A Complete Ranking Model for MCDA Based on Multi-Graded Dominance Relations

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Abstract: Based on multi-grade dominance relations, this study develops a new multi-criterion decision model, which can avoid losing information when decision table is changed into PCT and obtain complete ranking of alternatives. The model introduces PROMETHEE II into dominance-based rough set theory and with different preference grades for different criterions, describes binary preference relations of two objects. Meanwhile, the model resolves the problem of some alternatives having the same net flow value, which is a common problem when rough set method is applied to MCDA. Finally, an example illustrates the model is scientific and validity.

Key words: Dominance-based rough sets, PROMETHEE II model, synergic model, multi-criterion decision-making

INTRODUCTION

PCT (Pairwise Comparison Table) introduced by Greco has made rough set method be able to deal with choice and ranking problems, which extends the application range of rough set method (Greco et al., 2001, 2002a, b; Zaras, 2001). When studying the multi-criterion ranking problems with rough sets method from the point of view of ordinal relation, we need first describe the problem as PCT or change the traditional decision table into PCT and then derive rules. So, it becomes the focus paid more attention by the decision maker to change decision table possessing ordinal relation into PCT. But information loss in the changing process may bring new inconsistencies and finally influences on the validity of order rules; unfortunately the excessively simple changing method inevitably produces a certain information loss (Jiang et al., 2007; Wang et al., 2006a; Hua and Liu, 2004). Another question is that, when with the derived rules we rank some new instances, different alternatives have the same net flow values, which makes decision maker not be able to get the complete ranking. So, it is particularly important to change the traditional decision table into PCT meanwhile to avoid information loss and finally get the complete ranking.

PROMETHEE II is a valid one of multi-criterion decision analysis methods. It has such excellence as agility, adaptability to environment and stabilization of ranking result. And its ranking is based on pair comparison of alternatives and graded dominance relations so that the method need not standardize criterion values. Just for these good qualities, the method now is widely used in many fields such as selecting supplier, urban industrialization evaluation and optimal configuration of naval gun weapon systems (Sun and Qiu, 2007; Abbadi, 2004; Dulmin and Mininno, 2003; Wang and Yang, 2006; Wang et al., 2006a). Aiming at the localization that PROMETHEE II method do not deal with uncertain information and weight coefficients unknown, many scholars either properly change it or hybridize it with other methods such as AHP, fuzzy mathematic method, non-linear programming etc., to get an more effective decision analysis method which is more coincident with real environment (Goumas and Lygerou, 2000; Fernández-Castro and Jiménez, 2005; Macharis et al., 2004; Rekiel et al., 2002; Duvioler et al., 2007; Wang, 2005). Based on references, though PROMETHEE II is a valid analysis method for multi-criterion decision, we know still there are some certain problems. One is to get weights for criterions from exterior; the other is that the method cannot deal with the lingual criterion and uncertainty information.

Due to both PROMETHEE II and dominance-based theory based on preference dominance relation, so both they root the same theory base and have inherent contact. Therefore, this study hybrids the two methods and develops a new decision model that is more similar to natural reasoning process. The new model is designed to deal with ranking and choice in multi-criterion decision making problems.

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P2DRS DECISION MODEL

For short, the hybrid model developed from PROMETHEE II method and Dominance-based Rough Sets theory is shortened as P2DRS. The system flow figure of P2DRS is as Fig. 1. The process is that: first to change the decision table into PCT by criterion with multi-grade; then based on both the synthetic preference information of alternatives and the criterions' preference information, to assign weights of criterions with goal programming method and regarding the known ranking result of examples as the goal; finally, to derive decision rules from PCT and to rank the new objects with decision rules. Furthermore, the objects that have the same scores are compared with net flow value got with PROMETHEE II method. The new hybrid model can be used to deal with choice and ranking problems in multi-criterion decision fields, which inherits the virtue that PROMETHEE II method produces steady ranking result, meanwhile carry forward the characteristic that dominance-based rough sets method is good at dealing with uncertain problems.

