Guaranteed Cost Fault-tolerant Controller Design of Networked Control Systems under Variable-period Sampling

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Abstract: This study investigates the problem of integrity against actuator failures for networked control systems under variable-period sampling. Assuming that the distance between any two consecutive sampling instants is less than a given bound, by using the input delay approach, the networked control systems under variable-period sampling are transformed into the continuous-time networked control systems under time-varying delays. Then the existence conditions of guaranteed cost fault-tolerant control law is testified in terms of the Lyapunov stability theory combined with Linear Matrix Inequalities (LMIs). Furthermore, the guaranteed cost fault-tolerant controller gain and the minimization guaranteed cost can be obtained by solving a minimization problem. A numerical simulation example demonstrates the conclusions are feasible and effective. The proposed control method resolves the problems of variable-period sampling and actuator failures, which meets the requirements in industrial networked control systems.

Key words: Actuator failures, stability, control law, linear matrix inequalities, cost function

INTRODUCTION

Networked Control Systems (NCSs) are feedback control system wherein the control loops are closed through a real-time network (NCSs) (Zhang et al., 2001; Antsaklis and Baillieul, 2007; Hespanha et al., 2007). Components which are directly connected to network such as sensors and controllers are regarded as nodes of network. Compared with traditional point-to-point control, networked control systems have excellences such as shared resource, long range manipulation, low cost and easy of system maintenance. Hence, it has good application prospect. Long range manipulation, long rage teaching and experiment, wireless network robot and industrial Ethernet technology can all be ascribed to be control systems based on network. However, because of involved communication network, problems of time-delay, data packet dropout and disordereded time sequence arise, which can degrade performance of the systems and even breakdown the systems. So, it is significant to study the fault-tolerant control for the networked control systems.

At present, the research on the fault-tolerant control for the networked control systems is not very common both abroad and at home. A new platform for the fault tolerant control design in complex networked control systems is proposed and the system is formulated into a hybrid framework involving simultaneously decentralized and centralized topology and independent of the methodologies used to tackle the fault tolerant control design (Mendes et al., 2007). A procedure is proposed for controlling a system over a network using the concept of an NCS-information-packet which is an augmented vector comprising control moves and fault flags, then the problem of fault-tolerant control for networked control systems is studied by Klinkheo et al. (2006). A kind of networked control systems with random time-delays are modeled as a discrete-time jump linear system with Markov delay characteristics and the actuator failures of networked control systems are analyzed based on jump linear system theory and fault-tolerant control theory (Huo and Fang, 2006). The time-delay is obtained by the time-delay estimation method and online time-delay acquisition method and a new robust fault tolerant control algorithm is presented to deal with the sensor failure and the actuator failure (Zheng and Fang, 2004). The uncertainty of network-induced delays is converted to the uncertainty of the parameter matrix, the sufficient
conditions for closed-loop networked control systems with uncertain disturbance possessing robust integrity against sensor or actuator failures are given and the robust fault-tolerant controller is designed by Li et al. (2007). The conclusions of these references are based on the constant sampling period. However, when network resources are distributed dynamically in networked control systems, the systems may work by the time-varying sampling period. Additionally, the fluctuating load of the computer, the malfunction of the computer components or external disturbance may cause sampling period to vary. Therefore, it is essential to do research on networked control systems under variable-period sampling. Digital feedback control systems with time-varying sampling period consisting of an interconnection of a continuous-time nonlinear plant are considered by Hu and Michel (2000). Assuming that the control input u is constant between sampling instants, the problems of stability analysis/controller design for systems with time-varying sampling period and time delay are investigated by Lozano et al. (2004) and Sala (2005). Scheduling and control co-designs for networked control systems based on time-varying sampling period are proposed by Wang et al. (2008), Luo et al. (2004) and Ji et al. (2007). H∞ controller of network control systems under variable-period sampling is designed by Wang and Yang (2007) and Borges et al. (2008). However, as so far, the research on fault-tolerant control for networked control systems under variable-period sampling has not been found. So, it is significant to design the controller to ensure the good performance of the system, when faults happened in networked control systems under variable-period sampling.

