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## A Novel PCM/FM Multi-symbol Detection Algorithm for FPGA Implementation

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**Abstract:** In this research, a Baseband Quadrature Complex Rotation Multi-Symbol Detection (BQCR-MSD) algorithm is proposed. It can greatly reduce the computational complexity of MSD due to the partial correlation and complex rotation techniques applied, so is more suitable for implementation on large scale digital device such as Field Programmable Gate Array (FPGA). Simulation results also show that despite of the computational complexity decrease, the performance of BQCR-MSD algorithm is excellent and very close to that of MSD.

**Key words:** PCM/FM, multi-symbol detection, complex rotation, field programmable gate array

### INTRODUCTION

PCM/FM has been widely adopted as a main technique in current telemetry systems of strategic arms and launch vehicles due to its attractive features, such as flame interference mitigation, multipath fading mitigation, polarization interference mitigation and phase interference mitigation (Rice, 2000; Lee and Ra, 2008). PCM/FM actually is a kind of Continuous Phase Frequency Shift Keying (CPFSK) with PCM coding except that in PCM/FM the baseband symbols should be filtered by the premodulation filter (Takaishi *et al.*, 1997) first and then applied to frequency modulation. So, the frequency discriminators used in FSK can also be applied to PCM/FM (Petrovic and Molisch, 2000; Grayver and Daneshrad, 2001). There are two kinds of discriminators, the coherent ones and the noncoherent ones. The coherent demodulation has excellent performance but is hard to implement (Stefanovic *et al.*, 1999), while the noncoherent demodulation can be easily realized with relatively poor performance compared with the coherent demodulation (Simon and Alouini, 2003). As the transmission rate and distance increase rapidly in the launch vehicle telemetry systems nowadays, these methods become more and more unreliable.

In order to further improve the detection performance of PCM/FM, Multi-Symbol Detection (MSD) technique has been widely researched by Uhm and Cho (1999), Geoghegan (2000) and Cheng *et al.* (2007). MSD is first proposed to demodulate the CPFSK signal (Osborne and Luntz, 1974) and recently, it is widely used in the MSK and PCM/FM systems. Maximum likelihood sequence detecting (Yang and Zhang, 2004) algorithm and the memory inherent in the phase continuity of the waveform are employed in MSD. When a symbol is received, it will

not be decided immediately. Instead, the received signal of the following 3 to 5 symbols should be observed and all the possible transmitted waveforms over the observation interval should be correlated with the received signal, respectively and then compared to determine the sign of the symbol. Through MSD, the detection efficiency will increase by 3 dB when the Bit Error Rate (BER) is  $10^{-4}$ ; however, the computational complexity of it is extremely large and it is hard to implement in hardware especially when the number of observed symbols  $N$  is large and the premodulation filter is applied to the baseband symbols.

In this study, a Baseband Quadrature Complex Rotation Multi-Symbol Detection (BQCR-MSD) algorithm is proposed. The correlation is operated only on the current baseband quadrature symbol obtained from the Digital Down Converter (DDC) (Pasko *et al.*, 2001) and the corresponding correlation values of the previous symbols can be got by complex rotating operation. Then the correlation value of the whole observed interval can be obtained by adding the correlation values on these symbols together. The BQCR-MSD algorithm can obviously avoid large-scale correlation calculation and reduce the number of possible local waveforms. Computer simulations show that in the BQCR-MSD algorithm the computational complexity is reduced greatly with excellent performance close to that of the MSD, so it is more suitable for implementation on large scale digital device such as Field Programmable Gate Array (FPGA) (Herbordt *et al.*, 2007).

### MULTI-SYMBOL DETECTION

**CPFSK MSD:** The coherent detection is too complex to implement on digital devices for the phase of the carrier at the receiver should be estimated accurately, so in this

research only the noncoherent detection is discussed, in which the carrier phase need not be known. In noncoherent MSD, the detection problem consists of observing  $N$  symbols of a CPFSK signal and producing an optimum decision on one bit.

