Membership-Dependent Stability Conditions for Takagi-Sugeno Fuzzy Systems

Yashun Zhang and Jianzhen Li
School of Automation, Nanjing University of Science and Technology,
Nanjing, 210094, People’s Republic of China

Abstract: This study deals with the stability analysis and stabilization of fuzzy systems from the viewpoint of membership functions. Firstly, some linear matrix inequality conditions for the stability analysis and stabilization of Takagi-Sugeno (T-S) fuzzy systems are derived, which are dependent on the upper bound and the lower bound of the membership functions. Secondly, the local stability problem of the T-S fuzzy systems is considered and some sufficient conditions for the local stability analysis and stabilization are provided. Finally, some numerical examples are given to illustrate the effectiveness of the proposed results.

Key words: Fuzzy control, stability analysis, stabilization, membership function

INTRODUCTION

Much attention has been paid to the study of fuzzy systems which can model many nonlinear systems (Feng et al., 2005; Linfeng et al., 2009; Ajlouni and Al-Hamouz, 2004). The Takagi-Sugeno (T-S) fuzzy system is an important fuzzy model. Hence, many research efforts have been devoted to stability and performance analysis of Takagi-Sugeno (T-S) fuzzy systems over the past few years in the literature by Takagi and Sugeno (1985), Sugeno (1999) and Wang et al. (1996) and the references therein. A great number of the significant results were derived via the Linear Matrix Inequality (LMI) approach. Some basic stability conditions were presented by Wang et al. (1996) and some related results were provided by Guerra and Vemerein (2004), Kim and Lee (2000), Liu and Zhang (2003) and Tanaka et al. (1996, 2003). Currently, much attention is focused on reducing the conservativeness of the LMI stability and performance conditions. An important way to reduce the conservativeness is using the fuzzy or piecewise Lyapunov functional technique (Feng et al., 2005; Mozelli et al., 2009; Rhee and Won, 2006; Tanaka et al., 2007; Yuan et al., 2008; Zhou and Li, 2005; Zhou et al., 2007). However, some conservativeness still exists and the stability of many stable fuzzy systems can not be proved by the existing results. Take the system:

\[ x(t) = h_s(z(t))x(t) - (I - h_s(z(t)))x(t) \]

for example. It can not be proved stable by the aforementioned results. But it is stable when \(0 \leq h_s(z(t)) < 0.5\).

The LMI conditions in the aforementioned work do not depend on the membership functions, which may be a source of conservativeness. Indeed, the stability of fuzzy systems may depend on the bound of the membership functions sometimes, such as the system mentioned in the last paragraph. On the other hand, if some knowledge on the bound of the membership functions in a region around the equilibrium is known, some relaxed conditions may be derived on the local stability of the fuzzy systems. Motivated by this, Sala and Arino (2006, 2007) considered the local stability problem and the stability problem of fuzzy systems from the viewpoint of the membership functions.

In this study, the problems studied by Sala and Arino (2006, 2007) are considered and new results are proposed. Membership-dependent conditions on the stability of the fuzzy systems are derived, which are dependent on both the upper bound and the lower bound of the membership functions. Sufficient conditions for the local stability of the fuzzy systems are also derived.

For real symmetric matrices \(X\) and \(Y\), the notation \(X \preceq Y\) means that the matrix \(Y - X\) is positive definite. \(I\) is the identity matrix with appropriate dimensions. The superscript \(T\) represents the transpose.