Multi-grade dominance function: With respect to criterion α, the multi-grade dominance by pairwise comparison for alternatives are as follow:

When criterion is a numeric type, the preference grades produced by pairwise comparison for alternatives are as (Eq. 1):

\[
\begin{align*}
\forall x, y \in H \cup \{+\}, & \quad P^x (x, y) = \text{sgn}(x - y) \\
& \quad \text{subject to } \delta_k, \delta_l, \delta_n \\
& \quad \text{where, } \delta_k, \delta_l, \delta_n \text{ are grade thresholds, } k, l, n \in \{1, 2, ..., h, l, h\} \text{ is the set of dominance grades produced by pairwise comparison of two alternative with regard to criterion } x, x + y & \leq \delta_k \\
& \quad \text{or } x - y & \leq \delta_l \\
& \quad \text{or } |x - y| & \leq \delta_n \\
\end{align*}
\]

where, \(\delta_k\) is grade threshold, \(k \in H\), and \(H = \{1, 2, ..., h-1, h\}\) is the set of dominance grades produced by pairwise comparison of two alternative with regard to criterion \(x\), namely, it is actual range of dominance function \(P^x (x, y)\) and when \(P^x (x, y) = k\), it can be simply noted as \(P^x (x, y) = k\) or \(P^x (x, y) > 0\); when \(P^x (x, y) < 0\), it shows that with respect to criterion \(x\) alternative \(x\) is at least as good as alternative \(y\) and the degree is \(k\); when \(P^x (x, y) > 0\), it shows that with respect to criterion \(x\) alternative \(x\) is worse than alternative \(y\) and the degree is \(k\).

When criterion is a linguistic type, it can be considered as two instances.

The first is that: assume that \(x\) a linguistic criterion and its evaluation set of natural language is

\[\{v_1, v_2, ..., v_4\}\]

where, \(v_1\) satisfies order, namely if \(i \geq j\), then \(n_i \geq n_j\), meanwhile the neighbour grades are equal at interval. Now change the set of qualitative descriptive language into ordinal scales set \(V_N = \{1, 2, ..., 5\}\).

Example 1: An evaluation set of natural language is \(V = \{v_1\; Worst, v_2\; Medium, v_3\; Good, v_4\; Better\}\) and can be changed into the ordinal scales set as \(V = \{1, 2, 3, 4, 5\}\), where the size of element shows the grades of criterions and 1 shows that the value of criterion gets from the worst grade, 3 shows that the value of criterion gets from the third grade. In fact the ordinal scales set shows the same meaning as the evaluation set of natural language, which is just changed into numerical expression. Namely, ordinal grades of criterations are expressed by discrete numbers between which have equal interval. Then criterion with multi-grade still can be use.

The second instance for linguistic criterion is that: the decision makers have given preference information of pairwise comparison of alternatives with regard to that criterion, namely linguistic judgement matrix with regard to criterion is known. Assume that the evaluation set of natural language phrase is \(L = \{l_i | i \in \{1, 2, ..., T\}\}\), where, \(T\) is odd and satisfies:

- Ordinal, when \(i \geq j\), there is \(l_i \geq l_j\);
- Converse operation in existence, \(\neg(l_i = l_j) = T + 1 - i\);
- Maximizing operation, \(\max(l_i) = l_i\), where, \(l_i \geq l_j\);
- Minimizing operation, \(\min(l_i) = l_i\), where, \(l_i \geq l_j\);

Then change the set into quantitative set:

\[L = \{l_i = \text{Worse, } l_i = \text{Bad, } l_i = \text{Equal, } l_i = \text{Good, } l_i = \text{Better}\}\]
Example 2: When \( T = 5 \), there is an evaluation set of natural linguistic phrase \( L = \{ l_1 = \text{Worse}, l_2 = \text{Bad}, l_3 = \text{Equal}, l_4 = \text{Good}, l_5 = \text{Better} \} \) satisfies the above conditions, change it into the preference grade set showed as quantity set \( H_5 = \{-2, -1, 0, 1, 2\} \). If alternative \( x \) compares with alternative \( y \) very bad, then there is \( P_a^{+1}(x, y) \) or \( P_a^{-1}(x, y) \). If the extended preference function can be denoted as \( P(x, y) \) and simply denoted as \( P_a \). Where the minus value of preference function shows that with respect to criterion a alternative \( x \) is worse than \( y \).

