Logic Petri Nets and Equivalency

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Abstract: Logic Petri nets (LPN) can describe and analyze batch processing function and passing value indeterminacy in cooperative systems and its practical applications are shown with some nontrivial examples. This study focuses on the analysis of the modeling power of LPNs and the equivalency between LPNs and Petri nets with inhibitor arcs (IPN). The equivalency is proved formally and a constructing algorithm of equivalent IPNs from LPNs is proposed based on the disjunctive normal forms of logic input/output expressions. Moreover, the size of an LPN model is smaller than that of the equivalent IPN model.

Key words: Logic petri net, petri nets with inhibitor arcs, equivalency, constructing algorithm, logic expression

INTRODUCTION

Petri nets (PNs) (Murata, 1989) are a well-founded procedure modeling technique and have been used to model and analyze all kinds of procedures with applications ranging from protocols, hardware and embedded systems to flexible manufacturing systems and business processes. PNs are a graphical language. As a result, PNs are intuitive and easy to learn. Also, the semantics of PNs and several enhancements (color, time, hierarchy) have been defined formally (Wang et al., 2000).

PNs provide a solid framework for modeling and analyzing workflow processes (Vander Aalst, 2000; Derks et al., 2001). Workflow management is becoming a mature technology that can be applied within organizations and across organizational boundaries. It is very important to use an established framework for modeling and analyzing workflow processes (Vander Aalst, 1998), since processes are a main factor in workflow management systems. However, the existing Petri-net-based approaches are mainly used to investigate functional and secure properties of workflows. With rapid increase in the number of workflow processes where multiple organizations are involved, a batch processing function is urgently needed in cooperative systems, i.e., a batch data of the same type from different organizations need to be processed before a deadline. Also, the arrival times of these data are indeterminable, i.e., some data arrive before a given deadline, but others may not arrive. One usually requires that, to enhance the efficiency of workflows, all arrived data be processed at same time slice, while late received data after the deadline be processed in the next workflow cycle. This situation is called passing value indeterminacy (Yuyue and Changqium, 2002; Du et al., 2007). Therefore, a good formalism of cooperative systems should be able to model and analyze both the batch processing function and the passing value indeterminacy.

However, it is very difficult to apply Petri nets to the description of the batch processing function and the indeterminacy (Hao and Wang, 2003). Because the processing transition doesn’t been fired until all its input places contain tokens strictly. However, in that situation the transition needs a flexible trigger condition, i.e., some of its input places containing tokens are also able to fire the transition. Petri nets with weights of flow arcs may offer the batch processing function, but cannot easily represent the data arrival indeterminacy as the weights are not less than 1. Petri nets with inhibitor arcs (IPN) have the modeling power of a Turing machine (Suzuki and Lu, 1989). Although IPNs may model the batch processing function and the data arrival indeterminacy, the models of systems with these properties are very complex. To tackle the above problems, therefore, a new formal method-Logic Petri Net (LPN), an extend Petri net with logic transitions, is proposed and applied efficiently to the modeling and analysis of stock trading systems and electronic commerce systems (Yuyue and Changqium, 2002, 2003; Du et al., 2007). It is illustrated sufficiently that LPNs can model explicitly and represent compactly the batch processing function and the data arrival indeterminacy and mitigate the state explosion problem to some extent.

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LPNs sort the indeterminacy from interactive across organizational boundaries into the input and output indeterminacy and describe them respectively by logic input and output transitions. Firing them is subject to the constraints from logic expressions. In fact, the indeterminacy is implied by the logic expressions attached to logic transitions. Doubtless this will enhance the expression ability of PNs and cater to more application needs. Moreover, LPNs are able to reduce the size of system models. The objective of this study is to investigate the equivalence between LPNs and IPNs and to propose a constructing algorithm of an equivalent IPN from an LPN.

**LOGIC PETRI NETS**

To apply LPNs to the description of cooperative systems with the batch processing function and the passing value indeterminacy, two types of places in LPNs, control and data places are introduced. Control places are used to store control tokens within an organization, while data places to store data tokens forwarded from other organizations. Thereby, data places are also called interface places. Also, there are two kinds of logic transitions in LPNs, logic input and output transitions. Logic transitions are restricted by logic expressions that can efficiently describe the batch processing function and the passing value indeterminacy, such as the indeterminacy of arrival orders or actual trading quantities in electronic commerce. The transitions cannot be usually expressed in high-level PNs (Athuri and Huang, 2000), since the type of the data passed or generated is explicitly fixed by the variable symbol sums.

Assume that this research is based on the elementary net system, i.e., the capacity of each place is limited to 1. A brief introduction to PNs and IPNs is reviewed as follows.