PRELIMINARIES

Consider the T-S fuzzy control system described by the following rules:

Plant rule i: If \(z_i(T)\) is \(F_{i1}\), ..., and \(z_i(t)\) is \(F_{ip}\), then:
\( x(t) = A_0 x(t) + B_0 u(t) \)

where, \( x(t) \in \mathbb{R}^n \) is the state vector; \( u(t) \in \mathbb{R}^n \) is the control input; \( z_1(t), ..., z_s(t) \) are the premise variables; \( F_1, ..., F_s \) are the fuzzy sets; \( A_k \) and \( B_k \) are the system matrices. The normalized membership functions are then given by:

\[
    h_i(z(t)) = \frac{\prod_{j=1}^{s} \mu_i[z_j(t)]}{\sum_{j=1}^{s} \prod_{j=1}^{s} \mu_i[z_j(t)]} , \quad i = 1, ..., s
\]

where, \( \mu_i[z_j(t)] \) is the grade of membership function of \( z_j(t) \) in \( F_j \). In what follows, the argument of \( h_i(z(t)) \) will be dropped for simplicity. A more compact presentation of the fuzzy model is given by:

\[
    \dot{x}(t) = A(h)x(t) + B(h)u(t) \quad (1)
\]

Where:

\[
    A(h) = \sum_{i=1}^{s} h_i A_i, \quad B(h) = \sum_{i=1}^{s} h_i B_i
\]

The widely used controllers for T-S fuzzy systems are so-called Parallel Distributed Compensators (PDC) defined by:

\[
    u(t) = \sum_{i=1}^{s} h_i K_i x(t) \quad (2)
\]

where, \( K_i \) is a constant matrix to be designed. Combine Eq. 1 and 2 the closed-loop system becomes:

\[
    \dot{x}(t) = \sum_{i=1}^{s} \sum_{j=1}^{s} h_i h_j G_{ij} x(t) \quad (3)
\]

where, \( G_{ij} = A_i + B_i K_j \).

**Stability of fuzzy systems:** Let us review some existing results on the stability of system. When \( u(t) = 0 \), Eq. 3 becomes:

\[
    \dot{x}(t) = A(h)x(t) \quad (4)
\]

The following sufficient conditions are proposed by Tanaka *et al.* (1996) based on the Lyapunov function \( V(x(t)) = x^T(t)P x(t) \).

**Lemma 1:** Tanaka *et al.* (1996) system in Eq. 4 is asymptotically stable if there exists \( P > 0 \) such that:

\[
    A_i^T P + P A_i < 0 , \quad i = 1, ..., s \quad (5)
\]

To reduce the conservativeness of the conditions in Lemma 1, a fuzzy Lyapunov function approach is provided.

**Lemma 2:** Tanaka *et al.* (2003) assume that:

\[
    |h_i| \leq q_i , \quad i = 1, ..., s-1
\]

System in Eq. 4 is asymptotically stable if there exist \( P_i > 0 \) such that:

\[
    P_i \geq P_{i-1} , \quad i = 1, ..., s-1,
\]

\[
    \sum_{i=1}^{s} q_i (P_i - P_{i-1}) + \frac{1}{2} \left[ A_i^T P_i + P_i A_i + A_i^T P_{i-1} + P_{i-1} A_i \right] < 0 , \quad 1 \leq i < j \leq s
\]

**Stability of PDC fuzzy control systems:** For the stability of system, much more sufficient conditions have been reported. Let's review two main results in those works.

**Lemma 3:** Tanaka *et al.* (1998) system in Eq. 3 is asymptotically stable if there exist \( X > 0 \) and \( M_i \) such that:

\[
    A_i X + X A_i^T - M_i^T B_i^T - B_i M_i - M_i^T B_i^T < 0 , \quad i = 1, ..., s,
\]

\[
    A_i X + X A_i^T + A_i X + A_i X - B_i M_i - M_i^T B_i^T - B_i M_i - M_i^T B_i^T < 0 , \quad 1 \leq i < j \leq s
\]

In such case, the controller gain matrix can be given by \( K_i = M_i X^{-1} \).