This study aims to solve the problem of integrity against actuator failures for networked control systems under variable-period sampling. Assuming that the distance between any two consecutive sampling instants is less than a given bound, by using the input delay approach, the networked control systems under variable-period sampling are transformed into the continuous-time networked control systems with time-varying delays. Guaranteed cost fault-tolerant controller is designed and the minimization guaranteed cost solution is presented by use of Lyapunov stability theory and linear matrix inequality method.

**PROBLEM FORMULATION**

Network control systems studied in this research are shown in Fig. 1. The system has output time delay \( \tau_{o}(t) \) and control time delay \( \tau_{c}(t) \).

![Fig. 1: Structure of networked control systems](image)

Assume that the process to be monitored is an LTI system described by:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
\dot{y}(t) &= u(t - \tau_{o}(t))
\end{align*}
\]

(1)

where, \( x(t) \in \mathbb{R}^n \), \( y(t) \in \mathbb{R}^r \), \( u(t) \in \mathbb{R}^r \) and \( y(t) \in \mathbb{R}^r \) are state vector, input vector, control output vector and measure output vector, respectively. A, B and C are known matrices of compatible dimensions.

We assume:

- The sensor is time-driven, the controller and the actuator are event-driven.
- The sampling period of the control system is time-varying and the distance between any two sampling instants is bounded by \( \delta \), where \( \delta > 0 \).
- The total time delay of the system is denoted by \( \tau(t) \) and \( \tau(t) \) is bounded by \( \tau \), where \( \tau > 0 \). That means \( \tau(t) = \tau_o(t) + \tau_c(t) \) and \( \tau(t) \in (0, \tau] \).
- The system can be controllable.

For any sampling instant \( t_k \), we have:

\[
t_{k+1} - t_k \leq \delta, \quad \forall k \geq 0
\]

(2)

Aiming at Eq. 1, using state feedback control law as follows:

\[
u(t) = Kx(t_k - \tau_k^o)
\]

(3)

where, \( K \in \mathbb{R}^{m\times n} \).

According to Eq. 1 and 3, we can obtain:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + BKx(t_k - \tau_k^o - \tau_k^c) \\
\dot{y}(t) &= u(t - \tau_o(t))
\end{align*}
\]

(4)

Obviously,
\[
\bigcup_{k=0}^\infty [t_k + \tau_k^d + \tau_k^p, t_{k+1} + \tau_k^d + \tau_k^p, \lambda_{t_k}] = [t_0, \infty), t_k \geq 0
\]

and by using the input delay approach, we can obtain:
\[
t_k - \tau_k^d - \tau_k^p = t - (t - t_k) - \tau_k^d - \tau_k^p = t - \Theta(t) - \tau(t)
\]

where, \(\Theta(t) = t - t_k\) denotes the derivative time-varying delay, \(\tau(t) = \tau_k^p + \tau_k^d\) denotes the time-varying delay of the system.

Equation 4 can be rewritten as:
\[
x(t) = Ax(t) + BKx(\Theta(t) - \tau(t))
\]

where, \(0 < \Theta(t) \leq \delta\) and \(0 < \tau(t) \leq \tau\) and the system described by Eq. 6 is a continuous-time networked control systems with time-varying delays.

In order to formulate the possible actuator failure faults, the fault model must be established first. Considering possible actuator failure faults, we can introduce a switch matrix \(L\) to Eq. 6 and lay the matrix \(L\) between the matrix \(B\) and the feedback matrix \(K\), where, \(L = \text{diag} (I_1, I_2, \ldots, I_n)\) for \(i = 1, 2, \ldots, n\).

\[
I_i = \begin{cases} 1 & \text{the ith actuator normal} \\ 0 & \text{the ith actuator failure} \end{cases}
\]

The networked closed-loop fault system becomes:
\[
x(t) = Ax(t) + BLKx(\Theta(t) - \tau(t))
\]

Associating with the Eq. 7, we define the following cost function.
\[
J = \int_0^\tau [x^T(t)Qx(t) + u^T(t)Ru(t)]dt
\]

where, \(Q\) and \(R\) are given positive-definite symmetric matrices.

The purpose of this study is to design a guaranteed cost fault-tolerant control law for the Eq. 7 and to seek for a minimal guaranteed cost. To facilitate developments, we first introduce the following definition.

