Improved Design of Trellis Space-time Code for High Spatial-and Multipath Diversity in MIMO-OFDM Fading Channels

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Abstract: Trellis Coded Modulation (TCM) technique offering good coding gain without sacrificing any bandwidth for transmitting digital sequences over bandlimited channels and Space-Time Block Codes (STBC) from Coordinate Interleaved Orthogonal Designs (CIOD) allowing single-complex symbol decoding and offering higher data rates than traditional orthogonal STBC (OSTBC) have attracted considerable attention. In this study, for the purpose of achieving more high spatial- and multipath diversity in multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) fading channels with low decoder complexity, a trellis code CIOD system combining an inner STBC, a coordinate interleaver and an outer trellis code is proposed to enhance the transmission performance. Upper bound expression of a pairwise codeword error probability is derived for the proposed trellis code CIOD system and high spatial- and multipath diversities are achieved from this upper bound analysis. By means of the jointly optimization of the rotation angle of constellation and trellis codes using exhaustive search approaches, the proposed trellis code CIOD achieves higher diversity than the best benchmark system at the cost of reasonable increased complexities of encoding and decoding. Simulation results show that the proposed optimized \{4, 8, 16, 32\}-state rate-2/3 8-PSK trellis codes CIOD provides additional 2, 3, 2 and 1.5 dB gains in SNR at the CER of $10^{-3}$, when compared to their best counterparts, respectively.

Key words: Trellis coded modulation, space-time block code, single-complex symbol decoding, spatial diversity, multipath diversity, optimization

INTRODUCTION

In order to achieve reliable digital communications over fading channels, high signal energies and large bandwidth expansion factors should be required for time diversity signaling. Schlegel and Costello (1989) was used the Chernoff bounding technique to obtain performance bounds for bandwidth efficient trellis codes on fading channels with various degrees of side information and used the effective length and the minimum squared product distance to replace the minimum free squared Euclidean distance as code design parameters for fading channels with a substantial multipath component.

TCM (Ungerboeck, 1982) is one of the coded modulation techniques used in digital communications. It combines the choice of a modulation scheme with that of a convolutional code together for the purpose of gaining noise immunity over encoded transmission without expanding the signal bandwidth or increasing the transmitted power (Garello et al., 2002). The main advantage of TCM technique, which makes it suitable scheme for transmitting digital sequences over bandlimited channels, is providing good coding gain without sacrificing any bandwidth. TCM can be viewed as a combined coding and modulation technique wherein modulation is embedded into the encoding process and is designed in conjunction with a rate n/(n+1) convolutional code (Garello et al., 2002). Trellis codes specifically constructed for fading channels was presented by Schlegel and Costello (1989) and an efficient method to evaluate the Chernoff bound on the event error probability of these codes, their performance is analyzed for fading channels with different degrees of side information. Numerical results on event error probability demonstrate the importance of the effective length as a code design parameter for fading channels. A 4-state 8-PSK TCM code schemes for both speech and data

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transmission over mobile fading channels is addressed by Jamali and Le-Ngoc (1991), the design rules are based on that the minimum product of the squared branch distances along the shortest error event paths is maximized. Simulation results for light shadowed Rician fading channel showed that this code has better performance than the other 4-state 8-PSK TCM schemes for both data and speech bit error rates. A coordinate interleaving (Boule and Belfiore, 1992) based TCM code construction scheme designed for flat fading channels was introduced by Jelicic and Roy (1994). Due to finite interleaving and imperfect CSI effects, a significant portion of their coding gain is preserved. It was demonstrated that although coordinate interleaving schemes suffer relatively more impairments than corresponding symbol interleaving schemes. Code performance including the effects of finite interleaving and imperfect Channel State Information (CSI) was studied. The inherent coding gain of proposed coordinate interleaving based schemes are substantially by cut-off rate computations. A 4-PSK 16-state space-time coded OFDM scheme was proposed by Agrawal et al. (1998) for providing high data-rate delay-sensitive wireless communication over wideband channels and its performance was compared to the RS coded OFDM scheme. Simulation results showed that the proposed scheme is capable of robust reliable transmission at relatively lower SNRs in a variety of delay profiles. A simple space-time coded OFDM transmitter diversity technique for frequency selective fading channels was presented by Lee and Williams (2000). The proposed technique utilizes OFDM to transform frequency selective fading channels into multiple flat fading subchannels on which space-time coding is applied. In slow fading channels, the proposed transmitter diversity system achieves diversity gain equivalent to that of the optimal Maximal Ratio Combining (MRC) receiver diversity system. A two-branch transmitter diversity system is implemented without bandwidth expansion and with a small increase in complexity beyond that of a conventional OFDM system. Simulations verify that the two-branch transmitter diversity system achieves a diversity gain equivalent to that of the optimal MRC receiver diversity system. A trellis-structured STC designing scheme in OFDM systems for frequency-selective fading channels was proposed by Lu and Wang (2000), the Pairwise Error Probability (PEP) expression show that STC-OFDM systems can potentially provide a diversity order as the product of the number of transmitter antennas, the number of receiver antennas and the frequency selectivity order and also proposed that the large effective length and the ideal built-in interleaving are two most important elements in designing STC-OFDM system. Due to efficiently exploiting both the spatial diversity and the frequency-selective-fading diversity, the simulation results demonstrated the significant performance improvement of the proposed STC's over the conventional space-time trellis codes.