Meanwhile, based on the extended preference function, the calculating formula for net flow value is (Eq. 4).

\[
\alpha(x) = \sum_{j=1}^{n} w_j \sum_{l=1}^{m} P_l (x, x_l)
\]

And obviously both net flow values from the two preference functions are completely equal. The reason is (Eq. 5).

\[
\phi(x) = \theta^+(x) - \theta^-(x) = \sum_{l=1}^{n} \sum_{i=1}^{m} w_i P_l (x, x_i)
\]

After extending the range of preference function, it can be determined from the multi-grade dominance criterion in section I as:

\[
P_a(x, y) = \frac{1}{\delta_a} P_a^+(x, y)
\]

It can be seen that the preference function of each criterion in PROMETHEE II method can be obtained, when the traditional decision table is changed into PCT. Namely, for the same decision problem, the PCT information in dominance table is changed into PCT. 

Calculate weight of criterion: In the decision cases, the pairwise comparison information of alternatives with regard to each criterion is known and the comprehensive comparison information of alternative is also known. So a method can be developed for mining the relatively important information of each criterion in decision cases.

Assume that in PCT set \( A \) of condition criteria has card(A) and set \( E(=U) \) of alternatives has card(E). Then the comprehensive preference information of alternative set \( E \) has been known and the ranking of alternatives is considered as \( x_1 \succ x_2 \succ \cdots \succ x_n \). In PROMETHEE method, if \( x_i \succ x_j \), then net flow values hold \( \phi(x_i) > \phi(x_j) \).
\[
\phi(x) = \sum_{j=1}^{m} \sum_{i=1}^{n} w_{ij} P_{ij} (x, x_j)
\]

and the weights of criterions, \(w_{ij}, j = 1, 2, ..., m\), are unknown.

Now obtain weights of criterions with the following goal programming model (Eq. 6).

\[
\begin{align*}
\text{Min} Z &= P_1d_1^+ + P_2d_2^+ + \cdots + P_nd_n^+ \\
\sum_{j=1}^{m} w_j &= 1 \\
\phi(x_j^+) - \phi(x_j^-) + d_j^+ - d_j^- &= 0 \\
s.t.\ 
\phi(x_j^+) - \phi(x_j^-) + d_j^+ - d_j^- &= 0 \\
\vdots \\
\phi(x_m^+) - \phi(x_m^-) + d_m^+ - d_m^- &= 0 \\
w_j > 0, j = 1, 2, ..., n; d_j^+, d_j^- \geq 0, j = 1, 2, ..., m - 1
\end{align*}
\]

(6)

Weights of criterions are satisfactory solution obtained by solving the above goal-programming model. And weights obtained from PCT can reflect the comprehensive preference information of alternatives in PCT.

To derive decision rules: Assume that the comprehensive preference information of a part set of alternatives denoted as \(E(E \subseteq B)\) are known, then pairwise comparison table is a decision table that has complete information, namely \(D_{pct} = \{E \times E, \cup \{d\}, T \cup \{d_{g}\}\} = \{E \times E, \cup \{d\}, T \cup \{d_{g}\}\}\), where \(E\) is the exemplary objects set that their comprehensive preference relation or multicriteria evaluations are unknown and \(E \times E\) is pair comparison set.

For \(\forall (x,y), (w,z) \in E \times E\), with respect to criterion \(a, A\), \((x,y)D_{w}(w,z)\) mean that \((x,y)\) dominates \((w,z)\), or \(x\) is preferred to \(y\) at least as strongly as \(w\) is preferred to \(z\). Then the following holds: for \(\forall (x,y), (w,z) \in E \times E\), \(a, A, h_a, k_a, c_a \in H_a\), \(xP_a y\) and \((x,y)D_{w}(w,z)\) mean \(h_a \geq k_a\).

