Towards Common Acquaintance Immunization Strategy for Complex Network

Pan Liu, Huikou Miao and Jia Mei
School of Computer Engineering and Science, Shanghai University, Shanghai, 200072, China
Shanghai Key Laboratory of Computer Software Evaluating and Testing, Shanghai, 201112, China

Abstract: The study presents a new immunization strategy for computer networks and populations with board and, in particular, scale-free degree distributions. The proposed strategy calls for the immunization of common acquaintances of random nodes (individuals). Similar to acquaintance immunization, our strategy also requires no knowledge of the node degrees or any other global information. Firstly, we analyze the successful and unsuccessful probability of acquaintance immunization with a simple example and the strategy ineffective reasons. Then, we study the probability of looking for common neighbors and present common acquaintance immunization strategy. Next, to compare common acquaintance immunization with acquaintance immunization, we implement a series of experiments from different aspects. The result of experiments shows that common acquaintance immunization gains higher stability and reliability for protecting complex network and can detect the structure of the unknown network. The conclusions of the study are that, compared with other immunization strategy, our approach requires not the whole information of complex network and efficiently immunizes the HUBS in complex network.

Key words: Common acquaintance immunization, probability analysis, immune strategies, common neighbor, network detection

INTRODUCTION

Many models for describing the spread of infectious diseases in populations have been built mathematically, such as SIR and SIS (Anderson and May, 1992; Hethcote, 2000). Related immunization thresholds involving the basic epidemic parameters are also reviewed in the literature (Tsao et al., 2006; Dan et al., 2006). When complex network suffers attacks from external environment, it displays an unexpected robustness and weakness (Réka et al., 2000) due to the special topological structures itself (Paolo et al., 2004). Human society and Internet are two typical complex networks e.g., human society has been attacked by H1N1 in 2009 and SARS in 2003, which result in many people's death all over the world, however, human society network is still normal. Although, Internet had seriously influenced by Worm. Nimaya in 2006, which made millions of computers breakdown within a few days in the world. Internet work still normally. To prevent human society or Internet from infectious diseases or computer viruses, many immunization strategies were proposed.

The main immune strategies protecting complex network include random immunization (Anderson et al., 1992), targeted immunization (Romualdo and Alessandro, 2002), preferential targeted immunization (Dezsö and Albert-László, 2005), acquaintance immunization (Cohen et al., 2003), Distance-d Covering Immunization (Echenique et al., 2005; Gómez-Gardén et al., 2006), local immunization (Fu et al., 2008), random walk (Ke and Yi, 2006) and DeepCure (Xinli et al., 2007).

Random immunization requires immunizing a very large fraction of a computer network, or population, in order to arrest epidemics that spread upon contact between infected nodes. We define that the density of immunized nodes is \( g \), diseases' effective spread rate is \( \lambda \) and epidemic threshold is \( \lambda_c \), then conclude that immunized threshold for random immunization is:

\[
g_c = 1 - \frac{\lambda_c}{\lambda}
\]

Hence, the steady infection density is \( \rho_c = 0 \) where \( g \geq g_c \), or

\[
\rho_c = \frac{g_c - g}{1 - g_c}
\]

where, \( g \leq g_c \). Due to

\[
\lambda_c = \frac{< k >}{< k^2 >}
\]

(Romualdo and Alessandro, 2002), the epidemic threshold

Corresponding Author: Pan Liu, School of Computer Engineering and Science, Shanghai University, Shanghai, 200072, China
Tel: 86-21-56331902 Fax: 86-21-56331901

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In a scale-free network, since $\langle k^{\gamma} \rangle \rightarrow \infty$, $g_0 = 1$. As a result, random immunization requires immunizing almost 100% nodes in a scale-free network. Targeted immunization selects the most highly connected nodes as immunized nodes, i.e., the ones more likely to spread the disease. While this strategy is simplest solution to the optimal immunization problem in heterogeneous populations, its efficiency is comparable to the uniform strategies in networks with finite connectivity variance. The immunization threshold of targeted immunization is $g_k = e^{-2m/k}$, where $m$ is the minimum number of connections of any node and the epidemic threshold is $g_k = e^{2\alpha/k}$ in the scale-free network, that means, we just immune small nodes, with highly connected nodes and then avoid the virus spread in the scale-free network. However, this immunization strategy needs to know the global information about network in question and renders it impractical in many cases, such as human society and Internet. To conquer this obstacle, acquaintance immunization strategy is presented by Cohen et al. (2003). The main idea of this strategy is that we choose a random fraction $p$ of the $N$ nodes in a scale-free network and look for a random acquaintance with whom they are in contact. Only knowing the selected nodes and their neighbor nodes, need the global information (every node’s degrees) not to be known. In a scale-free network, the nodes with highly connected means that the nodes with lowly connected link them. If randomly select a node and then randomly select a neighbor of it, the probability of the highly connected nodes is larger than that of the lowly connected nodes. So the effect of acquaintance immunization strategy is better than that of random immunization strategy. If we define $p_i$ as the random selected fraction and immured fraction $f_i$ as stop epidemic thresholds, then we can obtain two formulas (Cohen et al., 2003) as follows:

$$\sum_{k \geq n} p(k) p(k-1) \frac{1}{k} e^{-2\alpha/k} = 1$$

and

$$f_i = 1 - \sum_{k} p(k) v_{k,i}$$

where, $P(k)$ is the regular distribution and $v_{k,i} = e^{\alpha k}$ is the probability for $v_{k,i} = v(k)$. Except for acquaintance immunization, other immunization strategies need to know the global or local information of complex network. In fact, it is hard to know the specific relation among people when an infectious disease spreads in populations. So acquaintance immunization can effectively prevent the spread of infectious diseases in human society when the global or local information does not be known. However, the way of directly choosing neighbors limits the application of this strategy in practice (Gomez-Gardenes et al., 2006; Xinli et al., 2007). According to the epidemic threshold $f_i \geq 0.16$ in the literature (Romualdo and Alessandro, 2002), if exist the threshold $f_i < 0.16$ in acquaintance immunization

Fig. 1: The structure of internet
strategy, it will be ineffective. Practically, exist some probabilities to satisfy the threshold $f_s<0.16$, resulting in the failure of this strategy. Additionally, it is hard to determine the fraction $p$ of Internet in Fig. 1. To overcome the limit of acquaintance immunization strategy (AIS), we design Common Acquaintance Immunization strategy (CAIS).

Table 1: The Probability for $\sigma(n)$

<table>
<thead>
<tr>
<th>Node</th>
<th>1/5</th>
<th>1/5</th>
<th>1/5</th>
<th>1/5</th>
<th>1/5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(n)$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2: The Probability for $\omega(n, n')$

<table>
<thead>
<tr>
<th>$(n, n')$</th>
<th>(1, 5)</th>
<th>(1, 3)</th>
<th>(3, 1)</th>
<th>(3, 5)</th>
<th>(2, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega(n, n')$</td>
<td>1/10</td>
<td>1/10</td>
<td>1/10</td>
<td>1/10</td>
<td>1/5</td>
</tr>
<tr>
<td>$(n, n')$</td>
<td>(4, 5)</td>
<td>(5, 1)</td>
<td>(5, 2)</td>
<td>(5, 3)</td>
<td>(5, 4)</td>
</tr>
<tr>
<td>$\omega(n, n')$</td>
<td>1/5</td>
<td>1/20</td>
<td>1/20</td>
<td>1/20</td>
<td>1/20</td>
</tr>
</tbody>
</table>

Let the fraction $p=2/5$ of all nodes in this network, that is, two nodes are randomly selected. The function $\sigma(i, j)$ denotes the probability of node pairs $(i, j)$ to be randomly selected, $\sigma(i, j)$, $(i', j')$ denotes the probability that the neighbor of random node pairs $(i, j)$ is $(i', j')$. The probabilities of both $\sigma(i, j)$ and $\sigma(i', j')$ are shown in both Table 3 and 4.

The successful probability of AIS by randomly selecting node pairs is as follows:

$$\sum_{i,j=(1,5)}^2 \sigma(i,j) + \sum_{i,j=(1,3)}^2 \sigma(i,j) + \sum_{i,j=(3,1)}^2 \sigma(i,j) + \sum_{i,j=(3,5)}^2 \sigma(i,j) + \sum_{i,j=(2,5)}^2 \sigma(i,j) = 18/200$$

The unsuccessful probability of AIS by randomly selecting node pairs is as follows:

$$\sum_{i,j=(1,5)}^2 \omega(i,j) + \sum_{i,j=(1,3)}^2 \omega(i,j) + \sum_{i,j=(3,1)}^2 \omega(i,j) + \sum_{i,j=(3,5)}^2 \omega(i,j) + \sum_{i,j=(2,5)}^2 \omega(i,j) = 19/200$$

From Eq. 2 and 4, there are certain probabilities to cause the failure of AIS in practice.