**Definition 1:** PN = (P, T, F, M₀) is a PN if and only if

- P is a finite nonempty set of places;
- T is a finite nonempty set of transitions;
- P∩T = ø;
- Fc(T×P)∪(P×T) is a set of arcs, i.e. flow relations;
- M₀: P → {0, 1} is the initial marking.

In the graphic representation, places are drawn as circles or ellipses, transitions as bars. The flow relations between the nodes (i.e., places and transitions) are represented as directed arcs and the tokens of the marking as dots inside places. This study uses the following symbols for the pre-set and post-set of a node x∈P∪T:

`x = {y | (y, x) ∈ F}, x' = {y | (x, y) ∈ F}`, respectively. R(M₀) represents a set of all markings reachable from M₀. A transition t is enabled if each p∈t contains one token at the current marking M ∈ R(M₀). If t is enabled, it can fire and a new marking M' is generated from the current marking M₀ represented by M(t>M'), where when p∈t⁺, M'(p) = M(p)+1; when p∈t⁻, M'(p) = M(p)-1; otherwise, M'(p) = M(p).

**Definition 2:** IPN = (P, T, F, I, O, M₀) is a PN with inhibitor arcs if and only if

- P is the set of places;
- T is the set of transitions;
- Fc(P×T)∪(T×P) is a set of the edges (flow relations);
- I=P×T is the inhibitor arc set, I∩F = φ and (p, t)∈I is represented as a non-directed arc with a small circle at t side;
- M₀: P → {0,1} is the initial marking of IPN;
- Transition firing rules: ∀t∈T, t is enabled if ∀p∈P: (p, t)∈F ⇒ M(p) ≥ 1 and (p, t)∈I ⇒ M(p) = 0. Firing t will result in a new marking M', denoted by M(t>M'). And ∀p∈t⁺ and (p, t)∈F: M'(p) = M(p)+1; ∀p∈t⁻, M'(p) = M(p)-1; otherwise: M'(p) = M(p).

In the following, LPNs are first defined formally. Then an example is given to show expressive ability of logic transitions.

**Definition 3:** LPN = (P, T, F, I, O, M₀) is a logic PN if and only if

- (P, T, F, M₀) is a PN;
- T includes three subsets of transitions, i.e., T = T₁∪T₂∪T₃, T ∈ T₁∪T₂∪T₃, "t" = φ where
  - T₁ denotes a set of traditional transitions;
  - T₂ denotes a set of logic input transitions;
  - T₃ denotes a set of logic output transitions;
- I is a mapping from a logic input transition to a logic input expression, i.e., ∀t∈T₁, O(t) = f(t);
- O is a mapping from a logic output transition to a logic output expression, i.e., ∀t∈T₃, O(t) = f(t);
- Transition firing rules:
  - ∀t∈T₂, the firing rules of t are the same as in PNs;
  - ∀t∈T₁, t is enabled only if f(t)ₙ = "T".
    - M(t>M')ₙ, ∀p∈t⁺, if M(p) = 1, then M'(p) = M(p)+1;
    - ∀p∈t⁻, M'(p) = M(p)-1;
    - ∀p∈t⁺, if M(p) = 1, then M'(p) = M(p)+1; ∀p∈t⁻, M'(p) = M(p).
  - ∀t∈T₃, t is enabled only if ∀p∈t⁺. M(p) = 1. Firing t will generate a new marking M', notation M(t>M')ₙ and ∃S∈t, S must satisfy f(t)ₙ = "T".
    - ∀p∈S, M'(p) = M(p)+1; ∀p∈S, M'(p) = M(p)+1; ∀p∈t⁻, M'(p) = M(p)-1;
    - ∀p∈t⁺ S or ∀p∈t⁻ S: M'(p) = M(p).
Fig. 1: Representation of logic input/output transitions in LPNs, (a) in put and (b) out put.

Usually such S is not unique, but only one case is selected and the selection is random. The randomness of S can describe the output indeterminacy of cooperative systems.

In Fig. 1, t_i is a logic input/output transition and it is restrained by a logic input/output expression f(t_i) = p_i ^ (p_i V p_j). p_i is the control place, p_j and p_k are the data places. According to logic input expression f(t_i) = p_i ^ (p_i V p_j), in Fig. 1a t_i is enabled if place p_i has one token and one or each of places p_j and p_k includes one token. In Fig. 1b, p_j is the control place, p_i and p_k are the data places. t_i is enabled after p_i includes one token and firing t_i will generate one token in place p_j and one or each of places p_i and p_k respectively based on logic input expression f(t_i) = (p_i V p_j) ^ p_k.