Using the dominance relation \(D_{w}\), let us introduce the positive dominance \(D_{w}^+(x,y)\) and the negative dominance \(D_{w}^-(x,y)\).

\[
D_{w}^+(x,y) = \{ (w,z) \in E \times E | (w,z)D_{w}(x,y) \}
\]

\[
D_{w}^-(x,y) = \{ (w,z) \in E \times E | (x,y)D_{w}(w,z) \}
\]

Considering the binary relation \(S\) and \(S^c\) confined to the decision criterion \(\{d\}\), we can respectively define lower and upper approximations of \(S\) with respect to condition criterion set \(C\) as follow:

\[
C(S) = \{(x,y) \in E \times E | D_{w}^+(x,y) \subseteq S \}
\]

\[
\bar{C}(S) = \{(x,y) \in E \times E | D_{w}^-(x,y) \cap S \neq \emptyset \} = \bigcup_{(x,y) \in S} D_{w}^-(x,y)
\]

Analogously we can defined the approximations of \(S^c\):

\[
C(S^c) = \{(x,y) \in E \times E | D_{w}^-(x,y) \subseteq S^c \}
\]

\[
\bar{C}(S^c) = \{(x,y) \in E \times E | D_{w}^-(x,y) \cap S^c \neq \emptyset \} = \bigcup_{(x,y) \in S^c} D_{w}^-(x,y)
\]

Exact definitions of the concept of upward cumulated preferences and downward cumulated preferences are the following: \(\forall (x,y) \in E \times E\), \(a, A, h, k \in H_a\), where, \(k \geq h\), if \(xP_{h} y\), then \(xP_{k} y\); where \(k \geq h\), if \(xP_{h} y\), then \(xP_{k} y\).

Using the above concepts, from the given PCT, three types of decision rules can be obtained:

- \(D_2\) decision rules: If \(xP_{h} y\) and \(xP_{k} y\) then \(xP_{h} y\) and these rules are supported by pairs of objects from \(C(S)\).
- \(D_3\) decision rules: If \(xP_{h} y\) and \(xP_{k} y\) then \(xP_{h} y\) and these rules are supported by pairs of objects from \(C(S^c)\).
- \(D_{st}\) decision rules: If \(xP_{h} y\) and \(xP_{k} y\) and \(xP_{h} y\) and \(xP_{k} y\) then \(xP_{h} y\) and \(xP_{k} y\) and these rules are supported by pairs of objects from \(b_{h}(S)\) (Greco et al., 2001).

To handle ranking problem with decision rules and PROMETHEE II method: After the application of the decision rules obtained on \(E\) to the whole object set \(U\) finally for each object \(x \in U\), to calculate Net Flow Score \(S(x)\), for handling ranking and choice, where:

\(S(x) = S^+(x) - S^-(x)\)

and

\[
S^+(x) = \text{Card}\left\{ y \in U \mid \text{There is at least one decision rule which affirms } xP_{h} y \right\};
\]

\[
S^+(x) = \text{Card}\left\{ y \in U \mid \text{There is at least one decision rule which affirms } ySx \right\};
\]

\[
S^+(x) = \text{Card}\left\{ y \in U \mid \text{There is at least one decision rule which affirms } yS^+ x \right\};
\]

\[
S^+(x) = \text{Card}\left\{ y \in U \mid \text{There is at least one decision rule which affirms } xS^+ y \right\};
\]

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The alternative \( x^* (x^* \in U) \), which satisfies \( S(x^*) \), is optimal (Greco et al., 2001).

In the ranking problems, when \( S(x) \), namely both object \( x \) and object \( y \) have the same Net Flow Score, we cannot distinguish the two objects with their Score. So we cannot completely rank all alternatives. Furthermore, we can calculate net flow \( \phi(x) \) by PROMETHEE method and then compare \( \phi(x) \) with \( \phi(y) \), if \( \phi(x) > \phi(y) \), then \( x \succ y \).

