An Approach to Rib Design of Injection Molded Product Using Finite Element and Taguchi Method

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Abstract: In this study, not only Taguchi Method but also ANSYS in providing an economical and effective advance to the optimum design of the rib for a plastic injected product are introduced. The analytical model of a rectangular thermoplastic Acrylonitrile Butadiene Styrene (ABS) plastic cover with rib of given thickness (2.8 mm) was selected and constructed based on design experiences and the dimensions as well as the width of the rib were selected as the control factors for Taguchi Method. The deflection under a constant force of 150 Newton at the back center of the cover was defined as quality characteristic. Additionally, the deformation experiment on a fixed thin beam was compared with the analytical result from ANSYS to verify the reliability of structure analysis from associated parameter setup and boundary condition operation. The L_{16}(4^{3}) orthogonal array from Taguchi Method was moreover arranged to establish nine sets of finite element analysis models. Through Taguchi Method, the optimum design parameters were furthermore received from minimum deformation at back center of the plastic cover analyzed by ANSYS. It is shown that the optimum structural parameters of a plastic rib can surely be effectively found with the integration of both Taguchi Method and ANSYS. Therefore, an Expert System of optimum design for various shapes of ribs can then be constructed through this study. This study exactly contributes a novel technique to the rib design for plastic injection industry in minimizing the development period of a new product.

Key words: Taguchi method, rib, finite element analysis, injection molding, deflection

INTRODUCTION

Rib is commonly designed and utilized in a plastic product to raise the strength (Chang, 1998). To minimize the volume as well as simplify the arrangement for the optimum strength without altering the shape of a product is definitely to be the critical issue of designing a rib. Practically, the experience rules are often followed to reduce the difficulties occurred in product formation and function, especially in the rib design, which will affect the strength and shape of a plastic product. Nevertheless, the existing researches hardly provide the solutions to this popular rib design and application.

The plastic rib not only increases the rigidity and strength of a product effectively, but also enhances the fluidity of the injected material to fill the mold as the function of an inter-runner. The optimal design of a rib surely will save plastic material, reduce weight, shorten design cycle time and minimize the problem by thick cross-section. The inconsistency wall thickness of a plastic injected product often delivers serious warp and size problem. Considering the product cost, it is rather utilizing the rib than increasing the wall thickness. When the rib is thicker comparing to the wall thickness, the sink marks will occurred easier. These sink marks have to be avoided or appeared on other side than the external shape of the product.

With these viewpoints above, the experience rules of injection molding design for a plastic rib in a product are selected to primarily layout the design parameters and the strength of the product is determined to be the objective of this study. With the parameter analysis through both Taguchi Method and the finite element analysis software ANSYS, the optimization of the structural dimensions of the rib can then be established. The period for product development in the plastic industry is then furthermore reduced using the technique proposed in this study.

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TAGUCHI METHOD

Taguchi design of experiments is an experimental method in improving the quality characteristic by the appropriate design parameters determined by practical operation on a site or virtual simulation from a computer (Li, 2000). To efficiently reduce the numbers of conventional experimental tasks, the orthogonal array (Chang, 2000; Wei, 2002) by using design parameter (control factor) in column and standard (level) quantity in row is proposed and further adopted. By using the analysis of parameters sensitivity to quality characteristic, the optimal parameters can thus be approached by Taguchi Method.

The illustration of orthogonal array is usually symbolized as L$_{b}^{c}$ where in L is the symbol of orthogonal array. By using the associated numbers (i.e., a, b, c), both of the format and range can therefore be understood clearly. Here, a representing the total numbers of experiment is demonstrated in the row of the orthogonal table. b, a level number, means that b numbers of design conditions are considered. c, the number of design parameters or factors, is presented in the column of the orthogonal array.

How to define the appropriate quality characteristic during analytical process in Taguchi Method is a very important kernel in Taguchi Method. According to the targeted function, the quality characteristic can be separated into numerous types (Li, 2000). The commonest four types are described and shown in Table 1. The main characteristic of Taguchi Method is trying to quantify the signal to noise ratio (S/N) as the quality index. The larger (S/N) means the quality of communication will become better. The lower ratio will be regarded as a better result when using the lowest expectation value. The related ratio of S/N is defined as:

\[
S/N = -10\log\sum_{i=1}^{n}\frac{y_{i}^{2}}{n} \text{ (dB)}
\]

(1)

Where:
- n = Testing numbers of experiment for each experimental set

On the contrary, the larger expectation value will own the better result for the characteristic of largest expectation value; therefore, by taking the inverse of quality characteristic into Eq. 1, the related S/N ratio can also be deduced as shown in Eq. 2.