The MIMO-OFDM systems can achieve significant error rate performance improvement by exploiting spatial and multipath diversities. The spatial diversity can be achieved with space-time coding at the transmitter and signal combining at the receiver, the multipath diversity can be achieved with channel coding through subcarriers in case of OFDM modulation. OFDM converts a frequency-selective fading channel into parallel flat-fading subchannels, thereby simplifying channel equalization and symbol decoding. However, OFDM's performance suffers from the loss of multipath diversity and the inability to guarantee symbol detectability when channel nulls occur. To achieve reliable wireless transmission over a slowly varying MIMO channel, Alamouti-based space-time Bit-Interleaved Coded Modulation (BICM) with Hybrid Automatic Repeat re-Quest (HARQ) transmission scheme is proposed by Wang et al. (2009a) to increases the efficiency of HARQ packet transmission by exploiting both the spatial and time diversity of the MIMO channel, which uses the full diversity of Alamouti STC and the added gain of the packet combining. An OFDM transmitters scheme using Linear Constellation Precoded (LCP) to achieves multipath diversity was introduced (Liu et al., 2003) for frequency-selective fading channels. Exploiting the correlation structure of subchannels and choosing properly system parameters, an optimal subcarrier grouping to partition the set of subchannels into subsets was performed. Relying on subcarrier grouping, the LCP-OFDM system was converted into a set of GLCP-OFDM subsystems and then designed LCPs for each subsystem. Subcarrier grouping enables the maximum possible diversity and coding gains while reducing the decoding complexity greatly and simplifying the precoder design. For multi-cell multi-antennas networks with uplink training, in order to mitigate corrupted uplink training and thus increase achievable rates of downlink transmission, a precoding techniques was proposed by Wang et al. (2009b), this precoding is the optimal solution of an optimization problem of the mean-square error of signals and the mean-square interference. A Space-Time-Frequency (STF) coding scheme achieving spatial and multipath diversities for MIMO-OFDM transmissions over frequency-selective Rayleigh fading channels was proposed by Liu et al. (2002). In order to simplify the design, convert the complex STF codes design into simpler GSTF designs per
group by incorporating subchannel grouping and choosing appropriate system parameters, both GSTF block codes and GSTF trellis codes was constructed based on the derived design criteria, which achieves the full diversity gain with low decoding complexity, this equals the product of the number of transmit and receive antennas times the channel length. A joint Error-Control Coding (ECC) and Complex-Field Coding (CFC) scheme to further increase the multipath diversity for MIMO space-time transmission over flat or frequency-selective fading channels was proposed by Wang et al. (2003). In this scheme, trellis encoding through OFDM subcarriers was used with serially concatenated outer convolutional code and inner LCP-STBC (SCLCF-STBC). Relying on OFDM transmitters to convert an FIR MIMO channel to correlated parallel flat fading MIMO channels, the CFC scheme for SISO flat fading channels was adapted to space-time flat fading MIMO channels. The diversity achieved by the joint system with transmit antenna-switching is the product of the free distance of the ECC, the complex-field encoder size and the number of receive antennas. Exploring the lattice sphere packing representation theory (Oggier and Viterbo, 2004), Sphere Decoding (SD) algorithm (Damen et al., 2000) was proposed for the V-BLAST multi-antenna system and the algebraic Space-Time (ST) codes over the Rayleigh fading channel. The simulation results shows that the algorithm obtains the Maximum-Likelihood (ML) performance with a computational complexity, It is does not depend on the constellation size. Hence, a very high spectral efficiency could be obtained along with ML performance. The simulation results also shows that the limitations of the V-BLAST detection algorithm over the uncoded system in taking advantage of the receive diversity. The algebraic ST codes can obtains the full diversity in a multi-antenna system without adding any redundancy and perform the ML decoding with low computational complexity.

It is known that by employing Space-Time-Frequency Codes (STFC) to frequency selective MIMO-OFDM systems, all the spatial, temporal and multipath diversity can be exploited. There exists STFBC designed using orthogonal designs with constellation precoder to get full diversity (Liu et al., 2002) for more than 2 transmit antennas, STFBC of rate-1 full diversity cannot be constructed using orthogonal designs due to orthogonal designs of rate-1 exists only for 2 transmit antennas. A rate-1 (complex symbols per channel use) STFBC scheme of rate-1 for 4 transmit antennas MIMO-OFDM systems with double symbol decoding designed using quasi-orthogonal designs combined with CIOD was presented by Gowrisankar and Rajan (2005), whereas all known schemes provide only rate 3/4 complex symbols per channel use. This is the first scheme using Quasi-Orthogonal Design to construct STFBC for MIMO-OFDM, the conditions on the signal sets used for the variables of the Quasi-Orthogonal design for full-diversity are obtained and simulation study shows that the BER performance is at least as good as that of the existing comparable scheme.

The STBC from CIOD offers advantages of full-diversity and single-symbol decidability (Khan and Rajan, 2006), symmetric structured CIOD are constructed by incorporating two same codeword-structured generalized linear complex orthogonal designs (GLCOD). STBC scheme from Symmetric structured CIOD for a quasi-static frequency-nonselective i.i.d. Nakagami-m fading channel was presented by Lee et al. (2009) and the performance analysis of average error rate (e.g., average Symbol Pairwise Error Rate (SPER), Symbol-Error Rate (SER), etc.), outage capacity, Information Outage Probability (IOP) and both average SER-based and IOP-based asymptotic diversity orders was presented. Tight union upper and lower bounds on the SER was derived from an accurate closed-form formula for the SPER. Closed form expressions for the outage capacity was provided using Gaussian and Gamma approximations. Accurate closed-form approximations for the IOP was derived using high Signal-to-Noise Ratio (SNR) and moment-matching approximation techniques. The proposed STBC offering full-diversity was proved by SER- and IOP-based asymptotic and instantaneous diversity orders.