**COMMON ACQUAINTANCE IMMUNIZATION STERALOGY**

The reasons of the failure of AIS lay on the two factors. Firstly, if the random node is a HUB, the probability that its random neighbor is a HUB in complex network is very low. Secondly, if the randomly selected nodes are the same one, the successful probability of AIS will be reduced. In order to solve this shortcoming of AIS, we look for the common neighbors of different random nodes in complex network as the immunized nodes. In fact, the common neighbors of different nodes are more likely to be the HUBs in complex network.

E.g., we use the scale-free network in Fig. 2 to verify our points. Since the node 5 is the only HUB in network, the common neighbor of the node pairs $(1,3), (1,2), (2,4)$.
Table 4: The Probability for \(\omega(j, i), (j', i')\)

<table>
<thead>
<tr>
<th>(j, i)</th>
<th>(j', i')</th>
<th>(\omega(j, i), (j', i'))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 3)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 4)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 5)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 6)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 7)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 8)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 9)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
<tr>
<td>(1, 10)</td>
<td>(5, 5)</td>
<td>1/100</td>
</tr>
</tbody>
</table>

\((1, 4), (3, 4)\) and \((2, 3)\) is the node 5. Let \(w(p, n)\) denote the probability that the neighbor of \(p\) is \(n\), where \(p\) is a random node pairs and \(n\) is a target node. So, \(w(p, 5)\) is the probability that the neighbor of \(p\) is the node 5. In the case of the node pairs \((1, 3)\), since the neighbors of the node 1 contain the nodes 3 and 5 and that of the node 3 contain the nodes 1 and 5, the node 5 is the only common neighbor of the node pairs \((1, 3)\). Hence, \(w((1, 3), 5) = 100\%\). Since, the probability that the random neighbor of the node 1 or 2 is the node 5 is \(1/2\), the probability that the common neighbor of the node pairs \((1, 3)\) are the HUBs is greater than that of the nodes 1 and 2. Similarly, we can get the same conclusion from other node pairs.

The probability for common neighbors: Let \(d_i\) denote the number of the nodes with the degree \(k\). The nodes with the degree greater than the HUBs of complex network and the maximum degree of the node is \(j\). According to the Power-law distribution, existing \(d_1, d_2, \ldots, d_j > 0\), and \(\sum_{i=1}^{j} d_i = j\). Since most of the HUBs do not connect with other HUBs, the probability that the neighbors of a random node are the HUBs of complex network is very low. We omit this probability in this section. Therefore, only when these randomly selected nodes are not the HUBs, are their neighbors likely to be the HUBs of complex network.

**Theorem 1:** Let \(P\) denote the probability that the neighbors of random nodes are the HUBs and let \(Q\) denote the probability that the common neighbors of random nodes are the HUBs. Then there must be satisfy:

\[ P < Q \]  \( (5) \)

**Proof:** Since, \(d_1\) is the number of the nodes with the degree \(k\) in complex network.

Hence, let

\[ \frac{d_k}{k \cdot \sum_{i=1}^{j} d_i} \]

denote the probability that the neighbor of the nodes with the degree \(k\) is a HUB in complex network.

Hence,

\[ \frac{d_k}{k \cdot \sum_{i=1}^{j} d_i} \]

denotes the probability that the neighbors of random nodes are the HUBs.

Hence,

\[ P \sum_{i=1}^{j} d_i \]

denotes the number of the selected HUBs with the probability \(P\).
Let

\[ L = \frac{d_i}{\sum_{k=1}^{i} d_k} + \frac{d_{i+1}}{(i+1) \sum_{k=1}^{i+1} d_k} + \ldots + \frac{d_j}{\sum_{k=1}^{j} d_k} \]  \hspace{1cm} (8)

and

\[ u = \frac{d_i + \ldots + d_{i+1}}{\sum_{k=1}^{i} d_k} \frac{1}{d_i + \ldots + d_j} \]  \hspace{1cm} (16)

\[ d_i + d_2 + \ldots + d_{i+1} \Rightarrow d_i + \ldots + d_j \]  \hspace{1cm} (17)

According to the Power-law Distribution, exist

\[ v << u \]  \hspace{1cm} (18)

From Eq. 15-17,

\[ v << u \]  \hspace{1cm} (19)

From Eq. 6 and 8, \( P < 1 \) and \( L < 1 \). Since, \( v << u \) and \( L < 1 \),

\[ P + L \frac{v}{u} < 1 \]  \hspace{1cm} (20)

