A New Approximation to Information Fields in Sensor Nets

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Abstract: A number of practical results event in a natural information potential field in the proximity of the phenomenon. A novel method is proposed in this study to approximate the information potentials fields in Wireless Sensor Networks (WSNs). The moving least-square method is introduced to solve related Partial Differential Equations (PDEs). This equation determines a smooth edition of the intrinsic potential field maintained major characters which is propitious to most applications of WSNs. For instance, navigation in parking yards, temperature, motion or pollutants, etc. The association between the nodes and the local information level are also considered in this method. Various information potentials fields can be integrated together naturally attribute to the well property of the PDEs. Efficiency and stability of our scheme are verified by numerical examples. The simulation results make clear that the presented way is advantageous to be used in WSNs.

Key words: Wireless sensor networks, moving least square, potentials fields

INTRODUCTION

In Wireless Sensor Networks (WSNs), one of the main design challenges is to save severely constrained energy resources and obtain long system lifetime. It involves lots of diverse applications. Such as, security (Gao et al., 2010; Xiao et al., 2009), routing and QoS topics, etc. Whatever any kind of topics, low cost merit of sensors enables us to randomly deploy a large number of sensor nodes. Thus, a potential approach to solve lifetime problem arises. That is to let sensors work alternatively by identifying redundant nodes in high-density networks and assigning them an off-duty operation mode that has lower energy consumption than the normal on-duty mode. In a single wireless sensor network, sensors are performing two operations: sensing and communication. Therefore, there might exist two kinds of redundancy in the network. The rapid development of WSNs brings universal computation to many applicable scenarios (Akyildiz et al., 2002, 2007). Current progress in WSNs reveals the potential of such hand-held device systems for revolutionizing the way we concern, interact with, and affect the real world. Preliminary application scenarios on distributed data gathering systems have already distinguished the merits of cheap sensor networks beyond traditional centralized sensing systems. As technologies become mature and as sensorsnets expand large in size and become inter-connected, we expect that sensor networks will move beyond military deployments and the monitoring of animal or other natural habitats to the places where humans work and live: homes, cars, buildings, roads, cities, etc. Note that in these human spaces sensor networks serve users embedded in the same physical space as the network. Furthermore, there is often the requirement to deliver related information (Kalantari and Shayman, 2004) with very low latency, such as to allow customers to react in a timely manner, as for example with first responders in disaster recovery scenarios (most is introduced from to be revised to prevent similarity). Information guidance has been explored before as a scalable approach for settings with high query frequency.

Most of these gradient-based approaches (Chu et al., 2002; Faruque and Helmy, 2004; Faruque et al., 2005; Liu et al., 2005) use the natural gradients of physical phenomena, since the spatial distribution of many physical quantities, e.g., temperature measurements for heat, follows a natural diffusion law. However, gradients imposed by natural laws can be far from perfect guides, as witnessed by the existence of local extrema or large plateau regions, forcing information guidance to deteriorate to a random walk.

In this study, we explore the potential of using a network of embedded sensors to aid information
discovery and navigation through a dynamic environment. This includes the navigation of packets (answering user queries from any node), as well as the navigation of physical objects (people or vehicles) moving in the same space such as users with hand-held devices communicating with nearby sensor nodes to get real time navigation information. For example, road-side sensors can monitor local traffic congestion; empty parking lots in downtown areas can be detected and tracked by sensors deployed at each parking spot.

Those application scenarios can be different from conventional scientific monitoring applications by several characteristics. The information is updated dynamically such as parking spaces are freed up or occupied over time, road conditions are changing at different periods of the day. Thus the navigation system needs to accommodate these environmental changes. The information and its change can be associated with a potential field. How to obtain this potential field efficiently is crucial to related application in WSNs.