The STBC using CIOD allow single-symbol complex decoding and offer higher data rates than traditional orthogonal STBC. An equivalent channel representation for CIOD codes enabling their decoding readily over MIMO channels was presented (Dao and Tellambura, 2008). A general ML metric is derived, enabling the computation of Symbol Pair-Wise Error probability (SPEP) and Union Bound (UB) on SER. The UB thus can be used to accurately predict and optimize the performance of CIOD codes, a signal design combining signal rotation and power allocation is presented for constellations with uneven powers of real and imaginary parts. the union bound can be used to accurately analyze the performance of CIOD codes and moreover, to optimize the signal rotation for arbitrary constellation.

For traditional Space-Frequency (SF) coded OFDM systems, in order to exploit the maximum achievable diversity order, we should be needed to design space-time trellis codes with high trellis complexity. A concatenated Trellis Coded Modulation (TCM) codes with STBC as an efficient SF coding scheme in coded OFDM system was proposed for high-speed transmission over wireless links (Gong and Letaief, 2003). The analytical expression for the
pairwise probability of the proposed system is derived in slow, space- and frequency-selective fading channels. The design criteria of TCM codes used in such SF OFDM systems are derived. Simulation results show excellent performance results in the space and frequency-selective fading channels. It is also shown that the proposed SF codes with low trellis complexity can easily exploit the full diversity order provided by the fading channel, as well as the excellent outage capacity properties of SF coded OFDM over multipath fading channels. An STBC concatenated with Differential Space-Time Block Code (DSTBC) designing scheme was proposed in frequency-flat Rayleigh-fading channels with and without perfect interferer (Tarasak and Bhargava, 2004). The design criteria of the proposed scheme are effective code length over span two symbol intervals and minimum product-sum distance over span two symbol intervals, which are exactly the same as that of TCM-STBC in perfectly known channels. Based on the design criteria, several rate-2/3 8-PSK systematic Ungerboeck’s TCM schemes (Ungerboeck, 1982) are found by computer search and shown to outperform the optimal codes design for AWGN channel and Rayleigh-fading channel without transmit diversity. The presented codes can be used with STBC in perfectly known fading channels. However, the computational complexities of a ML sphere decoder (Damen et al., 2000) used with LCP-STBC is very high and a turbo decoder used with SC-LCP-STBC leads to a search coded modulation techniques with lower decoding complexities. Such efficient technique with simple Viterbi decoding is a concatenated outer trellis coded and inner Space-Time Block Coded (TC-STBC) OFDM (Gong and Letaief, 2003). It is shown by Gong and Letaief (2003) that TC-STBC has a superior performance compared to other trellis coded MIMO-OFDM systems (Agrawal et al., 1998, Lu and Wang, 2000).

On the other hand, coordinate interleaving (Jelicic and Roy, 1994; Oggeri and Viterbo, 2004) is a powerful technique providing diversity by extending the considered signal constellation. A space-time-frequency block codes (STFBC) for MIMO-OFDM transmissions over frequency selective Rayleigh fading channels was presented (Jagannadha Rao et al., 2004). When the number of nonzero taps of the Channel Impulse Response (CIR) is equal to two, symbol-by-symbol decoding can be performed on these codes and these codes have reduced complexity for more than two channel taps. The STBC from CIOD and Orthogonal Designs (ODs) have been attracting wider attention due to their amenability for fast (single-symbol) ML decoding and full-rate with full-rank over quasi-static fading channels. These STBC codes are instances of single-symbol decodable codes. Linear STBC (Khan and Rajan, 2006) allow single-symbol ML decoding (not necessarily full-diversity) over quasi-static fading channels—calling them single-symbol decodable designs (SDD). The class SDD includes ODs and CIODs as proper subclasses, a class of those SDD that Full-rank SDD (FSDD) offering full-diversity are characterized and classified. For square designs, the maximal rate for square FSDDs was derived. For non-square designs, generalized CIOD (a superset of CIODs) are presented and analyzed. For rapid-fading channels, an equivalent matrix channel representation allows the results of quasi-static fading channels to be applied to rapid-fading channels, the rate of single-symbol decodable STBC are independent of the number of transmit antenna and inversely proportional to the block-length of the code. The CIOD for two transmit antennas is the only STBC that is single-symbol decodable over both quasi-static and rapid-fading channels. Importantly, the CIOD has a single symbol decodability feature (Khan and Rajan, 2006) ensuring low decoding complexities.

In this study, an outer trellis code concatenated with an inner CIOD OFDM system with low decoder complexity is proposed. This trellis code CIOD system combines a STBC, a coordinate interleaver and a trellis code to enhance the MIMO-OFDM performance. The analysis of trellis code CIOD codewords pairwise error probability (PEP) shows that trellis code CIOD provides full spatial and high multipath diversities and then extend the two symbol interleaved (Tarasak and Bhargava, 2004) case of TC-STBC (Gong and Letaief, 2003) to a symbol interleaved case of TC-STBC. Compared to the two symbol interleaved case (Gong and Letaief, 2003), the results of TC-STBC analysis show that the symbol interleaving doubles the achieved diversity of TC-STBC. Furthermore, the results of trellis code CIOD analysis show that the trellis code CIOD achieves diversity double that of the symbol interleaved TC-STBC. Due to perform rotating and coordinate interleaving operations on a trellis codeword symbols and then perform space-time block coding operation in trellis code CIOD, an additional signal space diversity is achieved, which doubles the multipath diversity achieved by symbol interleaved TC-STBC. Hence, the trellis code CIOD diversity is the same as that of SC-LCP-STBC (Wang et al., 2003) while preserving the low ML Viterbi decoding complexities, the diversity is four times higher than the two-symbol interleaved TC-STBC (Gong and Letaief, 2003). Subsequently, the trellis code CIOD trellis design rule is derived from analysis of codeword error probability and further applied to optimize the performance of trellis code CIOD for {4, 8, 16, 32}-state rate-2/3 8-PSK trellis codes. The simulation results show that the codeword decision error rate (CER) performances of trellis code CIOD is excellent than that of the benchmark systems.
SYSTEM MODEL

Here, we describe the channel model and the encoding and decoding operations of the proposed trellis code CIOD OFDM system.