Here, first assume that the received signal is CPFSK modulated at intermediate frequency (IF) and the  $(n+1)$ th symbol, which in the middle of the observation interval, is detected by observing  $N = 2n+1$  symbols every time. The waveform of the first symbol in the observation interval can be expressed as:

$$s(t) = \cos \left[ \omega_c t + \frac{a_1 \pi h t}{T} + \theta_1 \right], \quad 0 \leq t \leq T \quad (1)$$

where,  $a_1$  is the value of the first symbol assumed to be random  $\pm 1$ ,  $\theta_1$  is the phase of the IF carrier at the beginning of the observation interval assumed to be a random variable uniformly distributed between  $\pm \pi$ , the modulation index  $h$  is the peak-to-peak frequency deviation divided by the bit rate and  $T$  is the period of the baseband symbol. In accord with the continuity of phase, the waveform during the  $i$ th symbol of the observation interval can be written as:

$$s(t) = \cos \left[ \omega_c t + \frac{(a_i \pi h (t - (i-1)T))}{T} + \pi h \sum_{j=1}^{i-1} a_j + \theta_1 \right], \quad (i-1)T \leq t \leq iT \quad (2)$$

where,  $a_i$  is the value of the  $i$ th symbol in the observation interval. So, the received signal containing the channel noise is given by:

$$r(t) = s(t) + n(t) \quad (3)$$

where,  $n(t)$  is additive white Gaussian noise (AWGN), with a two-sided power spectral density of  $N_0/2$  W/Hz.

The objective of MSD is to design a receiver which observes  $N$  symbols of a CPFSK signal and uses the fact that the carrier phase during the  $i$ th symbol depends upon the data in the first symbol to minimize the probability of the BER. Let the signal waveform during the observation interval be denoted by  $s(t, a_{n+1}, A_k, \theta_1)$  where  $A_k$  represents a particular data sequence, i.e., it represents the  $2n$  tuple  $\{a_1, a_2, \dots, a_n, a_{n+2}, \dots, a_{2n+1}\}$  and the actual waveform is again given by Eq. 2. In MSD, all the possible transmitted local waveforms in the observation interval are generated, correlated with the received signal described in Eq. 3, squared and then compared to determine the sign of the  $(n+1)$ th symbol. The noncoherent IF MSD of CPFSK can be shown in Fig. 1.

In Fig. 1,  $m = 2^{2n} = 2^{N-1}$ . Considering that the detected symbol  $a_{n+1}$  can be either  $+1$  or  $-1$  and two local waveforms

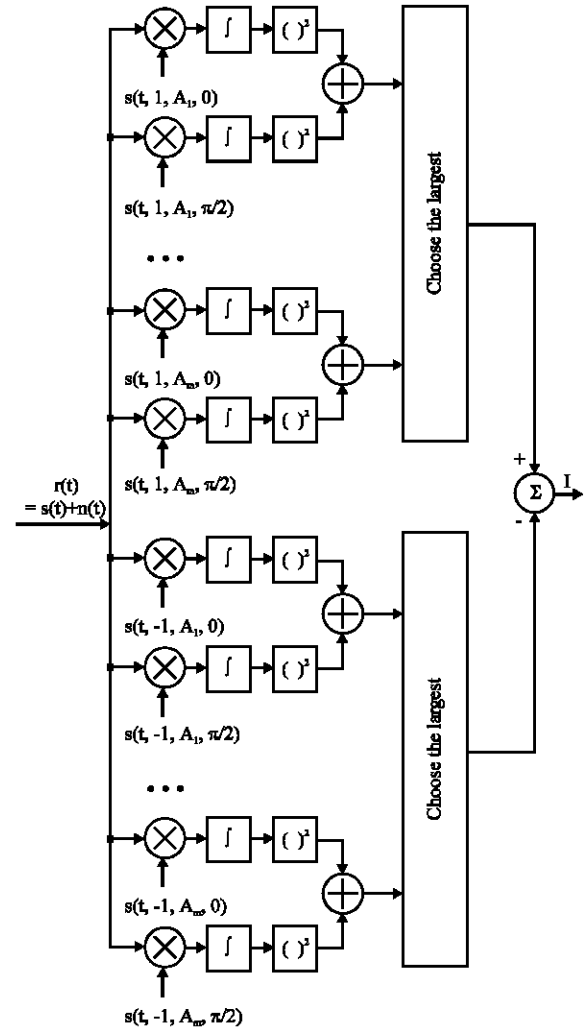


Fig. 1: Noncoherent IF MSD receiver

should be generated with initial phase 0 and  $\pi/2$  in every possible case of the symbols in observation interval,  $2^{N+1}$  possible local waveforms should be generated, correlated with the received signal in the observation interval, squared and then the determination can be made on the detected symbol. Assume the IF sample frequency  $f_s = 56$  MHz, the baseband symbol rate  $f_b = 2$  Mbps and the observation length  $N = 5$ , then 64 local waveforms of 140 samples each should be generated and correlated with the received signal and squared, respectively. In this case, the computational complexity is extremely large and it is difficult to implement in hardware.

