\). Firstly, we can not leave place \(p_i \in P\) through an arc \((p_i, I(t_i))\) as \(p_i\) is on a self-loop with \(t_i\). The condition of strongly connectedness is violated if we don’t have an arc \((p_i, I(t_i))\) such that \(t_i \in T_{s}(p_i)\) other than \(t_i \in T_{c}(p_i)\). So, ith component of transitions vector \(T_{c}\) must have at least two transitions. Secondly, we can not leave \(t_i\) which is on a self-loop, as there is only one outgoing arc from \(t_i\) to self-loop place \(p_i\), which violates the strongly connectedness condition. So, \(t_i\) must have coefficient at least 2 which indicates the outdegree of \(t_i\) at least 2, in order to find directed path from self-loop to any node of \(N\). So both or one of these two are necessary conditions for strongly connectedness.

Conversely, suppose \(p_i \in P\) on a self-loop with \(t_i \in T\) in cyclic PN \(N\), ith component of \(T_{c}\) has at least one transition other than \(t_i \in T_{s}(p_i)\) or and \(t_i\) has coefficient at least 2. Then we can leave place \(p_i\) through transition \(t_i \in T_{c}(p_i)\) other than \(t_i \in T_{s}(p_i)\) or and \(t_i\) can be left to enter a place other than \(p_i\). As given PN is cyclic having no zero entry in transition vectors \(T_{s}\) and \(T_{c}\), so from property 7, there exist directed path from any node \(x_i \in P \cup T\) to any other node \(x_i \in P \cup T\). Hence the given cyclic PN is strongly connected.

**Property 9:** If \(N\) be a PN without any source and sink place (and transition), then there is a directed cycle in \(N\).

**Proof:** According to the property 4, \(N\) given in the statement of property is cyclic which implies that transition vectors \(T_{s}\) and \(T_{c}\) both have all non-zero components. As all entries of \(T_{c}\) have at least one transition, so each place is the input of at least one transition i.e., \(p_i \in I(t_j)\) such that \(t_j \in T_{c}(p_i)\) \(\forall i = 1, 2, ..., |P|\) and \(\forall j = 1, 2, ..., |T|\). All entries of \(T_{s}\) also have at least one transition such that \(p_i \in O(t_j)\) \(t_j \in T_{s}(p_i)\) \(\forall i = 1, 2, ..., |P|\) and \(\forall j = 1, 2, ..., |T|\). Then we can leave every node \(x_i \in P \cup T\) through its outgoing arc and can enter to every node \(x_i \in P \cup T\) through its incoming arc. So, we stop as soon as we obtain a repeated node in directed path getting directed cycle and give the proof of the statement.

Directed cycles play a key role in the structure theory and analysis of modeled systems by PNs. The existence of directed cycle in the PN structure assists in verifying important structural properties. An algorithm is presented in order to find directed cycles in the cyclic PN structure using transition vectors.

**Algorithm to find a directed cycle:** INPUT: Transition vectors \(T_{s}\) and \(T_{c}\).

1. Select ith entry of \(T_{c}\), i.e., \(T_{c}(p_i)\) for \(i = 1\)
2. If selected place is same as previously selected place then, Go to step 6, else go to next step.
3. Do
   (i) Select first transitions in the ith component of \(T_{c}\).
   (ii) If selected transition is same as previously selected transition then go to step 6, else go to next step.
4. Do
   (i) Scan all components of \(T_{s}\) to locate selected transition in step 3 and select transition in first component having selected transition.
   (ii) Select an entry (place) in \(T_{s}\) corresponding to selected transition in step 4(i)
5. Go to step 2
6. OUT PUT: Directed cycle.
7. END

The algorithm given can efficiently be used to find directed cycle in cyclic PN. For example PN model given in Fig. 3, has transition vectors \(T_{s}\) and \(T_{c}\) obtained from transitive matrix \(L_{p_{i0}}\), given by \(T_{s} = [t_{s}, t_{i}, t_{i} + t_{i}]\) and \(T_{c} = [t_{s}, t_{s}, t_{i}, t_{i}, t_{i}]\).

First entry of \(T_{c}\) corresponds to place \(p_{i0}\) having transitions \(t_{i}\) and \(t_{s}\) in the first component of \(T_{c}\). After selecting \(t_{s}\), second component of \(T_{s}\) contains \(t_{s}\) which
corresponds to second entry i.e., \(p_{ii}\) getting directed path
\(p_{1} \rightarrow p_{2} \rightarrow p_{3}\). Augmenting the iterations of algorithm,
directed cycle is given by \(p_{1} \rightarrow p_{2} \rightarrow p_{3} \rightarrow p_{4} \rightarrow p_{1}\).
Similarly, the directed cycle \(p_{1} \rightarrow p_{2} \rightarrow p_{3} \rightarrow p_{4} \rightarrow p_{5} \rightarrow p_{1}\)
can be obtained using above mentioned algorithm.

**IDENTIFICATION OF PARTICULAR STRUCTURES USING TRANSITION VECTORS**

Certain PNs have structural characteristics and properties not possessed by the other PN structures. In
this section, transition vectors \(T_{N}\) and \(T_{C}\) are efficiently
used to identify these structural sub-classes of PNs.