Channel model: Assumed that the channel conditions remain constant during two consecutive OFDM frames transmission, the frequency domain fading coefficients $H_x[\alpha(k)]$ and $H_y[\alpha(k)]$ represent effects from the first and second transmit antennas during the transmission of $Y(k)$, respectively and denote $H_z[\alpha(k)]$ as $K_{\alpha(k)}$ ($\mu = 1,2$). Let, $r_\alpha$ be the received symbol from $\alpha$th subcarrier of the $i$th OFDM codeword symbol ($i=1,2$) and the subchannel noise variables $n_{k_i}$ are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variable with variance $1/2N_0$ per dimension, wherein $k = 0,1,...,K-1$ and $i=1,2$ the MIMO-OFDM codeword transmission can expressed as:

$$\begin{align*}
R(k) = Y(k)H(k)N(k)
\end{align*}$$

(1)

where $R(k) = (r_{k_1}, r_{k_2})^T$, $H(k) = (H_x[k], H_y[k])^T$ and $N(k) = (n_{k_1}, n_{k_2})^T$ and $(\ast)^T$ is the transpose operation. Assumed that a trellis code CIOD OFDM codeword $Y = \{Y(0), Y(1),..., Y(K-1)\}$ transmitting over $K$ subcarriers is partitioned into $1/2K$ CIOD groups. The corresponding reception and the additive Gaussian noise can be expressed as $R = \{R(0), R(1),..., R(K-1)\}$ and $N = \{N(0), N(1),..., N(K-1)\}$, respectively.

Encoding operation: Figure 1 shows the encoder block diagram of trellis code CIOD OFDM for two transmit antennas, the source bits are trellis encoded at rate-2/3 and mapped to trellis codewords of length $2K$ with symbols $x_k$ from 8-PSK signal constellation. and then, each 8-PSK symbol $x_k$ is rotated by $\theta$,

$$\begin{align*}
\tilde{x}_k = x_k \exp(j\theta)
\end{align*}$$

(2)

A vector of rotated symbols:

$$\begin{align*}
\tilde{X} = (\tilde{x}_0, \tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_{2K-1})
\end{align*}$$

(3)

is perfectly coordinate interleaved. A proper coordinate interleaver should be employed to achieve maximum diversity. $\tilde{x}_k$ is the $k$th element of vector $\tilde{X}$, the proposed coordinate interleaver performs the following operations:

![Fig. 1: Transmitter in the trellis code CIOD OFDM system](image)

Decoding operation: Assumed that the receiver have perfectly Channel State Information (CSI) $H = \{H(0), H(1),..., H(K-1)\}$, the decoding metric of the trellis codeword $X = (x_0, x_1, ..., x_{2K-1})$ can be expressed as:

$$\begin{align*}
m(R, X; H) = \sum_{k=0}^{K-1} m_k
\end{align*}$$

(8)

where, the CIOD group decoding metric:

$$\begin{align*}
m_k = m[R(k), R\left(\frac{k+K}{2}\right), \tilde{x}_{k}, \tilde{x}_{k+1}, \tilde{x}_{2K-k}, \tilde{x}_{2K-k+1}; H(k), H\left(\frac{k+K}{2}\right)]
\end{align*}$$

(9)
According to Eq. 2 and Eq. 4, the ML decoding rule can be expressed as X minimization of Eq. 8. Due to the elements of N are i.i.d., complex Gaussian random variables, each term in the right-hand side of Eq. 8 can be expressed as:

\[
m_k = |r_{x_k} - H_{x_k} \hat{x}_{2k} - H_{z_k} \hat{x}_{2k+1} |^2 + |r_{x_k+1} - H_{x_k} \hat{x}_{2k+1} - H_{z_k} \hat{x}_{2k+2} |^2 + |r_{x_k+2} - H_{x_k} \hat{x}_{2k+2} - H_{z_k} \hat{x}_{2k+3} |^2 + |r_{x_k+3} - H_{x_k} \hat{x}_{2k+3} - H_{z_k} \hat{x}_{2k+4} |^2,
\]

and have the metrics related to the symbols \(\hat{x}_{2k}, \hat{x}_{2k+1},\hat{x}_{2k+2},\hat{x}_{2k+3}\):

\[
m_{2k} = |\hat{x}_{2k} - \hat{x}_{2k} |^2 + (|H_{x_k} |^2 - 1)|\hat{x}_{2k+1}|^2
\]

where, \(i = 1, 2, 3, 4, 5, 6, 7\) and \(\xi = k+1/2K\). In Eq. 11, the estimation expressions of rotated and coordinate interleaved trellis codeword symbols:

\[
\begin{align*}
\hat{x}_{2k} &= r_{x_k} H_{x_k} + r_{x_k+1} H_{z_k} \\
\hat{x}_{2k+1} &= r_{x_k+1} H_{x_k} + r_{x_k+2} H_{z_k} \\
\hat{x}_{2k+2} &= r_{x_k+2} H_{x_k} + r_{x_k+3} H_{z_k} \\
\hat{x}_{2k+3} &= r_{x_k+3} H_{x_k} + r_{x_k+4} H_{z_k}
\end{align*}
\]

and the equivalent subchannel gains

\[
|H_{x_k} |^2 = |H_{x_k} |^2 + |H_{z_k} |^2
\]