**Definition 1**: Consider the Eq. 7, if there exist a control law \(u(t)\) and a scalar \(J^*\) such that the overall networked control system is asymptotically stable with \(L \in \Omega\) and the closed-loop value of the cost function (Eq. 8) satisfies \(J \leq J^*\), then \(J^*\) is said to be a guaranteed cost and the control law \(u(t)\) is said to be a guaranteed cost fault-tolerant control law for the Eq. 7, where, \(\Omega\) is a set which consists of all possible actuator failure faults switch matrix \(L\).

For the convenience of notations, (*) is denoted as an ellipsis for terms that are induced by symmetry in the rest of this study.

**GUARANTEED COST FAULT-TOLEERANT CONTROLLER DESIGN**

**Lemma 1**: **Schur Mendl theorem**: If \(A, P\) and \(Q\) are finite-dimensional constant matrices (Wang et al., 2007), then \(Q = Q^T, P = P^T > 0\), we have:

\[
A^T P A + Q < 0 \Leftrightarrow \begin{bmatrix} Q & A^T \\ A & -P \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -P & A \\ A^T & Q \end{bmatrix} < 0
\]

**Lemma 2**: Given matrices \(W, M, N\) of appropriate dimensions and with \(W\) symmetric (Wang et al., 2007), then:

\[
W + N^T F(k) M F(k) N < 0
\]

For all \(F(k)\) satisfying \(F(k) F(k) \leq I\), if and only if there exists a scalar \(\varepsilon > 0\) such that:

\[
W + \varepsilon M M^T + \varepsilon^2 N N < 0
\]

**Theorem 1**: Consider the Eq. 7, if there exist symmetry positive-definite matrices \(\bar{P} > 0, \bar{Q} > 0\), matrices \(\bar{X}, \bar{Y}\) and \(\bar{Y}\) and a scalar \(\varepsilon > 0\), such that:

\[
\begin{bmatrix}
PA^T + \bar{X} + \bar{X}^T & -\bar{X} + \bar{Y}^T & -\bar{X} & \bar{X} + \bar{X}^T & 0 & 0 & 0 & 0 \\
* & -\bar{Y} - \bar{Y}^T & * & * & * & * & * & * \\
* & & (\delta + \varepsilon) (T - 2\bar{P}) & * & * & * & * & * \\
* & & & * & * & * & * & * \\
* & & & & * & * & * & * \\
* & & & & & * & * & * \\
* & & & & & & * & * \\
* & & & & & & & * \\
\end{bmatrix} < 0
\]

539
then the Eq. 7 is asymptotically stable with the guaranteed cost fault-tolerant controller gain $K = \bar{K}^{-1}$ and the associated cost function satisfies $J = J^*$, where:

$$J^* = x^T(0)P^*x(0) + \int_{t=0}^{\infty} x(\alpha)T^*x(\alpha)\,d\alpha, \quad (10)$$

**Proof:** Let $\gamma(t) = \theta(t) + \tau(t)$, obviously, we have $0 < \gamma(t) < \delta + \tau$. Then defining the following Lyapunov-Krasovskii function:

$$V(t) = x^T(t)Px(t) + \int_{t=0}^{\infty} x(\alpha)Tx(\alpha)\,d\alpha$$

where, $P$ and $T$ are symmetry positive-definite matrices. Then the whole time derivative of $V(t)$ yields:

$$\dot{V}(t) = x^T(t)Px(t) + x^T(t)Px(t) + (\delta + \gamma)x^T(t)Tx(t) - \int_{t-\gamma}^{t} x(\alpha)Tx(\alpha)\,d\alpha$$

We have:

$$\dot{V}(t) \leq 2x^T(t)P(Ax(t) + BLKx(t - \theta(t) - \tau(t))) + (\delta + \gamma)x^T(t)Tx(t) - \int_{t-\gamma}^{t} x(\alpha)Tx(\alpha)\,d\alpha, \quad (11)$$

Where:

$$\Xi(t, \alpha) = 2x^T(t)P(Ax(t) + BLKx(t - \theta(t) - \tau(t))) + (\delta + \gamma)x^T(t)Tx(t) - (\theta(t) + \tau(t))x(\alpha)Tx(\alpha)$$

By the Newton-Leibniz formula, we have:

$$\int_{t-\gamma}^{t} x(\alpha)\,d\alpha = x(t) - x(t - \theta(t) - \tau(t))$$

Then, for any matrices $X$ and $Y$, we can obtain:

$$\Lambda = \frac{1}{\theta(t) + \tau(t)} \int_{t-\gamma}^{t} x(t - \theta(t) - \tau(t))\,Y\,x(t)\,X\,x(t)\,y(t - \theta(t) - \tau(t)) = 0$$