**PCM/FM MSD:** The noncoherent IF MSD for CPFSK modulation is introduced, and here, MSD will be applied to PCM/FM. PCM/FM actually is a kind of CPFSK with PCM coding except that in PCM/FM the baseband

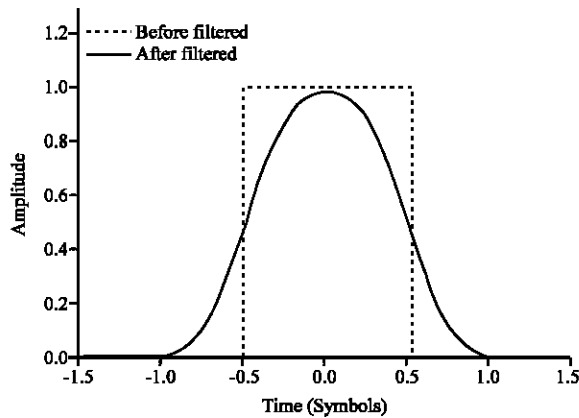


Fig. 2: The waveforms of a square wave before and after the premodulation filter

symbols should be filtered by the premodulation filter first and then applied to frequency modulation.

A square wave will become smooth and last longer, from  $T$  to  $LT$ , after filtered by the premodulation filter. In PCM/FM systems, generally  $L$  is equal to 3. The waveforms of a square wave before and after the premodulation filter are compared in Fig. 2.

In Fig. 2, a square wave is extended from  $T$  to  $3T$  after filtered. Through the premodulation filter, the channel band width is decreased and transmitting power is saved; however, inter-symbol interference is introduced and it will make MSD more sophisticated to be implemented. The waveform of every symbol will be affected by the symbols before and after it, for the length of a symbol has been extended to  $3T$ . For example, assume that the observation length  $N = 5$  and the waveform in the observation interval of these 5 symbols will be affected by the symbols just before and after the observation interval, so the number of possible local waveforms will be four times of those in CPFSK MSD, i.e., 256 local waveforms and the computational complexity therefore becomes even larger.

### BASEBAND QUADRATURE MSD

The performance of the noncoherent IF MSD is excellent; however, it is too complex to implement in hardware, especially when the premodulation filter is considered in PCM/FM systems. Therefore, a baseband quadrature MSD (BQ-MSD) algorithm is proposed here. In BQ-MSD, IF signal is converted to the in-phase and quadrature baseband signals through DDC (Kim and Lee, 2004) and then the MSD is carried out upon these two branches of signals.

The received IF signal in the observation interval can be simplified following Eq. 1 and 2 as:

$$s(t) = \cos[\omega_c t + f(t) + \theta_1] \quad (4)$$

In the DDC,  $s(t)$  is multiplied by the local quadrature oscillators  $\cos(\omega_c t)$  and  $\sin(\omega_c t)$  and then filtered, respectively. The obtained baseband quadrature signals are as follows:

$$I = \cos(f(t) + \theta_1) \quad (5)$$

$$Q = -\sin(f(t) + \theta_1) \quad (6)$$

Which can be written in the complex form as:

$$R = I + j \cdot Q = \cos(f(t) + \theta_1) - j \cdot \sin(f(t) + \theta_1) \quad (7)$$

The local waveform containing symbols the same as the received signal in the observation interval is given by:

$$L = \cos(f(t) + \theta_2) + j \cdot \sin(f(t) + \theta_2) \quad (8)$$

The baseband complex signal in Eq. 7 is multiplied by the local waveform described in Eq. 8 in the observation interval and the result can be expressed as:

$$\begin{aligned} R \cdot L &= \cos(\theta_2 - \theta_1) + j \cdot \sin(\theta_2 - \theta_1) \\ &= I_B + j \cdot Q_B \end{aligned} \quad (9)$$

Equation 9 shows that the complex waveform obtained through multiplying the baseband complex signal by the local waveform containing symbols the same as the received signal in the observation interval remains unchanged. By contrast, the complex waveforms got through multiplications by all the other possible local waveforms change continuously.