During the past decades, as an effective numerical method, Finite Element Method (FEM) is convenient to be applied in various fields, but there are still lots of difficulties in solving large deformation problems. The grid generation of complex structures is also difficult and time-consuming (Zhang et al., 2003a). In recent years, meshless methods (Belytschko and Kröner, 1996) have been developed dramatically and applied to various fields. This method is a node-based numerical method and it can effectively overcome those shortcomings. The current meshless methods are all based on the Galerkin method (such as the EFG (Zhou and Kou, 1998), FUM, Hp-Clouds, MLPG (Atluri and Zhu, 1998), etc.) and the collocation method (such as SPH (Zhang, 1996), FPM, PCM, etc.). Numerical integration is necessary for Meshless methods based on the Galerkin method and it will lead to large amount of computation. Additionally, the background grid should be introduced advance (Zhang and Liu, 2004). Meshless methods based on the collocation method require no integration and fewer computation, but hold low precision and poor stability. Based on the least-square method (Zhang et al., 2003a) proposed the meshless weighted least-squares method (MWLS) and overcome the drawbacks mentioned above. Moving Least-Square approximation (MLS) is one of the cores in this method.

In this study meshless method is introduced to approximate the information potentials fields in WSNs. First, Poisson equations is proposed to determine a smooth version of the original potential field which keep the main features. These features are useful to most applications. Then, the moving least-square method is applied to solve above PDEs. The relationship among the nodes and the local information level are also considered in our model. Contribute to the well property of those PDEs, more than one information potentials fields can be intergraded together naturally. The presented method is more efficient to obtain the proper potential field.

**POTENTIAL FIELDS AND POISSON EQUATION**

As the above mentioned, information potential fields are introduced to describe the information distributions of nodes and be remunerative to different applications. Most of those applications benefit from the potential fields contribute to the applied of some gradient algorithms. It is crucial for these algorithms to build up a proper field suitable. More exactly, a smooth field with most original features is expected. The smoothness will be beneficial to the gradient calculation and algorithm implemented, while the reservation of features is a necessary guarantee for purpose accomplished.

Many mathematical methods and techniques can be introduced to potential fields surfaces approximation such as polynomial fitting, non-linear fitting, polynomial interpolation, least-squares fitting, Radial Basis Function (RBF) interpolation, and so on. They have been applied in various fields. With the development of those methods, the error of the surface becomes smaller gradually and the smoothness is increased steadily.

But, these methods can not be applied in our scenario directly. As show in following Fig. 1, the x,y coordinate of each figure separately stands for the related space position. In Fig. 1a, the u stands for the information quantity. In Fig. 1c, the |Vu| stands for the specific modulus of gradients. In the universal surface of traditional methods hold good smoothness and high accuracy almost everywhere. But the gradient modules are reduced rapidly to zero nearly the maxima. Figure 1b and c show the gradients and modules field. The contours are displayed in the left Fig. 1d.

If the information potential field is approximated like the above process, users will be guided by a low-level gradient near the local maximal points in later applications such as navigation in parking lots. It means the expected potential field should remain high-level gradient close to the targets.

With the above-mentioned purpose, the expected potential field u(x) is assumed as a harmonic in domain Ω and the Laplace equation \( \nabla^2 u(x) = 0 \) is introduced by Lin et al. (2008). It is acknowledged that the value of any internal point can be determined uniquely if the values of the information field function on the boundaries are given. Furthermore, based on the maximum principle, the certain extreme points must lie the boundary. Namely, with the Dirichlet condition and Neumann condition, the field can be obtained by solve the Laplace problem. Figure 2a-d shows several potential fields obtain by this method.
Fig. 1: Universal results of traditional methods, (a) Field, (b) Gradients, (c) Gradient modules and (d) Contours

Fig. 2: Results of Laplace problems in several Non-convex fields (a) One extrema, (b) Two extremas, (c) Three extremas and (d) Four extremas
However, the hypothesis is too strong to generate a proper field in some extents. So, the Poisson equation is introduced as following:

Compared with Gao's work (Lin et al., 2008), present method can determine a potential field with weaker constraint.

**MOVING LEAST-SQUARE APPROXIMATION TO POTENTIAL FIELDS**

There are many numerical methods can be applied to solve the Poisson problem such as Finite Difference Method (FDM), Finite Element Method (FEM) and so on. As an effective numerical method, FEM is convenient to be applied in solving PDEs. However, because of the irregular distribution of nodes, it is inconvenient to create grid for FEM. In recent years, meshless methods have been developed dramatically with better merits. They are node-based numerical method and require no grid. The proposed Meshless Weighted Least-Squares method (MWLS) (Zhang et al., 2003b) can overcome the difficulties of FEM. Moving Least-Square approximation (MLS) is one of the cores in this method.