THE PROCESS OF P2DRS

The process of P2DRS decision model is as following:

- **Step 1**: According to criterion with multi-grade, change decision table of exemplary objects into Pairwise Comparison Table (PCT) and meanwhile get the preference function in PROMETHEE II method.
- **Step 2**: With goal programming method, calculate weights for criterions, based on the known ranking information in the decision table and preference information obtained by preference functions with respect to condition criterions.
- **Step 3**: Derive decision rules from PCT.
- **Step 4**: Use the decision rules to deal with alternative ranking and choice. To the alternatives that still cannot be distinguished, we can judge them with PROMETHEE II to get the completely ranking for all objects.

AN APPLICATION OF P2DRS DECISION MODEL IN SELECTING SUPPLIERS

A selecting suppliers problem: there are 6 successful purchasing cases and now need rank 5 new suppliers (Table 1). Condition criterion set is:

\[
A = \{ \text{Price, Delivery Date, Ability of R and D} \}
\]

and decision criterion set is \( \{ d \} = \{ \text{Ranking Number} \} \). First change range \{ good, medium, poor \} of linguistic criterion Ability of R and D into \{ 3, 2, 1 \}; then with the model in this study, change the decision table into PCT.

Confirm the criterion with multi-grade and preference functions for 3 condition criterions as following:

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Price ($)</th>
<th>Delivery Date (Month: Days)</th>
<th>Ability of R and D</th>
<th>Ranking No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>180</td>
<td>4</td>
<td>Good</td>
<td>1</td>
</tr>
<tr>
<td>S2</td>
<td>160</td>
<td>5</td>
<td>Medium</td>
<td>4</td>
</tr>
<tr>
<td>S3</td>
<td>165</td>
<td>6</td>
<td>Good</td>
<td>2</td>
</tr>
<tr>
<td>S4</td>
<td>145</td>
<td>7</td>
<td>Medium</td>
<td>3</td>
</tr>
<tr>
<td>S5</td>
<td>145</td>
<td>7</td>
<td>Medium</td>
<td>5</td>
</tr>
<tr>
<td>S6</td>
<td>135</td>
<td>8</td>
<td>Poor</td>
<td>6</td>
</tr>
</tbody>
</table>

To get more abundant conclusion, just with PROMETHEE II method, we evaluation all new objects and can get their net flow values, respectively as \( \phi(S7) = 0.22, \phi(S8) = -0.1367, \phi(S9) = 0.4733, \phi(S10) = 0.4967 \) and \( \phi(S11) = 1.0533 \). The result is consistent with the ranking result obtained with decision rules derived with dominance-based rough set method. To some degree, it is feasible to hybrid the two methods of PROMETHEE II and dominance-based rough set both based on binary dominance relation.

The decision table of suppliers

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Price ($)</th>
<th>Delivery Date (Month: Days)</th>
<th>Ability of R and D</th>
<th>Ranking No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S7</td>
<td>165</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>S8</td>
<td>150</td>
<td>5</td>
<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>S9</td>
<td>145</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>S10</td>
<td>135</td>
<td>5</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>S11</td>
<td>135</td>
<td>7</td>
<td>1</td>
<td>-8</td>
</tr>
</tbody>
</table>

**Table 2**: The decision table of new suppliers

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Price ($)</th>
<th>Delivery Date (Month: Days)</th>
<th>Ability of R and D</th>
<th>Ranking No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S7</td>
<td>165</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>S8</td>
<td>150</td>
<td>5</td>
<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>S9</td>
<td>145</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>S10</td>
<td>135</td>
<td>5</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>S11</td>
<td>135</td>
<td>7</td>
<td>1</td>
<td>-8</td>
</tr>
</tbody>
</table>
CONCLUSION

By hybridizing the two methods of PROMETHEE II and dominance-based both based on binary dominance relation, this study builds a new P2DRS model to deal with ranking problem in multi-criterion decision field. On one side, due to the excellence of PROMETHEE II having steady ranking result, this model can easily deals with the problems of translating PCT and equal scores of different objects. On the other side, the model can not only derive decision rules from PCT, but also obtain weights of criterions with goal programming method based on PCT. In short, the model built in this study, which not only inherits the virtue of PROMETHEE II having steady ranking result, but also develops the characteristic of dominance-based rough set being good at dealing with uncertain problems, provides a new effective method for dealing with choice and ranking problem in multi-criterion decision field.

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