\[
S/N = -10\log\sum_{i=1}^{n}\frac{1/y_{i}^{2}}{n} \text{ (dB)}
\]

(2)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>H (long length) (mm)</td>
<td>30.0</td>
<td>40.0</td>
<td>50.0</td>
</tr>
<tr>
<td>B</td>
<td>h (short length) (mm)</td>
<td>20.0</td>
<td>15.0</td>
<td>10.0</td>
</tr>
<tr>
<td>C</td>
<td>t (width) (mm)</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
</tr>
<tr>
<td>D</td>
<td>c (back angle) (degree)</td>
<td>3.0</td>
<td>2.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In this study, the deformation of plastic product under constant force is considered as the quality characteristic; therefore, the characteristic of lowest expectation value will be selected in this research.

FINITE ELEMENT METHOD

Finite Element Method (FEM) (Chen, 2003; Tsai, 2003) is one of the modern calculation techniques in engineering structural analysis. It disconnects the whole system from continuum into many finite individual elements. Each element is connected with several nodes. By using interpolation function (or shape function), the nodes’ relationship to the neighborhood can then be clearly constructed. By taking the geometry information into the minimum potential energy theory and stationary function theory, the linear equation set or matrix form will be constructed, thereafter, all local finite elements are reassembled into the existed global system. Consequently, the locations of nodes can thus be obtained.

The steps of Finite Element Analysis (FEA) in solid mechanics are presented and described as below:

**Step 1**: Local Equilibrium Equation

\[
[k]^{e} \{u\}_{n}^{e} = \{f\}_{n}^{e}
\]

(3)

Where:
- \([k]^{e}\) = Local stiffness matrix of the element
- \(\{u\}_{n}^{e}\) = Displacement of the element’s nodes

**Step 2**: Global Equilibrium Equation

\[
[k]_{n}^{\text{stiff}} \{u\}_{n}^{\text{stiff}} = \{f\}_{n}^{\text{ext}}
\]

(4)

Where:
- \([k]^{\text{stiff}}\) = Structural (global) stiffness matrix
- \(\{u\}_{n}^{\text{stiff}}\) = Structural (global) displacement vector at nodes
- \(\{f\}_{n}^{\text{ext}}\) = Structural (global) external force vector at nodes

**Step 3**: Boundary Condition of Displacements at Nodes

Taking the given displacements \(\{u\}_{n}^{\text{stiff}}\) of boundary nodes into Eq. 4.
Step 4: Boundary Condition of External Forces at Nodes
The given values of external forces \( \{f\} \) at boundary nodes are taken into Eq. 4 to simplify the calculation.

Step 5: Solution of Linear Equations
Using the information from Steps 2 and 4, n numbers of the linear equations can be solved simultaneously. Both the displacements and related forces of the residual nodes will thus be obtained.

Step 6: Postprocessor
Through the displacement and force from Step 5, the relevant curves of structural displacement and location as well as other stress and strain can be furthermore plotted evidently.

THE EXPERIMENT

Deflection measurement: A designed instrument in measuring the deflection of material has been set up and shown in Fig. 1. The adjustable chucks equipped at both ends of the platform are used to fix the plastic material for the deflection measurement purpose. Two kinds of testing sheets (named A and B) shown in Fig. 2 and 3 were selected. Applying a constant force of 150 Newton, an average force applied by human body, at the back centre point of the thin ABS beams, the precision measuring dial gauge (measuring range: 0–10 mm, precision: 0.01 mm) can thus sense the deflection \( \delta \) from the two types ABS test pieces (tensile stress 15000 psi) individually. The experimental results are therefore used to verify the accuracy of the settings of parameters and boundary condition in the established ANSYS model.

The total weight of the counterweight and related forcing pole are measured five times and the average value of these data is then derived. The resultant weight is totally in 328 g. The test piece is placed on the deflection-measuring platform, locked with four screws and compressed with two clips. As the effective length of the fixed thin beam is 366 mm, the boundary condition along the testing piece can therefore be regarded as totally fixed (i.e., all degree of freedom is zero in ANSYS). Therefore, the instrument is located below the test piece and calibrated later to be zero. Applying the loading of total weight (counterweight plus forcing pole) at the upper side of test piece and then recording the deflection from the dial gauge, the resultant deflection values from sheet A and sheet B are found to be 5.43 and 1.03 mm, respectively.