From Eq. 19,

\[ u > uP + Lv \]  \hspace{1cm} (21)

From Eq. 20,

\[ \frac{u}{uP + Lv} > 1 \]  \hspace{1cm} (22)

From Eq. 14 and 22,

\[ P < Q \]  \hspace{1cm} (23)

**Algorithm for CAIS:** Based on the above analysis, we design a novel algorithm for CAIS by choosing the common neighbors of different nodes as follows:

1. **Step 1:** Choose a random fraction \( p \) of the nodes in complex network.
2. **Step 2:** By looking for the neighbors of randomly selected nodes, build a set \( \tau_i \).
3. **Step 3:** Repeat step 1 and step 2 for six times and get six sets \( \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6 \).
4. **Step 4:** Count the number of occurrences of all the same nodes among sets \( \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6 \). And then pick up those nodes whose number of occurrences isn’t less than \( k \) to build a new set \( \tau \). \( \tau \) is the target of CAIS.
In this algorithm, there are three key parameters: the fraction $p$ in step 1, the number of repeated times in step 3 and $k$ in step 4. We determine the value of $p$ depending on our experience, set the number of repeated times based on the property of six degrees of separation (Peter et al., 2003) in global social network and evaluate $k$ according to the analysis of the experimental result. Since, select the common neighbor of different nodes, $P<Q$ according to theorem 1.

**EXPERIMENTS**

To compare the experimental results of both AIS and CAIS, we take the American Air Network (Albert-László, 2005) in Fig. 3 as an experimental map, which is marked serial numbers for all nodes. The red nodes including 1, 6, 7, 17, 24, 33 and 39 are the HUBs in Fig. 3 and the black nodes are the non-HUBs.

**The ideas of the experiment:** Firstly, by the serial numbers of the nodes, we analyze the properties of American air network. Secondly, we simulate the process of both CAIS and AIS by a program written in Java in the Jbuilder2005 compile environment. Thirdly, based on the analysis of the experimental data, some important conclusions are obtained.

**Statements:** In the experiment, we must know the global information of the network because the computer needs utilizing them to generate random nodes or random neighbors for the implementation of two strategies. If CAIS is proved to be successful, we may apply this approach to complex network without knowing the global information of complex network.

Figure 4 shows that the America air network is a scale-free network because it satisfies the power-law distribution in the logarithmic coordinate system. So we can utilize the data of the America air network to carry out some experiments for both CAIS and AIS by the computer.

**The experiment for AIS:** Here, we do an experiment to evaluate AIS. First, randomly select ten nodes for ten times and then output ten suites of data into Table 5. Next, look for the random neighbors of random nodes in Table 5 and then output them into Table 6. The red numbers in both Table 5 and 6 are the HUBs of the American Air Network. Red numbers in Table 5 indicates that some HUBs are randomly selected as the random nodes. That confirms our point, that is, random nodes may contain a few HUBs of complex network. In Table 6, the same HUB is selected as the neighbor

![Fig. 3: The American air network](image)

![Fig. 4: The power-law distribution](image)

![Fig. 5: The results of AIS](image)

of random nodes, i.e., red No. 7 occurs two times in second experiment. According to the content of Table 6, we construct the immunization result of AIS shown in Fig. 5.

Figure 5 shows that only one HUB is selected in second experiment, resulting in threshold $\tau<0.16$ and the fluctuation of the curve is acute from 1 to 6, which indicates that AIS is unstable.

**The experiment for CAIS:** Hence, we do another experiment for evaluating CAIS. First, we set the parameter $k=3$ of the algorithm for CAIS. Second, randomly select ten nodes for ten times by the computer and then build a set $\tau$ in Table 7.

Figure 6 shows that the average number of HUBs in Table 7 is almost greater 4 and Fig. 7 shows that the size
Table 5: Random nodes for ten times

<table>
<thead>
<tr>
<th>Times</th>
<th>Node</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
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<tr>
<td>2</td>
<td>34</td>
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<tr>
<td>3</td>
<td>27</td>
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<tr>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
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<tr>
<td>6</td>
<td>5</td>
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<tr>
<td>7</td>
<td>19</td>
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<tr>
<td>8</td>
<td>26</td>
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<tr>
<td>9</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
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</table>

Table 6: Ten random neighbors of ten random nodes in Table 5

<table>
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<th>Times</th>
<th>Node</th>
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<tbody>
<tr>
<td></td>
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<td>1</td>
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<td>9</td>
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</table>

Table 7: The results of the CAIS

<table>
<thead>
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<th>Times</th>
<th>Node</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>1</td>
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<td>8</td>
<td>1</td>
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<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>

Fig. 6: The results of CAIS

of the final set $\tau$ is almost less 10. Comparing with the experimental results of both AIS and CAIS shown in Fig. 5-7, some important conclusions can be drawn.