**Property 10:** PN structure is said to be state machine if
and only if there does not exists any transition \(t \in T\) such that

\[
\begin{align*}
& t \in T_{N}(p_{i}) \text{ and } t \in T_{N}(p_{j}) \forall i, j = 1, 2, ..., |P|, i \neq j \text{ and} \\
& t \in T_{C}(p_{i}) \text{ and } t \in T_{C}(p_{j}) \forall i, j = 1, 2, ..., |P|, i \neq j
\end{align*}
\]

In words, if every component of \(T_{N}\) and \(T_{C}\) is a
distinct transition(s), then state machine structure of PN
can be identified.

**Property 11:** A PN \(N\) is said to be a marked graph if and
only if \(\#(T_{N}(p_{i})-\#(T_{C}(p_{i}))=1 \forall i = 1, 2, ..., |P|\); i.e.,
single transition (not necessarily distinct) is appearing in all
components of transition vectors \(T_{N}\) and \(T_{C}\) then PN
structure is called as marked graph.

**Note:** \(\#(T_{N}(p_{i}))\) (respectively, \(\#(T_{C}(p_{i}))\) is the number of
transitions in ith component of \(T_{N}\) (respectively, \(T_{C}\)).

**Property 12:** A PN \(N\) is said to be a conflict free or
persistent if and only if

\[
\begin{align*}
& \#(T_{C}(p_{i})) \leq 1 \forall i = 1, 2, ..., |P|, \text{ or} \\
& \text{If } \exists T_{C}(p_{i}) \text{ such that } \#(T_{C}(p_{i})) > 1 \text{ then } \forall t \in T_{C}(p_{i}), t \in T_{N}(p_{i})
\end{align*}
\]

In words, a structure is said to be a conflict free Petri
net if and only if every component of \(T_{C}\) has at most one transition, or if any component of \(T_{C}\) has more than one transition then these transitions must appear in the
corresponding same component of \(T_{N}\). A PN structure has
conflict or choice if at least one component of vector \(T_{C}\)
has more than one transition. For \(T_{C}\) having multiple transitions at ith entry, provided self-loop exception of
place \(p_{i}\), with these transitions, the case corresponds to
the situation of conflict.

**Property 13:** A PN structure is said to be simple Petri net
if and only if

\[
\forall i \in T_{C}(p_{i}) \Rightarrow \forall t \in T_{C}(p_{i}) \forall i, j = 1, 2, ..., |P|, i \neq j
\]

In words, if all the transitions of ith component are
included in jth component of \(T_{C}\) or all the transitions of jth
component are included in the ith component of \(T_{C}\), then
simple PN structure can be identified.

PN structure represents to have concurrent or parallel
activity if and only if at least one entry of vector \(T_{C}\) has
coefficient \(2\) or greater than two. If every component of \(T_{C}\)
of PN structure has distinct transition(s) i.e., every
transition \(t \in T\) has exactly one input place, then PN
structure has Basic Parallel Process (BPP). In formal
way, there does not exist \(t \in T\) such that \(t \in T_{C}(p_{i})\) and
\(t \in T_{N}(p_{j}) \forall i, j = 1, 2, ..., |P|, i \neq j\). A PN structure having BPP is
called BPP-net (Esparza and Nielsen, 1994).

For example, all the components of both the transition
vectors \(T_{N}\) and \(T_{C}\) of PN model shown in Fig. 3 are non-
zero and no corresponding same entries of \(T_{N}\) and \(T_{C}\) have
identical transition, so given PN structure is cyclic and
self-loop free. Property 7 implies that given PN is strongly
connected. As all components of transition vectors \(T_{N}\) and
\(T_{C}\) are distinct transition(s), so state-machine
structure of PN having basic parallel process (BPP) must
be identified. First component of transition vector \(T_{C}\) has
two transitions i.e., \(\#(T_{C}(p_{i})) = 2\) such as \(t_{i}, t_{j} \in T_{C}(p_{i})\).
Property 12 implies that transitions \(t_{i}\) and \(t_{j}\) are on conflict
or have choice to fire when \(p_{i}\) is marked. So, either firing
sequence \((t_{i}, t_{j})\) or \((t_{j}, t_{i})\) can occur.

**CONCLUSION**

In this study, novel concept of transition vectors has
been introduced to analyze the structure of PN. It has
been shown that transition vectors provide complete
information about the structure of PNs is symmetrical
manner as they are bijection of the set of places P to their
sets of input and output transitions which can not be
observed in transitive matrix. Some structural classes of
PNs have been decided and basic concepts about the
structure of PN have been established by using the
simplified approach of transition vectors. Simple algorithm
to find a directed cycle in PN structure has been
presented using transition vectors. Transition vectors
have efficiently been used to identify the structural
subclasses of PNs. New representation of place transitive
matrix for acyclic PN has also been introduced which
provides the complete list of transitions and elaborate the
place-transition relationship in transitive matrix, whenever
acyclic PN have source or/sink transition(s). It has
been established that transition vectors provide a simplified and more adequate approach than transitive matrix towards the analysis and depiction of important properties of PN structure. Some fundamental marking-independent properties of PN model of system e.g. boundedness, liveness and conservativeness etc, will be studied using transition vectors.

ACKNOWLEDGMENTS

This research was financially supported by National Natural Science Fund of China with grant No. 10771030 and grant No. 60435020 and National High-Tech R and D program (863 Program) under grant No. 2006AA01Z197.

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