In the application situation, when the control place in Fig. 1a contains a token, starting the business processing (transition t_i) requires at least one of the data places whose tokens come from other organizations contains tokens, instead of each of the data places must contain tokens rigidly in general Petri nets. The situation in Fig. 1b is similar to that in Fig. 1a. So, LPNs can describe clearly the batch processing function and the indeterminacy of data arrival with the corresponding logic expressions. In fact, the logic expressions can enhance the representation ability of LPNs. With respect to LPN expressivity, this study will prove that there is the equivalence between LPNs and IPNs.

**EQUIVALENCE OF LPNS AND IPNS**

In order to prove the equivalence between LPNs and IPNs, their isomorphism and equivalent definitions are introduced first.

**Definition 4 (Isomorphism):** Let \( \Sigma_i = (P, T, F, I, O, M_0) \) be an LPN and \( \Sigma_j = (P, T', F', I', O', M_0) \) an IPN. RG (\( \Sigma_i \)) is the reachable marking graph of \( \Sigma_i \) and \( R_i \) is the vertex set of RG (\( \Sigma_i \)), i = 1, 2. If there exists a bijective function \( f : R_i \rightarrow R_j \), such that \( V M_f, M_j \in R_i, t \in T, M_j[|> M_f t \rightarrow f(t), f(M_j)[|> t \rightarrow f(M_j) \). Then RG (\( \Sigma_i \)) and RG (\( \Sigma_j \)) are isomorphic.

**Definition 5 (Equivalence):** Let \( \Sigma_i = (P, T, F, I, O, M_0) \) be an LPN and \( \Sigma_j = (P, T', F', I', O', M_0) \) an IPN. \( \Sigma_i \) and \( \Sigma_j \) are equivalent if and only if RG (\( \Sigma_i \)) and RG (\( \Sigma_j \)) are isomorphic.

**Theorem 1:** For any LPN, there exists an equivalent IPN. Given an LPN \( \Sigma_i = (P, T, F, I, O, M_0) \) and IPN \( \Sigma_j = (P, T', F', I', O', M_0) \) is equivalent to \( \Sigma_i \).

For each transition of \( \Sigma_i \) into a corresponding transition or a subnet of \( \Sigma_i \) on the basis of the following three cases:

\( t \in T_i = T_i \cup T_j \cup T_{ij} \); Let \( t \in T_i \); \( \forall p \in P \), if \( (p, t) \in F \) then \( (p, t) \in F' \); and if \( (t, p) \in F \), then \( (t, p) \in F' \).

\( t \in T_j \); Let \( \ast = \{p_1, p_2, \ldots, p_n\} \), \( f(t) \) is its logic input expression. \( t \) is enabled once the value of \( f(t) \) is true on the basis of the token change of places \( p_i \). In fact, there may be several situations of places \( p_i \) where \( f(t) \) is true. However, \( f(t) \) can be transformed into a disjunctive normal form and the disjunctive normal form is unique, i.e., each disjunctive clause in the disjunct is a conjunct which consists of all positive or negative places \( p_i \) and each conjunct corresponds to a situation of places \( p_i \) where \( f(t) \) is true.

Thus, the disjunctive normal form of \( f(t) \) can be represented as follows:

\[
f(t) = \bigwedge_{i=1}^{n} \bigvee_{t \in T_i}
\]

where, \( \bigwedge_{i=1}^{n} \) is p_i or \( \neg p_i \). \( f(t) \) is true if and only if one of m disjunctive clauses is satisfied. Any two disjunctive clauses are mutually exclusive. In the following, m transitions in \( \Sigma_i \) corresponding to m kinds of the true value states are constructed in the disjunctive normal form respectively. That is, a logic input transition \( t \) with logic expression \( f(t) \) in \( \Sigma_i \) can be represented equivalently by a subnet including m transitions in \( \Sigma_i \) and m transitions correspond to m disjunctive clauses in the disjunctive normal form of \( f(t) \) respectively. The subnet is constructed in detail as follows:

For any disjunctive clause \( \sum_{i=1}^{n} \bigwedge_{t \in T_i} \in \{1, 2, \ldots, m\} \)

Assume that the disjunctive clause corresponds to a transition \( t_i' \) in \( \Sigma_i \), i.e., \( t_i' \in \Sigma_i \). Then the arc set of this subnet is defined. \( \forall p \in H_i \), if \( c_i = p_o \) then \( (p_o, t_i') \in \Sigma_i \); if \( c_i = \neg p_o \) then \( (p_o, t_i') \notin \Sigma_i \).