**Lemma 4:** Liu and Zhang (2003) system in Eq. 3 is asymptotically stable if there exist matrices \( X > 0 \) and \( Y_i \) such that:

\[
    A_i X + X A_i^T - M_i^T B_i^T - B_i M_i - Y_i < 0 , \quad i = 1, ..., s,
\]

\[
    A_i X + X A_i^T + A_i X + A_i X - B_i M_i - M_i^T B_i^T - B_i M_i - M_i^T B_i^T - B_i M_i - Y_i - Y_i < 0 , \quad 1 \leq i < j \leq s,
\]

\[
    Y_{i1} \quad ... \quad Y_{i1} \\
    Y_{i2} \quad ... \quad Y_{i2} \\
    \vdots \quad \vdots \quad \vdots \\
    Y_{is} \quad Y_{is} \quad ... \quad Y_{is}
\]

In this case, the controller gain matrix can be given by \( K_i = M_i X^{-1} \).

**Membership-dependent stability conditions:** When the above conditions are infeasible, we should seek some other conditions. Note that the membership functions do not appear in the aforementioned LMI conditions. This may be a source of conservativeness sometimes. In fact,
some conservativeness may be reduced if the knowledge on the membership functions for a particular T-S fuzzy system is introduced in the LMI framework. Motivated by this, some researches have been done to seek the stability condition for system in Eq. 3 and 4 with the assumption that the upper bound of $h_i$ and $h_i$ is known. The main results on this problem are restated in the following.

**Lemma 5**: Suppose that $0 \leq h_i \leq \beta_i$, system is asymptotically stable if there exist symmetric positive matrices $P_i$ and $N_i$ such that:

$$
A_i^TP_i + P_iA_i - N_i + \sum_{j=1}^{s} \beta_j N_j < 0, \quad i = 1, \ldots, s
$$

(6)

**Lemma 6**: Suppose that $0 \leq h_i h_i \leq \beta_i$, system in 3 is asymptotically stable if there exist matrices $X_i$, $X_i = X_i^T$, symmetric matrix $P_i$, such that:

$$
X_i > 0,
$$

$$
R_i \geq 0,
$$

$$
A_i x + X_i A_i + \sum_{j=1}^{s} \beta_j (X_i A_i + A_i^{T} X_i) + \sum_{j=1}^{s} \beta_j N_j < 0, \quad i = 1, \ldots, s
$$

$$
X_i + X_i^T + A_i X_i + X_i A_i^T - B_i M_i - B_i R_i + \sum_{j=1}^{s} \beta_j N_j < 0, \quad i = 1, \ldots, s
$$

$$
+2\Delta - X_i - X_i < 0, \quad i < j \leq s,
$$

$$
X = \begin{bmatrix}
X_1 & \cdots & X_s \\
\vdots & \ddots & \vdots \\
X_s & \cdots & X_s
\end{bmatrix} < 0
$$

(7)

Where:

$$
\Delta = \sum_{i=1}^{s} \sum_{j=1}^{s} \beta_j R_{ij}
$$

In this case, the controller gain matrix can be given by $K_i = M_i X_i^{-1}$.

**Lemma 5 and 6** are corollaries of Theorem 2 and 3 in Sala and Arino (2006, 2007), respectively. It is easy to see that if there exists $P_i$ satisfying, let $N_i = 0$, and $P_i$ satisfy if $\varepsilon$ is enough small. So, it is more relaxed than. By a similar process, one can see that the conditions in Lemma 6 are less conservative than those in Lemma 4. It is worthy pointing out that the results in Lemma 5 can be extended to a fuzzy Lyapunov functional approach to further reduce the conservativeness.

When $A_i^TP_i + P_iA_i$ is unfeasible for some $i$, system in Eq. 3 cannot be proved stable by Lemma 1 and 2. But it may be proved stable by Lemma 5. It is noted that the condition in Lemma 5 depend on the upper bound of $h_i$ only, but has no relationship with the lower bound of $h_i$.

In the next section, we will seek to introduce the lower bound of the membership functions into the LMI stability conditions of the fuzzy systems.

**RESULTS**

Here, we presents some sufficient conditions on the local stability and stabilization of system in Eq. 3 and 4 when both the lower bound and the upper bound of $h$ are known.