Adding $\Lambda + \Lambda^T$ to Eq. 11, we have:

$$\dot{V}(t) \leq \frac{1}{\theta(t) + \tau(t)} \int_{t-\gamma}^{t} [\Theta(t, \alpha)\Phi(t, \alpha)]\,d\alpha$$

Where:

$$\Theta(t, \alpha) = \begin{bmatrix} x^T(t) & x^T(t - \theta(t) - \tau(t)) & x^T(\alpha) \end{bmatrix} \Phi = \begin{bmatrix} A^TP + PA + (\delta + \gamma)A^TDA + X + X^T & PBLK + (\delta + \gamma)A^TBBLK - X + X^T & -(\theta(t) + \tau(t))X \\ * & (\delta + \gamma)(BLK)^TBLK - Y + Y^T & -(\theta(t) + \tau(t))Y \\ * & * & -(\theta(t) + \tau(t))T \end{bmatrix}$$

For $Q > 0$, $K^T RK > 0$, $(\theta(t) + \tau(t)) < \delta + \tau$, $V(t) < 0$ is equal to $\Phi < 0$. Based on lemma 1, $\Phi < 0$ can be equivalently rewritten as:

$$\begin{bmatrix} A^TP + PA + X + X^T + Q & PBLK - X + Y^T & X \\ * & -Y - Y^T + K^T RK & -Y \\ * & * & -((\delta + \gamma)T)^T \end{bmatrix} < 0$$

Obviously, we have $L^T L \leq I$. In light of lemma 2 and using the lemma 1, the above inequality is true for all admissible uncertain matrices $L$ if and only if there exists a constant scalar $\varepsilon > 0$ such that:

540

\[
\begin{bmatrix}
A^TP + PA + X + X^T + Q & -X + Y^T & -X & A^T & 0 & PB \\
* & -Y - Y^T + K^T R K & -Y & 0 & K^T & 0 \\
* & * & * & (\delta + \gamma)^T & 0 & 0 \\
* & * & * & * & * & -el_0 \\
* & * & * & * & * & -e_0^T \\
\end{bmatrix} < 0
\]

(12)

Pre- and post-multiplying both sides of Eq. 12 with \( \text{diag} \{ P^{-1}, P^{-1}, P^{-1}, I, I, I \} \) and its transpose, we can obtain:

\[
\begin{bmatrix}
P A^T + A F + X + X^T + P^{-1} Q P^{-1} & -X + Y^T & -X & P A^T & 0 & e B \\
* & -Y - Y^T + K^T R K & -Y & 0 & K^T & 0 \\
* & * & * & (\delta + \gamma)^T & 0 & 0 \\
* & * & * & * & * & -el_0 \\
* & * & * & * & * & -e_0^T \\
\end{bmatrix} < 0
\]

where, \( \tilde{P} = P^{-1}, \tilde{T} = T^{-1}, \tilde{X} = P^{-1} X P^{-1}, \tilde{Y} = P^{-1} Y P^{-1}, \tilde{K} = K P^{-1} \).

For \( T > 0 \), we have:

\[-P^{-1} T P^{-1} \leq -2 P^{-1} + T^{-1}\]

(13)

From Eq. 13 and lemma 1, we can obtain Eq. 9.

Now we shall prove that Eq. 3 guarantees the associated cost function less than or equal to \( J^* \), where, \( J^* \) is defined theorem 1. From Eq. 8, we have:

\[
J = \int_{t_0}^{t_1} \left[ x^T(t) Q x(t) + u^T(t) R u(t) \right] dt
\]

\[
d_t = \int_{t_0}^{t_1} \left[ x^T(t) Q x(t) + x^T(t - \theta(t) - \tau_c(t)) K^T R K x^T(t - \theta(t) - \tau_c(t)) \right] dt,
\]

For \( \tau_c(t) \leq \tau(t) \), we can obtain:

\[
J \leq \int_{t_0}^{t_1} \left[ x^T(t) Q x(t) + x^T(t - \theta(t) - \pi(t)) K^T R K x^T(t - \theta(t) - \pi(t)) \right] dt
\]