Assume that u is known on nodes $x_i$ (i = 1, 2, ..., N) in domain $\Omega$ and the basis functions are denoted as $P(x) = [p(x), p(x), ..., p(x)]$.

Then the approximation of $u$ in $O(x, \delta)$ can be denoted as $u^*(x) = A(x)\Gamma P(x)$ (2).

Where, $A(x) = [a_1(x), a_2(x), ..., a_N(x)]$ is the coefficient to be determined. The weight function at every $x_i$ is defined as $\omega_i(x) = (x-x_i/d)$ and $d$ denotes the radius of support field. The total error functional of $u^*(x)$ is:

$$E = \sum_{i=1}^{N} \omega_i(x)[P(x_i)^T \Phi(x) - u(x_i)]^2$$

(3) in $O(x, \delta)$ can be denoted as Minimizing this functional, then we can obtain:

$$u^*(x) = P(x)B(x)^{-1}C(x)U$$

where

$$B(x) = [P(x)^T P(x)]^{-1/2} \sum_{i=1}^{N} \omega_i(x)$$

$$C(x) = [p(x), p(x), ..., p(x)]\Phi[\omega_1(x), \omega_2(x), ..., \omega_N(x)]$$

and

$$U = [u(x_1), u(x_2), ..., u(x_N)]^T$$

**NUMERICAL EXAMPLES AND SIMULATIONS**

Here, quadratic monomial basis functions $[1, x, y, x^2, xy, y^2]$ and Gaussian weight function

$$\omega (r) = \frac{\exp(-r^2)}{1 - \exp(-r^2)}$$

are adopted to solve several numerical examples. The results are compared with the FEM solutions and exact solutions. Then a wireless sensor network with random information distribution is generated, and our method is applied to obtain the expected potential field.

Let

$$\Omega = [-1,1] \times [-1,1], \tau_g = \partial \Omega, \Omega = \{x \mid \Omega, \partial \Omega, \partial g, a(x) = 1, b(x) = 0, c(x) = -1, f(x) = 0\}$$

$$f(x) = (-6(x^2 + y^2) - 4(x^2 + y^2)\exp(-x^2 + y^2)) - 1$$

Then the numerical solution and the error can be obtained as shown in Fig. 3a and b. The maximal relative error is less than 0.76%.

A distribution of some information nodes in domain $\Omega$ is shown in Fig. 4a. The information quantities on nodes (-1, 1), (1, -1), (0, 0) are set to be 2, 2, 5. Set $\Omega = (-1, 1), (1, -1), (0, 0), \Omega = \{x \mid \Omega, \partial g, a(x) = 1, b(x) = 0, c(x) = -1, f(x) = 0\}$. Then we can obtain the corresponding boundary conditions, a smooth potential field is determined as shown in Fig. 4b for advanced application.

Another information distribution is shown in Fig. 5a. The information quantities on four vertexes of region $[-1, 1] \times [-1, 1]$ are set to be 3, 1, 6, 2. Set $f(x) = -1$ and give proper boundary conditions, we can obtain the smooth potential field as shown in Fig. 5b.

Figure 6a shows a random information distribution of WSNs. Similar boundary conditions are set for the Poisson problem. The expected potential field is approximated by the proposed method. The result is shown in Fig. 6b. The smoothness and reservation of main feathers can be advantageous to various applications.
Fig. 3: Solution and the error of example 1

Fig. 4: Potential field determined by three nodes

Fig. 5: Potential field determined by five nodes

Fig. 6: Potential field determined by a random distribution
CONCLUSIONS AND FUTURE WORK

In this study, we explore the potential of using a network of embedded sensors to aid information discovery and navigation through a dynamic environment. A novel method is proposed in this study to approximate the information potentials fields in WSNs. The moving least-square method is introduced to solve relative PDEs which determines a smooth version of the original potential field preserved major features. It is advantageous to most applications of WSNs such as navigation in parking lots, environment monitoring, etc. Efficiency and stability of the proposed method are verified by several numerical as described earlier. The simulation results show that our method is convenient to be applied in WSNs. Such as users with hand-held devices communicating with nearby sensor nodes to get real time navigation information; road-side sensors can monitor local traffic congestion; empty parking lots in downtown areas can be detected and tracked by sensors deployed at each parking spot.

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