Using Eq. 11, rewrite Eq. 10 as:

\[
m_k = m_{2k} + m_{2k+1} + m_{2k+2} + m_{2k+3}
\]

In order to obtain the Viterbi decoding metrics dependent on rotated trellis codeword symbols \(\hat{x}_{1}, \hat{x}_{2k+1}, \hat{x}_{2k+2},\hat{x}_{2k+3}\) and \(\hat{x}_{2k+4}\), separately. By combining \(\hat{m}_{2k}, \hat{m}_{2k+1},\)
\(\hat{m}_{2k+2}, \hat{m}_{2k+3}\), and \(\hat{m}_{2k+4}\), and simplifying Eq. 14, the CIOD group decoding metric can be expressed as:

\[
\hat{m}_k = \hat{m}_{2k} + \hat{m}_{2k+1} + \hat{m}_{2k+2} + \hat{m}_{2k+3}\]

where, the decoding metrics expressions of rotated trellis codeword symbols:

\[
\hat{m}_{2k} = (\text{Re}(\hat{x}_{2k}) - \text{Re}(\hat{x}_{k}))^2 + (|H_{x_k} |^2 - 1)(\text{Re}(\hat{x}_{k}))^2 + (\text{Im}(\hat{x}_{2k}) - \text{Im}(\hat{x}_{k}))^2 + (|H_{z_k} |^2 - 1)(\text{Im}(\hat{x}_{k}))^2
\]

\[
\hat{m}_{2k+1} = (\text{Re}(\hat{x}_{2k+1}) - \text{Re}(\hat{x}_{k+1}))^2 + (|H_{x_k} |^2 - 1)(\text{Re}(\hat{x}_{k+1}))^2 + (\text{Im}(\hat{x}_{2k+1}) - \text{Im}(\hat{x}_{k+1}))^2 + (|H_{z_k} |^2 - 1)(\text{Im}(\hat{x}_{k+1}))^2
\]

\[
\hat{m}_{2k+2} = (\text{Re}(\hat{x}_{2k+2}) - \text{Re}(\hat{x}_{k+2}))^2 + (|H_{x_k} |^2 - 1)(\text{Re}(\hat{x}_{k+2}))^2 + (\text{Im}(\hat{x}_{2k+2}) - \text{Im}(\hat{x}_{k+2}))^2 + (|H_{z_k} |^2 - 1)(\text{Im}(\hat{x}_{k+2}))^2
\]

\[
\hat{m}_{2k+3} = (\text{Re}(\hat{x}_{2k+3}) - \text{Re}(\hat{x}_{k+3}))^2 + (|H_{x_k} |^2 - 1)(\text{Re}(\hat{x}_{k+3}))^2 + (\text{Im}(\hat{x}_{2k+3}) - \text{Im}(\hat{x}_{k+3}))^2 + (|H_{z_k} |^2 - 1)(\text{Im}(\hat{x}_{k+3}))^2
\]

The Viterbi Algorithm using the branch metrics (Eq. 16) can be easily applied in the ML decoding of the proposed trellis code CIOD.

**DESIGN OF TRELLIS CODE**

Here, the codeword error probability of the proposed trellis code CIOD will be derived and then jointly optimize the rotation angle \(\theta\) and trellis code to obtain the best CER performances.

**Codeword error probability of proposed trellis code CIOD:** Assumed that the receiver have perfectly Channel State Information (CSI), the Pairwise Error Probability (PEP) between the decoder selects erroneous trellis codeword \(Z\) and the transmitted codeword \(X\) can be expressed as:

\[
P(X \rightarrow Z | H) = P[m(R,X,H) > m(R,Z,H)]
\]

where, \(R = (R^0, ..., R^{n_{P}})\), the received symbols from the vth antenna:

\[
R^v = [R^v(0), R^v(1), ..., R^v(K-1)]
\]

where, \(v = 1, ..., n_{P}\) and \(R^v(k) = (r_{x_k}^v r_{z_k}^v)^T\). Similarly, \(H = [H^0, ..., H^{n_{P}}]\):

\[
H^v = [H^v(0), H^v(1), ..., H^v(K-1)]
\]

and \(H^v(k) = (H_{x_k}^v H_{z_k}^v)^T\). By substituting \(m(R,X,H)\) (Eq. 8) the CIOD group decoding metrics (Eq. 14 and 11) and the corresponding decoding metrics for \(m(R,Z,H)\) in Eq. 17. And assuming that the subchannel noise variables \(n_{i,x,u}\) are i.i.d., zero-mean complex Gaussian distributed with variance 1/2N_{0} per dimension, have:

\[
P(X \rightarrow Z | H) = \prod_{i=1}^{n_{P}} \frac{1}{\sqrt{2\pi N_0}} \exp \left( -\frac{(d_i^v - d_{i,x,u})^2}{2N_0} \right)
\]