- CAIS can find more HUBs than AIS at the same fraction $p$ of all nodes

Fig. 7: The node sizes of CAIS

- The fluctuation of the curve in Fig. 6 is less than that of Fig. 5
- An effective immunization set can be obtained by CAIS

Experimental evaluations: In order to further evaluate our strategy, we do more experiments from different aspects,
Fig. 8(a-j): Ten suites of experiments for both CAIS and AIS

including immunization efficiency, failure times, the size of immunized nodes, network detect and final experiment conclusions. The experimental settings for both CAIS and AIS are as follows:

- $6/42 \leq n \leq 15/42$, that is, the number of the chosen nodes is from 6 to 15 ($6 \leq n \leq 15$)
- $k = 3$ (for $n = 6$)
- Ten suites of experiments and ten times for each suite for both CAIS and AIS, respectively

To avoid bias, we carry out the experiments for 200 times without any modification and selection.

**Immunization efficiency:** Figure 8a-j show that the immunization efficiency of CAIS is almost better than that of AIS under the same experimental conditions. In Fig. 8a, there is zero HUB in third suite of experiment of CAIS when $k = 3$ and $n = 6$. This because the experimental parameters $n = 6$ (the fraction $p = 1/7$ of all nodes)

Fig. 9: The failure times in experiments of AIS

and $k = 3$ (the common neighbor of three random nodes) result in the failure of CAIS. So we reset the experimental parameters $k = 2$ and $n = 6$ and the experimental result of CAIS shown in Fig. 8a is satisfying. From Fig. 8c-e, i and j, exist the situation that the number of HUBs is 1 in the experiments for AIS, which causes the failure of AIS.

**Failure times:** Figure 9 shows the failure times in each of the ten suites of experiments for AIS and the experimental results conform to our points and the probability
Fig. 10(a-j): The size of immunized nodes

Fig. 11: Analysis of trend for CAIS and AIS

In the experiments of AIS, some same HUBs are randomly chosen, resulting in the decreasing of the number of the efficient nodes and the increasing of the failure times of AIS. In the experiments of CAIS, the common neighbors of random nodes are HUBs of America Air Network, so the failure times of this strategy is zero.

The size of the immunized nodes: Figure 10a-j show the final size of the selected nodes for both CAIS and AIS in ten suites of experiments when k = 3 and 6 ≤ nodes ≤ 15. Generally, the size of the immunized nodes from CAIS is less than that from AIS.

Analysis for the trend: From Fig. 11, the immunization result of CAIS is better than that of AIS. As well as, with the increasing of the fraction P of all nodes of network, CAIS can almost immunize all HUBs of America Air Network.

Network detection: In the following experiment, we detect the internal structure of the America Air Network by counting the number of occurrences of all the nodes in the tenth experiment. Figure 12 shows the degree distribution of each node in the America Air Network, while Fig. 13 shows the number of occurrence of each node of the network in the tenth experiment for CAIS where nodes = 15 and k = 3. Compared Fig. 12 with Fig. 13, there are similarities between two curves. First, most of inflection points of two curves are alike. This indicates that all HUBs of the America Air Network are the common neighbors of other non-HUBs. Second, nearly 65% nodes
AIS and then analyzes the probability of the common neighbors of random nodes. To compare CAIS with AIS, a series of experiments are implemented based on the America Air Network map. The experimental results indicate CAIS has higher immunization efficiency than AIS under the same conditions and can detect the unknown network structure. In the future, we will apply our approach to real networks, such as Internet, software testing, drug attack and epidemic prevention.

ACKNOWLEDGMENTS

The authors thank the anonymous reviewers for their valuable remarks and comments. This study is supported by National Natural Science Foundation of China (NSFC) under grant No. 60673115 and 60433010, National High-Techology Research and Development Program (863 Program) of China under grant No. 2007AA01Z144 and the National Grand Basic Research Program (973 Program) of China under grant No. 2007CB310800, the Research Program of Shanghai Education Committee under grant No. 07ZZ06 and Shanghai Leading Academic Discipline Project, Project Number: J50103.

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