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If \( \forall p \in \tau^*, \) then \((t', p) \in F'\). Therefore, firing a logic input transition of \( \Sigma_\iota \) corresponds to firing a transition of \( \Sigma_\iota \), i.e., if a logic input transition is enabled in \( \Sigma_\iota \), then there must be an enabled transition in the corresponding subnet of \( \Sigma_\iota \) and it is unique.

**t \in T_\iota:** Let \( t^* = \{p_1, p_2, \ldots, p_n\} \). \( f_\iota(t) \) is its logic output expression. \( t \) is enabled once its input place \( p \) includes a token. Firing \( t \) will generate a token in all or some output places and must assure that \( f_\iota(t) \) is true at the next state of places \( p_1 \ldots p_n \).

Similarly, \( f_\iota(t) \) can be transformed into a unique disjunctive normal form. Without loss of generality, assume that the disjunctive normal form consists of \( r \) disjunctive clauses. Thus,

\[
f_\iota(t) = \bigvee_{i=1}^{r} \bigwedge_{j} \tau_{ij}
\]

where \( \tau_{ij} = p_i \) or \( \neg p_i \). \( f_\iota(t) \) decides all possible states of places \( p_1 \ldots p_n \) and each disjunctive clause corresponds to a possible state in which \( f_\iota(t) \) is true. Thus \( r \) transitions in \( \Sigma_\iota \) corresponding to \( r \) transitions of the possible states are constructed in the disjunctive normal form respectively. That is, a logic output transition \( t \) corresponds to a transition \( t' \) in \( \Sigma_\iota \) if \( f_\iota(t) \) and \( f_\iota(t') \) are equivalent by a disjunction including \( r \) disjunctive clauses of \( f_\iota(t) \) respectively. The subnet is built as follows.

For any disjunctive clause,

\[
\bigwedge_{i=1}^{k} \tau_{ij}
\]

\( i \in \{1, 2, \ldots, r\} \). Assume that the disjunctive clause corresponds to a transition \( t' \) in \( \Sigma_\iota \), i.e., \((t', p_i) \in F'\). Then the arc set of this subnet is defined. \( \forall j \in \bar{M}, \) if \( \tau_{ij} = p_i \), then \((t', p_i) \in F'\); if \( \tau_{ij} = \neg p_i \), then \((t', p_i) \notin F'\). \( \forall p \in \tau^* \), then \((p, t') \in F'\). Therefore, firing a logic output transition of \( \Sigma_\iota \) corresponds to the occurrence of a transition in \( \Sigma_\iota \), i.e., firing a logic output transition in \( \Sigma_\iota \) is functionally equivalent to the occurrence of one of \( t, t' \).

Therefore, the place set \( P \) in \( \Sigma_\iota \) is the same as that in \( \Sigma_\iota \). But \( T = T', F \neq F' \) and \([T] \neq [T]\), \([F] \neq [F]\), where \([T]\) denotes the size of set \( T \). Since \( \Sigma_\iota \) and \( \Sigma_\iota \) have the same initial marking \( M_\iota \), the equivalence between \( \Sigma_\iota \) and \( \Sigma_\iota \) are now proved based on the reachable marking graph. In \( \Sigma_\iota \), for \( \forall M_\iota, M'_\iota \in R(\Sigma_\iota) \), \( t \in T \) and \( M_\iota \rightarrow M'_\iota \). Then there is a mapping function \( f: R(\Sigma_\iota) \rightarrow R(\Sigma_\iota) \) based on the construction of \( \Sigma_\iota \) such that \( f(M_\iota) = M_\iota \) and \( f(M_\iota) = M'_\iota \) and in \( \Sigma_\iota \) if \( t \in T_\iota \), then \( \exists t' \in T': t' = t \) and \( f(M_\iota) = t \rightarrow f(M_\iota) \).

Accordingly, \( f \) is an identity mapping and satisfies injective and surjection requirements at \( M \in R(M_\iota) \). That means that \( \Sigma_\iota \) and \( \Sigma_\iota \) have the same behavior characteristics. Moreover, the structure of \( \Sigma_\iota \) is unique since the disjunctive normal form is only one. So \( f \) is a bijective function and \( R(\Sigma_\iota) \) and \( R(\Sigma_\iota) \) are isomorphic.

Consequently, \( \Sigma_\iota \) and \( \Sigma_\iota \) are equivalent based on definition 5. The conclusion holds.

From Theorem 1 and the construction of \( \Sigma_\iota \), the following constructing algorithm of an equivalent IPN from an LPN can be obtained.