where,

\[
d_v^i = \sum_{i=1}^{2N_0} (|H_{x_k}^v |^2 + |H_{z_k}^v |^2) \hat{x}_{2k+1} - \hat{x}_{2k+1}^2
\]
where $\xi = \{ k, k+1/2K \}$. In Eq. 18, $(d_1^2 + d_2^2) \cdot \pi$ is the modified Euclidean distance of a CIOD group pair represented by $(x_k, x_{k+1/2K}, x_k, x_{-k+1/2K})$ and $(\bar{x}_k, \bar{x}_{k+1/2K}, \bar{x}_k, \bar{x}_{-k+1/2K})$ using the inequality:

$$Q(x) \leq \exp \left( -\frac{x^2}{2} \right).$$  \hspace{1cm} (20)

upper-bound of Eq. 18 can be expressed as:

$$P(X \rightarrow Z | H) \leq \exp \left[ -\frac{E_e}{4N_0} d^2(X, Z) \right]$$  \hspace{1cm} (21)

where, the modified Euclidean distance between pair of trellis codewords X and Z.

$$d^2(X, Z) = \sum_{k=1}^{2K} \sum_{n=1}^{3} (|H_{k,n}^1|^2 + |H_{k,n}^2|^2) |x_{k+n} - \bar{x}_{k+n}|^2$$  \hspace{1cm} (22)

Let $n_k = 1$ for simplicity, rewrite Eq. 22 as:

$$d^2(X, Z) = \sum_{k=1}^{2K} \sum_{n=1}^{3} (|H_{k,n}^1|^2 + |H_{k,n}^2|^2) |x_{k+n} - \bar{x}_{k+n}|^2$$  \hspace{1cm} (23)

The rotated trellis codewords corresponding to X and Z are:

$$\bar{x} = (x_0, x_1, x_2, x_3, x_4, x_5, x_6)$$

and

$$\bar{z} = (z_0, z_1, z_2, z_3, z_4, z_5, z_6)$$

respectively. Let, $\bar{x}$ and $\bar{z}$ differ only inside the short part with length $k$, i.e., only $(x_{k+1}, x_{k+2}, \ldots, x_{2K})$ differs from $(z_{k+1}, z_{k+2}, \ldots, z_{2K})$. Equation 23 becomes:

$$d^2(X, Z) = \sum_{k=1}^{2K} \sum_{n=1}^{3} (|H_{k,n}^1|^2 (\text{Re}(x_k) - \text{Re}(z_k))^2 + |H_{k,n}^2|^2 (\text{Im}(x_k) - \text{Im}(z_k))^2)$$  \hspace{1cm} (24)

where, $\eta = \{ s+1, s+2, \ldots, s+K \}$

$$f(k) = d_1^2 \cdot \text{Re}(\pi(k)) t, \text{g}(k) = d_2^2 \cdot \text{Im}(\pi(k)) t$$

and $a(k)$ takes the integer operation. The coordinate interleaver $\pi$ can be represented by a pair of permutations for real parts $\text{Re}\{\pi(k)\}$ and imaginary parts $\text{Im}\{\pi(k)\}$ of the input vector. According to Eq. 4:

$$\text{Re}\{\pi(k)\} = \begin{cases} 2k, & 0 \leq k < K \\ 2k - 2K + 1, & K \leq k < 2K \\ \end{cases}$$

and

$$\text{Im}\{\pi(k)\} = \begin{cases} 2k + K + 1, & 0 \leq k < \frac{K}{2} \\ 2k - K, & \frac{K}{2} \leq k < \frac{3K}{2} \\ 2k - 3K + 1, & \frac{3K}{2} \leq k < 2K \\ \end{cases}$$

Perfect coordinate interleaving means that $f(\xi_i) \cdot g(\xi_i)$ for every pair $\xi_i$, $\omega \in \eta$ and $f(\xi_i) \cdot g(\omega)$, $g(\xi_i) \cdot g(\omega)$, for every pair $\xi_i$, $\omega \in \eta$ when $\xi \neq \omega$. Hence, after performing perfect coordinate interleaving, there is no repeated subcarrier fading coefficients $H_{k,n}$ in Eq. 24. Assuming that the subcarriers are also perfectly interleaved and transmit antennas are well separated, hence, the subcarrier fading coefficients $H^r_{k,n}$ in Eq. 24 are zero-mean i.i.d., complex Gaussian random variables with variance 1/2 per dimension. Taking the expectation of Eq. 21 over Rayleigh distributed random variable $|H_{k,n}|$ using Eq. 24, obtain:

$$P(X \rightarrow Z) \leq \prod_{k=1}^{2K} \prod_{\text{Re}(\eta)} \left[ 1 + \frac{E_e}{4N_0} (\text{Re}(x_k) - \text{Re}(z_k))^2 \right] \prod_{\text{Im}(\eta)} \left[ 1 + \frac{E_e}{4N_0} (\text{Im}(x_k) - \text{Im}(z_k))^2 \right]$$  \hspace{1cm} (25)

In general, rotation angle $\theta$ can be selected such that both of real and imaginary components of $x_k$ and $z_k$ do not differ in the case of $x_k \neq z_k$. Define two different sets of k values, $\text{Re}\{\eta\}$ and $\text{Im}\{\eta\}$ for which real and imaginary components of rotated trellis codeword symbols $x_k$ and $z_k$ differ, respectively. Assuming in the high SNR region, Eq. 25 can be expressed as:

$$P(X \rightarrow Z) \leq \prod_{k=1}^{2K} \prod_{\text{Re}(\eta)} \frac{E_e}{4N_0} (\text{Re}(x_k) - \text{Re}(z_k))^2 \prod_{\text{Im}(\eta)} \frac{E_e}{4N_0} (\text{Im}(x_k) - \text{Im}(z_k))^2$$  \hspace{1cm} (26)