Operation verification for ANSYS: In this study, we use Pro/E to construct the 3-D graphic model for the thin beam and then transfer the geometric drawing into IGES with face-frame structure. The file is then transmitted into ANSYS (Kan and Chen, 2004; Hong et al., 2002; Moaveni, 2001; Chen and Tsai, 2002). The Solid 92 three-dimensional element, which is often used for the isotropic solid problem, is selected as the analytic element type in ANSYS package. This can be utilized for engineering applications in large displacement, large strain, plasticity and creep. Additionally, the free mesh is selected as meshing process for time saving purpose in this study.
Before ANSYS simulation being performed, the unit transformation of force from gram to Newton is required. In this study, the elastic deformation is considered and the deflection is analyzed in stead of stress distribution. Introducing the related loading and boundary conditions ($E = 2.1$ Gpa, $\nu = 0.35$, All DOF = 0) into ANSYS, the simulated deflection data at the center point can subsequently be obtained. Through the deflection comparison between experimental and simulated data, the accuracy of parameter setting and package operation can be recognized and verified. The simulation results in test pieces A and B are shown in Fig. 4 and 5, respectively.

It is shown that the simulated results are found reasonable compared to the real cases. It is shown that the accuracy of ANSYS Model of test piece A reaches 95% approximately. In addition, the accuracy of test piece B also settles at roughly 93%. Based on the verifications in the two experiments, the package operation, parameter setting and boundaries are found to satisfactorily meet the real world. Besides, it is proved that the reliability of ANSYS in structural analysis is absolutely high and acceptable.
OPTIMUM APPROACH

In this study, the parameter design of an ABS rectangular plastic cover with a 2.8 mm thickness which is commonly used for livelihood is introduced to the optimization approach. Under the circumstance of room temperature, the back centre of the cover is loaded vertically with a constant force of 150 Newton. The deflection of plastic cover at the back centre point is considered as the quality characteristic.

The purpose of this study is trying to approach the optimum design parameters of the reinforced rib for strengthening the plastic product appropriately. Besides, the steady quality (minimal deflection) which is essential in plastic material is expected and assured under the optimal design. In this study, the selected four parameters (control factors), including the dimensions and width of the rib, are shown in Fig. 6 and described as below:

Control factor A = H-Long length of the rib
Control factor B = h-Short length of the rib
Control factor C = t-Width of the rib
Control factor D = ε-Back angle of the rib

The deflection of the plastic cover under 150 Newton is served as the quality characteristic. Table 1 describes the four design parameters (control factors) and their levels.

This research adopts \( L_0(3^4) \) orthogonal array (Table 2) to establish nine sets of experiment. The cubic element of Solid 92 is determined as the analytic element for deflection simulation in ANSYS. Under various combinations of parameters, nine sets with different control factors are selected, accordingly. The related description of the setup is listed below:

Material Property (Beer and Johnston, 1999; MatWeb, 2004):

- Isotropic material: ABS
- Elastic coefficient: \( E = 2.1 \text{ GPa} \)
- Poisson’s ratio: \( v = 0.35 \) (at 23°C)
- DOF Constraints and loading Type

All the degrees of freedom are constrained. It means all \( DOF = 0 \) (assuming the rear cover is fixed). Besides, a centrifugal and constant force of 150 Newton is acted at the back centre of the cover.

RESULTS AND DISCUSSION

In this study, the deflection of a rectangle plastic cover is selected as the quality characteristic for parameter optimization. The dimensions and width of the rib are selected as the four control factors into Taguchi experiment, including A (the long length of rib), B (the short length of rib), C (the width of rib) and D (the back angle of rib). Besides, three levels of each parameter are chosen to establish the experimental in \( L_0(3^4) \) orthogonal array. The values of parameters for the nine sets of experiments are thus established and shown in Table 3. Through the numerical simulation in ANSYS, the results with respect to each experiment are obtained and shown as Y1 in Table 4. By using lowest expectation, the signal to noise ratio (S/N) is also calculated and listed in Table 4.

Moreover, the responses of each factor to the quality characteristic is shown in Table 5. As indicated the higher slope describes the higher effect of factor’s level to the quality characteristic. It is found that factor B has the largest effect to quality characteristic. Factor A has the second influence to quality characteristic. However, factor D (the back angle of rib) has almost no influence to quality characteristic. This can be explained that the back angle is often designed for the convenience in pulling the product out of the mold only.
Table 3: Experiments for L_9(3^4) orthogonal Array