**Algorithm 1 (Constructing Algorithm):** Transforming an LPN into an equivalent IPN

**Input:** an LPN \( \Sigma_\iota = (P, T, F, I, O, M_\iota) \)

**Output:** an equivalent IPN \( \Sigma_\iota = (P, T', F', I', O, M_\iota) \)

1. LPN \( P = LPN \, P \)
2. For each transition \( t \) in \( LPN, T_\iota \)
3. \( LPN.T' = LPN.T' \cup \{t\} \)
4. For each \( p \) in \( t \)
5. \( LPN.F' = LPN.F' \cup \{ (p, t) \} \)
6. End for
7. For each \( p \) in \( T_\iota \)
8. \( LPN.F' = LPN.F' \cup \{ (t, p) \} \)
9. End for
10. End for
11. For each \( t \) in \( LPN, T_\iota \)
12. For each disjunctive clause \( \bigwedge_{i} \tau_{ij} \) of \( f_\iota(t) \)
13. \( LPN.T' = LPN.T' \cup \{ \bigwedge_{i} \tau_{ij} \} \)
14. End for
15. For each \( p \) in \( t \)
16. If \( \tau_{ij} \) \( p \) then \( LPN.F' = LPN.F' \cup \{ (p, t) \} \)
17. else \( LPN.I' = LPN.I' \cup \{ (p, t) \} \)
18. End for
19. For each \( p \) in \( T_\iota \)
20. \( LPN.F' = LPN.F' \cup \{ (t, p) \} \)
21. End for
22. End for
23. For each \( t \) in \( LPN, T_\iota \)
24. For each disjunctive clause \( \bigwedge_{i} \tau_{ij} \) of \( f_\iota(t) \)
25. \( LPN.T' = LPN.T' \cup \{ \bigwedge_{i} \tau_{ij} \} \)
26. End for
27. For each \( p \) in \( t \)
28. If \( \tau_{ij} \) \( p \) then \( LPN.F' = LPN.F' \cup \{ (t, p) \} \)
29. End for
30. For each \( p \) in \( T_\iota \)
31. \( LPN.F' = LPN.F' \cup \{ (p, t) \} \)
32. End for
33. End for

Example 2: Transforming a logic output transition in LPNs to an equivalent subnet in IPNs. In Fig. 3a, $t_1$ is a logic output transition in an LPN and $f_3(t_1) = (p_1 \lor p_2) \land p_3$ is its logic output expression. $t_1_2 = \{p_1, p_2, p_3\}$ and $t_1_2 = \{p_4\}$. The disjunctive normal form of $f_3(t_1)$ is deduced as follows.

\[
f_3(t_1) = (p_1 \lor p_2) \land p_3
\]
\[
= (p_1 \lor p_2) \lor (p_1 \land p_3)
\]
\[
= ((p_1 \lor p_2) \land (p_1 \lor p_3)) \lor ((p_2 \lor p_3) \land (p_1 \lor p_3))
\]
\[
= (p_1 \lor p_2) \lor (p_1 \land p_3) \lor (p_2 \lor p_3)
\]

Similarly, Fig. 3b shows the equivalent LPN subnet of $t_1$. By the constructing method of IPNs, three disjunctive clauses of $f_3(t_1)$ correspond with three transitions in the IPN subnet. Therefore, the disjunctive normal form of $f_3(t_1)$ decides fully the structure of the corresponding LPN subnet.

From the examples, it is a fact that the size of an IPN model can be reduced in its equivalent LPN model, since many transitions are encapsulated into an equivalent logic transition. Their modeling power and behaviors are same, because LPNs and IPNs have the same reachable marking graph in essence and IPNs and LPNs are isomorphic.

**CONCLUSION**

With the rapid progress of cooperative and network extent of interorganizational workflows, batch processing function and passing value indeterminacy of workflow processes need often to be considered in the design of workflow systems. Moreover, formal modeling and analysis technologies should have the modeling ability of these functions. To deal with the problems, a new extension of PNs, i.e., LPNs, is proposed in (Yuyue and Changqun, 2002; Du et al., 2007) and its practical application is illustrated with some nontrivial examples. This study focuses on the analysis of the modeling power of LPNs and the equivalency between LPNs and IPNs. The equivalency is proved formally and a constructing algorithm of equivalent IPNs from LPNs is proposed based on the disjunctive normal forms of logic input/output expressions. Moreover, the size of LPN models is smaller than that of the equivalent IPN models. Further study will be the investigation of fundamental properties of LPNs, such as liveness, state equivalency and reachability. Moreover, the analysis technologies of LPNs will be studied and its developmental tool is considered and designed.

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