Assume the sample number in the observation interval is  $N_c$  and then integrate the complex waveform in Eq. 9 of the observation interval and the integral value is given as:

$$M = N_c \cdot I_B + j \cdot N_c \cdot Q_B \quad (10)$$

and its squared absolute value can be expressed as:

$$S = N_c^2 \cdot (I_B^2 + Q_B^2) \quad (11)$$

In Eq. 11,  $S$  is the squared absolute value of the correlation between the baseband complex signal from the DDC and the local waveform containing symbols the same as the transmitted data in the observation interval. In

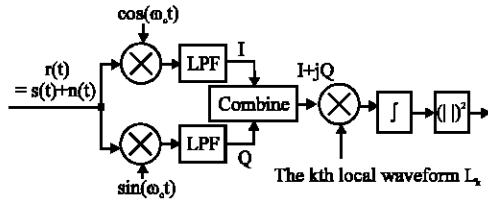


Fig. 3: The kth branch BQ-MSD receiver

contrast, as the products of multiplying the baseband complex signal by all the other possible local signals change continuously, the squared absolute values of the correlation are all smaller than that in Eq. 11 and the sign of the detected symbol can be determined.

A part of the receiver based on the BQ-MSD algorithm is shown in Fig. 3.

To be simple, only the operation of the kth local waveform is depicted in Fig. 3, all the other cases are similar to the kth branch. The processing results of all the local waveforms in the observation interval are compared to decide the sign of the detected symbol.

The required processing speed is of the BQ-MSD algorithm is low for the detection is at baseband, so it is easily to implement in digital devices and can achieve high performance. However, the computational complexity of it is still not reduced, the same as that of the IF MSD algorithm.

### BASEBAND QUADRATURE COMPLEX ROTATION MSD

In order to reduce the computational complexity of BQ-MSD, a baseband quadrature complex rotation MSD (BQCR-MSD) algorithm is proposed. As affected by the premodulation filter, there exists inter-symbol interference in the PCM/FM signal. Hence, there are  $2^3 = 8$  different waveforms for only one symbol. Besides, as the detection is operated at baseband, the variation in phase of each kind of waveform over one symbol is fixed. Assume, the sample frequency  $f_s = 56$  MHz, the baseband symbol rate  $f_b = 2$  Mbps, the modulation index  $h = 0.7$ , the 3 dB cutoff bandwidth is  $0.63 f_b$ . In this case, the possible transmitted one-symbol waveforms and their corresponding phase variations are shown in Table 1.

In Table 1, the middle value of every symbol sequence is the current symbol and the two values just beside it are the two interfering symbols. So, there are only 8 local baseband complex waveforms with zero initial phase,  $L_k (k = 1, 2, \dots, 8)$  and the difference between  $L_k$  and the local waveform in Eq. 8 is that  $L_k$  is the local signal over only one symbol. Then correlate them with the baseband complex signal from DDC over one symbol according to Eq. 9 and 10, respectively and Eq. 8 complex data  $M_{k0}(1) (k = 1, 2, \dots, 8)$  are obtained.

Table 1: Possible one-symbol waveforms and their phase variations

No.	Symbol sequence	Phase variation ( $\pi$ )
1	-1-1-1	-0.70
2	-1-1+1	-0.58
3	+1-1-1	-0.58
4	+1-1+1	-0.47
5	+1+1+1	0.70
6	+1+1-1	0.58
7	-1+1+1	0.58
8	-1+1-1	0.47

Assume the observation interval  $NT = 5T$ , so there are totally  $2^{N+2} = 128$  possible cases. According to the phase continuity of the waveform and the data in Table 1, the initial phases of the last four symbols in the observation interval can be calculated, so all the possible initial phases of  $L_1$  to  $L_8$  can also be calculated. Rotate the complex data  $M_{k0}(1) (k = 1, 2, \dots, 8)$  with corresponding angles and all the correlation results between the 8 local waveforms with different initial phase and the received baseband quadrature signal  $M_{kp}(1) (k = 1, 2, \dots, 8, p = 1, 2, \dots, P)$  can be obtained, where,  $P$  is the number of possible initial phases except zero of the 8 symbol waveforms in Table 1. Then the data  $M_{kp}(1)$  are fed to shift register to be processed.