**Stability of fuzzy systems**: The following theorem gives sufficient stability conditions of system in Eq. 4.

**Theorem 1**: Assume that $0 \leq h_i \leq \beta_i$, system in Eq. 4 is asymptotically stable if there exist matrices $P_i > 0$ and $i = 1, \ldots, s$ such that:

$$
A_i^TP_i + P_iA_i - Y_i < 0, \quad i = 1, \ldots, s
$$

(8)

$$
\sum_{i=1}^{s} \beta_i Y_i + \sum_{i=1}^{s} \beta_i (A_i^TP_i + P_iA_i - Y_i) < 0
$$

(9)

**Proof**: Choose a Lyapunov function as:

$$
V(x(t)) = x^T(t)P(t)x(t)
$$

Note that:

$$
\dot{V}(x(t)) = 2x^T(t)P(t)x(t)
$$

$$
\dot{V}(x(t)) = x^T(t)\left(A_i^TP_i + P_iA_i - Y_i + \sum_{i=1}^{s} \beta_i (A_i^TP_i + P_iA_i - Y_i)\right)x(t)
$$

$$
\leq x^T(t)\left[\sum_{i=1}^{s} \beta_i Y_i + \sum_{i=1}^{s} \beta_i (A_i^TP_i + P_iA_i - Y_i)\right]x(t)
$$

where, $\dot{V}(x(t))$ denotes the time derivative of $V(x(t))$ along the trajectory of Eq. 4. It follows from Eq. 7 and 8 that $\dot{V}(x(t)) < 0$ for any $x(t) > 0$. So, system in Eq. 4 is asymptotically stable if and are satisfied. This completes the proof.

Theorem 1 provides a new condition to check the stability of system. When $\alpha > 0$, Eq. 7 and 8 may guarantee larger stability region than sometimes. This will be illustrated in the following example.

**Example 1**: Consider the fuzzy system with the following rules:

- $R_1$: If $x_i(t)$ is $M_i$, then $\dot{x}_i(t) = A_i x(t)$
- $R_2$: If $x_i(t)$ is $M_i$, then $\dot{x}_i(t) = A_i x(t)$

Where:
\[ A_1 = \begin{bmatrix} -5 & -4 \\ -1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & -4 \\ 20 & a \end{bmatrix} \]

Assume that \( 0 < \alpha \leq \beta \leq 1 \), then \( 1 - \alpha \leq h_i \leq 1 \). Let \( \alpha = 4 \). Using the LMI tool box, one has that the LMI conditions in both Lemma 1 and 2 are infeasible. When \( \alpha = 0.4 \), solving, one has:

\[
P = \begin{bmatrix} 1.9382 & 0.4497 \\ 0.4497 & 0.8972 \end{bmatrix}
\]

\[
N_1 = \begin{bmatrix} 1.5267 & -0.8059 \\ -0.8059 & 0.7936 \end{bmatrix}
\]

\[
N_2 = \begin{bmatrix} 33.2259 & 19.1956 \\ 19.1956 & 11.1163 \end{bmatrix}
\]

Solving Eq. 7 and 8, one has:

\[
P = \begin{bmatrix} 0.2747 & 0.0795 \\ 0.0795 & 0.1705 \end{bmatrix}
\]

\[
N_1 = \begin{bmatrix} 1.5508 & -0.3428 \\ -0.3428 & 1.8394 \end{bmatrix}
\]

\[
N_2 = \begin{bmatrix} 3.3249 & 2.2121 \\ 2.2121 & 1.5189 \end{bmatrix}
\]

When \( \alpha = 0.3 \), by solving Eq. 7 and 8, the following results can be obtained:

\[
P = \begin{bmatrix} 0.2729 & 0.0761 \\ 0.0761 & 0.1527 \end{bmatrix}
\]

\[
N_1 = \begin{bmatrix} 1.4312 & -0.3177 \\ -0.3177 & 1.7192 \end{bmatrix}
\]

\[
N_2 = \begin{bmatrix} 2.9768 & 1.9183 \\ 1.9183 & 1.2579 \end{bmatrix}
\]

Next we consider the stability of system. The following theorem provides sufficient conditions for the stability of system when the bound of \( h \) is known.