<table>
<thead>
<tr>
<th>Experiments</th>
<th>length H (mm)</th>
<th>length h (mm)</th>
<th>Width t (mm)</th>
<th>Back angle C (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp.1</td>
<td>30</td>
<td>20</td>
<td>1.2</td>
<td>3</td>
</tr>
<tr>
<td>Exp.2</td>
<td>30</td>
<td>15</td>
<td>1.4</td>
<td>2</td>
</tr>
<tr>
<td>Exp.3</td>
<td>30</td>
<td>10</td>
<td>1.6</td>
<td>1</td>
</tr>
<tr>
<td>Exp.4</td>
<td>40</td>
<td>20</td>
<td>1.4</td>
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<tr>
<td>Exp.5</td>
<td>40</td>
<td>15</td>
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<td>3</td>
</tr>
<tr>
<td>Exp.6</td>
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<td>Exp.7</td>
<td>50</td>
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<td>2</td>
</tr>
<tr>
<td>Exp.8</td>
<td>50</td>
<td>15</td>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td>Exp.9</td>
<td>50</td>
<td>10</td>
<td>1.4</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4: Experimental results of the S/N ratio

<table>
<thead>
<tr>
<th>Exp</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Y1</th>
<th>S/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2.859</td>
<td>-9.10</td>
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<tr>
<td>2</td>
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<td>3.480</td>
<td>-10.83</td>
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<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5.076</td>
<td>-14.10</td>
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<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1.898</td>
<td>-5.56</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2.643</td>
<td>-8.44</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4.928</td>
<td>-13.85</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1.338</td>
<td>-2.53</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2.606</td>
<td>-8.32</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>4.119</td>
<td>-12.30</td>
</tr>
</tbody>
</table>

Avg. 3.216-8.4

Table 5: Result of factor responses to quality characteristic

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>3.805</td>
<td>2.032</td>
<td>3.464</td>
</tr>
<tr>
<td>Level 2</td>
<td>3.156</td>
<td>2.909</td>
<td>3.165</td>
</tr>
<tr>
<td>Level 3</td>
<td>2.698</td>
<td>4.707</td>
<td>3.019</td>
</tr>
</tbody>
</table>

Effect 1.117 2.675* 0.445 0.058

Table 6: Result of factor responses to S/N

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>-11.34</td>
<td>-5.73</td>
<td>-10.42</td>
</tr>
<tr>
<td>Level 3</td>
<td>-7.71</td>
<td>-13.41</td>
<td>-8.36</td>
</tr>
</tbody>
</table>

The responses of factors with respect to S/N are furthermore presented in Table 6. As the aforementioned results indicated, the optimum quality characteristic can be achieved by using the parameter set of A3 (50 mm), B1 (20 mm), C3 (1.6 mm) and D2 (2 mm) correspondingly. Under the optimum combination, the resulting deflection of plastic product from 150N back center loading will be 1.338 mm and thus the optimum design criteria can be reached.

CONCLUSION

The mold design for a plastic injected product is deemed to be complicated. This study proposes an optimal approach to the rib design on a plastic injected product by using both Taguchi method and ANSYS. A practical design task of plastic injection molded product with rectangular cover has been fully discussed and exemplified. In accordance to the experience rule, the reinforced rib that efficiently strengthens the plastic product has been chosen as the optimal target. Additionally, the dimensions and width of rib are considered as the four control factors. The back center deflection of the plastic cover under a constant centrifugal force has been determined as the quality characteristic. By using the orthogonal array from Taguchi method, the numerical simulations by ANSYS, as well as the effect analysis of control factors to quality characteristic, the results reveal that control factors A and B (the long length and short length of the rib) will highly dominate the deflection. It is also found that either length of the rib plays more essential role than the width of the rib.

Through the propose approach, various shapes of ribs on injection molded products can be furthermore optimized. This study definitely not only provides a novel technique to the optimal design of reinforced rib for the plastic injection industry, but also minimizes the development period as well as cost for a new plastic injected product.

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NOMENCLATURE

This study is constructed on the basis of the following notations.

- $L_9(b')$: Form of orthogonal array in Taguchi Experimental Method
- $S/N$: Ratio of signal to noise
- $A$: Total numbers of experiment
- $b$: No. of design conditions
- $c$: No. of design parameters or factors
- $y$: Measured deformation of a product (mm)
- $[k]^e$: Local stiffness matrix of the element e (N mm$^{-1}$)
- $[u]^e$: Displacement of the element's nodes (mm)
- $[k]^{g}$: Structural (global) stiffness matrix at nodes (N mm$^{-1}$)
- $[f]^{g}$: Structural (global) displacement vector at nodes (mm)
- $[f]^{e}$: Structural (global) external force vector at nodes (N)
- $\delta$: Deflection of the product (mm)
- $H$: Control factor A, long length of the rib (mm)
\( h \) = Control factor B, short length of the rib (mm)

\( t \) = Control factor C, width of the rib (mm)

\( \phi \) = Control factor D, back angle of the rib (mm)

\( E \) = Elastic coefficient (GPa)

\( \nu \) = Poisson's ratio

**REFERENCES**