$M_{k0}(1) (k = 1, 2, \dots, 8)$  and  $M_{kp}(i) (k = 1, 2, \dots, 8, p = 1, 2, \dots, P, i = 2, 3, 4, 5)$  in the  $i$ th symbol are used to calculate correlation values of all the 128 cases in the observation interval of  $5T$  approximately. For example, assume the sequence containing the local symbols in the observation interval and the symbols just beside them is -1+1-1-1-1+1 (the last symbol is on the right). In this case, the correlation value over the observation interval of 5 symbols can be obtained by complex rotation and adding, as follows:

$$M_n = M_{7a}(5) + M_{6b}(4) + M_{3c}(3) + M_{1d}(2) + M_{20}(1) \quad (12)$$

where,  $a, b, c$  and  $d$  are integers in  $[1, P]$ .

So, the squared absolute values of the complex data in 128 cases obtained in Eq. 12 can be calculated as in Eq. 11 and then decide the sign of the current detected symbol in the middle of the observation interval.

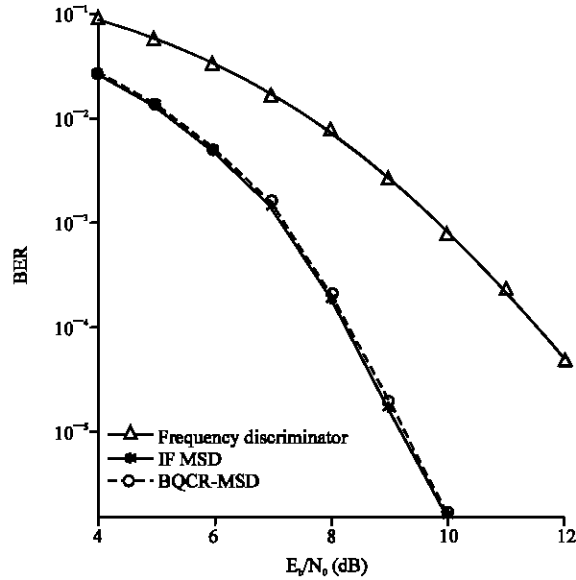
As stated above, the correlation over 5 symbols can be approximately be achieved by correlation over only one symbol using the 8 local symbols in Table 1, some complex rotation and adding calculations. Therefore, in BQCR-MSD algorithm, large-scale correlation calculation is avoided and the computational complexity is obviously reduced.

### EXPERIMENTAL RESULTS

As described above, the computational complexity of detecting one symbol using conventional

Table 2: Computational complexity comparison

Computational complexity	MSD	BQCR-MSD
Local signals	256×140 samples	16×28 samples
Multiplication	35840 times	896 times
Addition	35584 times	1392 times
Square	128 times	256 times
Complex rotation	0	40~80 times

Fig. 4: Bit error rate vs.  $E_b/N_0$ 

MSD algorithm and BQCR-MSD algorithm, respectively is compared in Table 2.

Table 2 shows that the storage memory, times of multiplication, times of addition and times of square in MSD algorithm are 80, 40, 23.4 and 0.5 times of those in BQCR-MSD algorithm. Besides there are additional 40 to 80 times of complex rotation in BQCR-MSD algorithm, which can be easily implemented on FPGA using CORDIC algorithm (Sunmanasena, 2008). The times of complex rotation depends on the phase accuracy. So, the computational complexity of the BQCR-MSD algorithm is extremely small compared with that of MSD and it is more suitable for implementation on FPGA.

While the computational complexity greatly decreases, the performance of the BQCR-MSD algorithm does not degrade and it is almost the same as that of the conventional MSD algorithm. The BER performance of different receivers is compared in Fig. 4 when the observation length  $N = 5$ .

As shown in Fig. 4, the BER performance of the BQCR-MSD receiver is very close to that of the MSD receiver and the value of  $E_b/N_0$  of these two receivers is 3 dB lower than that of the conventional frequency-discriminator receiver when BER is  $10^{-4}$ .

## CONCLUSION

In this research, a novel multi-symbol detection algorithm, BQCR-MSD, is proposed. Through partial correlation over only one symbol at baseband and complex rotation technique, the BQCR-MSD algorithm can reduce the computational complexity of MSD algorithm to a great extent and is more suitable to be implemented on FPGA. Simulation results have shown that the performance of the BQCR-MSD algorithm is excellent, which is almost the same as that of the MSD algorithm and much better than that of conventional frequency discriminator.

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