The cardinality of sets $\text{Re}\{\eta\}$ and $\text{Im}\{\eta\}$ are denoted as $|\text{Re}\{\eta\}|$ and $|\text{Im}\{\eta\}|$, respectively, Eq. 26 becomes:

$$P(X \rightarrow Z) < \frac{E_e}{4N_0} \left( |\text{Re}\{\eta\}| \cdot |\text{Im}\{\eta\}| \right) \times \left[ \prod_{k=1}^{2K} (\text{Re}(x_k) - \text{Re}(z_k))^2 \right] \times \left[ \prod_{k=1}^{2K} (\text{Im}(x_k) - \text{Im}(z_k))^2 \right]$$  \hspace{1cm} (27)
From Eq. 27, under the assumptions of perfect coordinate and channel interleaving, the achievable diversity of the system for \( n_b = 1 \) is:

\[
G_d = 2 \times \min_{\eta \in \mathbb{Z}^2} (|\text{Re}(\eta)| + |\text{Im}(\eta)|) \tag{28}
\]

The maximum value of \( \min |\text{Re}(\eta)| \) and \( |\text{Im}(\eta)| \), is \( 2\Gamma \), wherein, \( \Gamma \) is the effective code length of the trellis code. The achievable diversity of trellis code CIOD OFDM is \( G_d = 4\Gamma \), which is four times of the two-symbol interleaved TC-STBC OFDM (Gong and Letawe, 2003). The trellis code CIOD coding gain is:

\[
G_c = \min_{\eta \in \mathbb{Z}^2} \left\{ \prod_{b \in \mathbb{Z}} (\text{Re}(X_b) - \text{Re}(Z_b)) \right\}^{1/2} \times \left\{ \prod_{b \in \mathbb{Z}} (\text{Im}(X_b) - \text{Im}(Z_b)) \right\}^{1/4} \tag{29}
\]

The codeword error probability can be expressed in the form of the PEP:

\[
P_e = \sum_X P(X) \sum_{Z \in X} P(X \rightarrow Z) \tag{30}
\]

where, \( P(X) \) is the probability of the codeword \( X \) yielded from the trellis encoder and \( P(X \rightarrow Z) \) is the PEP.

**Jointly optimization of rotation angle and trellis code:**

There are two possible approaches for exhaustive trellis code optimization. The one approach is to optimize the trellis code and rotation angle \( \theta \) to maximize the achievable diversity of trellis code CIOD OFDM system \( G_d \) (Eq. 28) and the trellis code CIOD coding gain \( G_c \) (Eq. 29). The other approach is to optimize the trellis code and rotation angle \( \theta \) to minimize the upper-bound of codeword error probability, which can be obtained by substituting Eq. 27 in 30. We joint rotation angle \( \theta \) and trellis code to settle with minimization problem of the upper-bound of codeword error probability over all possible trellis generator polynomials using exhaustive search technique as described by Schlegel and Costello (1989). After performing an exhaustive search minimizing the upper-bound of codeword error probability over all possible trellis code pairs \( X \) and \( Z \) starting and merging at the common trellis states in the case of \( \{4, 8, 16, 32\} \)-state rate-2/3 8-PSK trellis code, respectively, the values of rotation angle \( \theta \) ranging from 0° to 22.5° with step of 2° are selected, wherein, the high SNR case of \( E_b/N_0 = 14 \text{ dB} \) is considered, the value of \( E_b/N_0 \) is selected to find the optimum trellis codes at the CER of 10^{-2} during the exhaustive search and let \( \kappa = 3 \), in order to contain the critical codeword pairs having considerable effect on the CER performance, the value of \( \kappa \) is selected larger for trellises with larger number of states during the exhaustive search.

From Fig. 2, the upper-bound of codeword error probability with optimal trellis codes for different values of \( \theta \) are considered for \( \{4, 8, 16, 32\} \)-state trellises. The upper-bounds of codeword error probability for the optimum trellis code decrease with values of rotation angle \( \theta \) and achieve their minimum value \( \theta = 22.5° \). For 8-PSK constellation is used, rotation angles between 22.5°±45°, it result in the same codeword error probability upper-bound values for \( \theta = 45°-0° \). The octal generator polynomials calculated by an exhaustive code search technique for the optimum trellis codes minimizing Eq. 30 are shown in Table 1, wherein the optimum rotation angle \( \theta = 22.5° \) and the values of achievable diversity gain \( G_d \) (Eq. 28) and coding gain \( G_c \) (Eq. 29) are also given.

In Table 1, the 4-state trellis code is given by minimizing the upper-bound of codeword error probability for \( E_b/N_0 = 18 \text{ dB} \) with \( \kappa = 4 \), the 8, 16 and 32-state trellis codes are given for \( E_b/N_0 = 14 \text{ dB} \) with \( \kappa = 6 \).