So, we have:

\[
J = \int_{t_0}^{t_1} \left[ x^T(t) Q x(t) + u^T(t) R u(t) \right] dt \leq \int_{t_0}^{t_1} \Theta^T(t, \alpha) \left( \Phi + \text{diag}(Q, K^T R K, 0) \right) \Theta(t, \alpha) d_t - \int_{t_0}^{t_1} \Theta^T(t, \alpha) \Phi \Theta(t, \alpha) d_t
\]

Then:

\[
J \leq \int_{t_0}^{t_1} \Theta^T(t, \alpha) \left( \Phi + \text{diag}(Q, K^T R K, 0) \right) \Theta(t, \alpha) d_t - V(\alpha) + V(0) \leq \int_{t_0}^{t_1} \Theta^T(t, \alpha) \left( \Phi + \text{diag}(Q, K^T R K, 0) \right) \Theta(t, \alpha) d_t + V(0)
\]

From Eq. 9, we know \( \Phi + \text{diag}(Q, K^T R K, 0) = 0 \). So, we have Eq. 10. The proof is completed.

In terms of theorem 1, the upper bound Eq. 10 of guaranteed cost is apparently not a convex function in \( \tilde{P} \) and \( \tilde{T} \). Hence, in order to obtain a controller feedback gain \( K = K P^{-1} \), which achieves the least guaranteed cost value \( J^* \) among all possible choices of \( \tilde{P}, \tilde{T}, \tilde{K}, \) and \( \tilde{e} \), finding the minimum of this upper bound can be formulated into an LMI generalized eigenvalue minimization problem subject to LMI (linear matrix inequality) constraints.

Defining \( \Gamma = \int_{t_0}^{t_1} x^T(t) x(t) d_t \) for \( \text{tr}(AB) = \text{tr}(BA) \), we have:

\[
\int_{t_0}^{t_1} x^T(t) x(t) d_t = \text{tr}(T^2 T^2)
\]

By introducing new variables \( \gamma > 0 \) and \( U = U^T \), which satisfy \( x^T(0) F^{-1} x(0) < \gamma \) and \( \Gamma^T T^2 T^2 < U \), according to lemma 1, we have:

\[
\begin{bmatrix}
-\gamma & x^T(0) \\
* & -P
\end{bmatrix} < 0
\]

(14)

\[
\begin{bmatrix}
-U & T^2 \\
* & -\Gamma
\end{bmatrix} < 0
\]

(15)

Then, the guaranteed cost fault-tolerant controller gain and the minimization guaranteed cost can be obtained by solving the following minimization problem:

\[
\min_{P, T, X, F, K} \left[ \gamma + \text{tr}(U) \right]
\]

(9)

s.t. (14)

(15)

**SIMULATION EXAMPLE**

Consider a system described by Eq. 1, where:
In cases of actuator normal and possible failures, the switch matrices $L_0 = \text{diag}(1, 1)$, $L_1 = \text{diag}(0, 1)$ and $L_2 = \text{diag}(1, 0)$ indicate actuator normal and actuator 1, 2 failure, respectively. Within the Matlab/Simulink environment, in the case of $L_{in}$, $L_1$, $L_2$, zero-input response of state $x_1$, $x_2$ are shown in Fig. 2 and 3. The curves of zero-input response state $x_1$, $x_2$ in Fig. 2 and 3 show that the networked control system against possible actuator failure faults is asymptotically stable. It reveals that the presented method makes the networked control system possess integrity against actuator failures and the minimization guaranteed cost is $J^* = 1.8391$.

CONCLUSION

Aiming at the networked control systems under variable-period sampling, assuming that the distance between any two consecutive sampling instants is less than a given bound, by using the input delay approach, the networked control systems under variable-period sampling are transformed into the continuous-time networked control systems with time-varying delays. Then, the problem of guaranteed cost fault-tolerant control for the networked control systems against actuator failures is investigated based on Lyapunov stability theory and linear matrix inequality. The advantage of the presented fault-tolerant control method is considering integrity and the optimization problem of the system performance meanwhile, so it has practical significance to the application of the networked control systems. In addition, the method that the networked control systems under variable-period sampling are transformed into the continuous-time networked control systems with time-varying delays by using the input delay approach provides a new approach to study networked control systems.

REFERENCES