**NUMERICAL RESULTS**

Assumed that MIMO-OFDM channel is static during two consecutive OFDM frames transmission, the receiver have perfect CSI, perfect random channel interleaving,
OFDM subcarriers \( K = 128 \) and a multipath channel taps \( L = 32 \) with equal power assignment. In Fig. 3, the simulation results show the CER performances of the proposed trellis code CIOD OFDM system outperforms that of the two-symbol interleaved TC-STBC OFDM system by Gong and Letaief (2003), wherein, the 4-state 8-PSK trellis code proposed by Jamali and Le-Ngoc (1991). Both of the systems are configured with two transmit and one receive antennas and their bandwidth efficiency equal to 2 bits/s/Hz, when trellis code termination and OFDM cyclic prefix are removed. All of the trellis codewords of length \( 2K = 256 \) were terminated. Figure 3 shows the CER performances of the proposed trellis code CIOD OFDM system with different values of the rotation angle \( \theta \), the best performance is achieved at \( \theta = 22.5^\circ \), it agree with the trellis code search results given earlier. Nevertheless, the 8-PSK CIOD (Khan and Rajan, 2006) lost its optimality (previously optimal \( \theta = 4.87^\circ \)) when trellis code is used. The proposed trellis code CIOD provides 10 dB SNR gain at the CER of \( 10^{-3} \) compared to the two-symbol interleaved TC-STBC (Gong and Letaief, 2003), the achieved diversity of trellis code CIOD is greater than that of the TC-STBC from the slopes of the CER curves. When a symbol interleaver is used with TC-STBC, the set size equal to the effective length of the trellis code, thus, the maximum achievable diversity of TC-STBC doubles. Figure 3 shows that 2-level block type symbol interleaved TC-STBC outperforms the two-symbol interleaved TC-STBC (Gong and Letaief, 2003) by 7.5 dB in SNR at the CER value of \( 10^{-3} \) and trellis code CIOD outperforms the symbol interleaved TC-STBC by 2.5 dB in SNR at the CER value of \( 10^{-3} \).

In order to evaluate the performance improvement contributed by the trellis code design as earlier using the trellis code (Jamali and Le-Ngoc, 1991) and the trellis code PT4 (Table 1) to simulate trellis code CIOD OFDM system. Both of the 4-states trellises using 8-PSK signal constellation provide bandwidth efficiency of 2 bits/s/Hz. From Fig. 4, the CER performance of trellis code CIOD improves about 0.4 dB SNR gain when PT4 is used. Therefore, the proposed optimized trellis code CIOD OFDM offers approximately 10.7 dB SNR gain at the CER of \( 10^{-3} \) compared to two symbol interleaved TC-STBC OFDM (Gong and Letaief, 2003). The performance of CIOD OFDM (Jagannadh Rao et al., 2004) is nearly equal to that of the LCP-STBC (Liu et al., 2002), the CIOD is single-symbol decodable while LCP-STBC requires more complex sphere decoding. In Fig. 4, the number of OFDM subcarriers used with 16-state 4-PSK Space-Frequency Trellis Code (SFTC) is 256, hence SFTC has an OFDM codeword with the same number of symbols as the other systems and Fig. 4 shows that the proposed 4-state trellis code CIOD outperforms the 16-state SFTC (Agrawal et al., 1998) by 6 dB in SNR at the CER of \( 10^{-3} \). Figure 4 also
Fig. 4: The CER performance of proposed vers. benchmark OFDM systems

Fig. 5: The CER performance of proposed trellis code CIOD OFDM vers. benchmark two-symbol interleaved TC-STBC OFDM systems [GL] with {8, 16, 32}-state trellis codes

shows that the CER performance of the proposed 4-state trellis code CIOD is 2 dB better than that of LCP-CIOD (Jagannadha Rao et al., 2004). The 4-state symbol interleaved TC-STBC offers a diversity gain $G_d = 4$ while offering approximately additional coding gain of 3 dB, but its CER slope is same as CIOD and LCP-STBC. The performance of STBC OFDM (Lee and Williams, 2000) shows that the proposed 4-state trellis code CIOD OFDM system has superior performance compared to all of the benchmark systems.

In Fig. 5 and 6, assumed that the interleaved multipath channel is perfect, the receiver have perfect
knowledge of CSI and OFDM subcarriers $K = 256$. Figure 5 shows the CER performances of $[8, 16, 32]$-state two-symbol interleaved TC-STBC systems (Gong and Letaief, 2003), wherein, the trellis codes are denoted by C8, C16 and C32, by Schlegel and Costello (1989) and the optimized $[8, 16, 32]$-state trellis codes (Table 1) are used in the proposed trellis code CIOD OFDM. Figure 5 shows that the proposed $[8, 16, 32]$-state trellis codes CIOD OFDM system provides additional 10.2, 4.8 and 4.2 dB gains in SNR at the CER of $10^{-3}$ compared to the two-symbol interleaved TC-STBC (Gong and Letaief, 2003).

Figure 6 shows the CER performances of $[8, 16, 32]$-state symbol interleaved TC-STBC, the trellis codes are denoted by C8, C16 and C32, by Schlegel and Costello (1989) and the optimized $[8, 16, 32]$-state trellis codes (Table 1) are used in the proposed trellis code CIOD OFDM. the proposed system provides additional 3.2 and 1.5 dB gains in SNR at the CER of $10^{-3}$ compared to the symbol interleaved TC-STBC with the $[8, 16, 32]$-state trellis codes, respectively. the CER performances curve of the 8-state trellis code CIOD extraordinary agree with that of the 32-state symbol interleaved TC-STBC.

CONCLUSIONS

A robust trellis code CIOD OFDM system offering high diversity gain is proposed to considerably boost up the CER performances compared to the benchmark systems. The proposed trellis code CIOD system combines the trellis code, the coordinate interleaver and the STBC to enhance the MIMO-OFDM transmission performance. The Viterbi decoding branch metrics of proposed trellis code CIOD is derived from PEP analysis. the optimum $[4, 8, 16, 32]$-state rate-2/3 8-PSK trellis codes for trellis code CIOD are found by exhaustive search, the simulation results shows that the performances of proposed trellis code CIOD considerably outperforms benchmark systems.

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