**Theorem 2**: Suppose that \( 0 \leq \alpha \leq \beta \leq 1 \). System (3) is asymptotically stable if there exist matrices \( X \succ 0 \), \( Y_i \succ 0 \) and \( M_i \) such that:

\[
A_i X + X A_i^T - M_i B_i^T - B_i M_i - Y_i < 0
\]

\[ i = 1, \ldots, s \]

\[ j = 1, \ldots, s \]

\[
\sum_{i=1}^{s} \alpha_i (A_i X + X A_i^T - M_i B_i^T - B_i M_i - Y_i) + \sum_{i=1}^{s} \beta_i Y_i < 0
\]

\[ j = 1, \ldots, s \]

![Proof: Define \( P = X^{-1} \) and \( \bar{V}_i = PV_i P \). Pre-multiply and post-multiply \( P \) to Eq. 9 and 10, respectively, one has:

\[
G_i P + PG_i - Y_i < 0, \ i = 1, \ldots, s
\]

\[
\sum_{i=1}^{s} \alpha_i (G_i P + PG_i - Y_i) + \sum_{i=1}^{s} \beta_i Y_i < 0
\]

Choose the Lyapunov function as \( V(x(t)) = x^T(t)Px(t) \). From, one has that:

\[
V(x(t)) = x^T(t)Px(t)
\]

\[
= x^T(t) \left( \sum_{i=1}^{s} \sum_{j=1}^{s} h_{ij} (G_i P + PG_i - Y_i) \right) x(t)
\]

\[
= x^T(t) \left( \sum_{i=1}^{s} \sum_{j=1}^{s} h_{ij} (G_i P + PG_i - Y_i) \right) x(t)
\]

\[
\leq x^T(t) \left( \sum_{i=1}^{s} \sum_{j=1}^{s} \alpha_{ij} (G_i P + PG_i - Y_i) \right) + \sum_{i=1}^{s} \beta_i Y_i x(t)
\]

\[ \leq x^T(t) \left( \sum_{i=1}^{s} \sum_{j=1}^{s} \alpha_{ij} (G_i P + PG_i - Y_i) \right) + \sum_{i=1}^{s} \beta_i Y_i x(t) \]

It follows from Eq. 12 and 13 that for any \( \dot{V}(x(t)) < 0 \). Therefore, system (3) is asymptotically stable if Eq. 9-10 are satisfied. This completes the proof.

**Example 2**: To show the effectiveness of Theorem 2, we consider the fuzzy control system with the following rules:

- **R:** If \( x_i(t) \) is \( M_i \) then,

\[
\dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, 2, 3
\]

where:

\[
A_i = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

\[
A_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix}
\]

\[
A_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 6 + b \\ -1 \end{bmatrix}
\]

Assume that \( \alpha_i \leq h_i \leq \beta_i \), \( \alpha_i \leq h_i \leq \beta_i \), \( \alpha_i \leq h_i \leq \beta_i \) then:
\[ \alpha_1, \alpha_2 \leq h, h_i \leq \frac{1}{4} (1 - \alpha_i)^2, \quad \alpha_1, \alpha_2 \leq h, h_i \leq \frac{1}{4} (1 - \alpha_i)^2 \]

and

\[ \alpha_1, \alpha_2 \leq h, h_i \leq \frac{1}{4} (1 - \alpha_i)^2 \]

When \( \alpha_1 = \alpha_2 = \alpha_3 = 0.1 \text{ and } \beta_1 = \beta_2 = \beta_3 = 0.8 \), it can be shown that Theorem 2 can guarantee the stability of the closed-loop system and the controller can be designed.

**Local stability conditions:** Unfortunately, the lower bound of \( h \) is usually equivalent to 0. In such a case, the LMIs in Theorem 1 and 2 are not feasible. However, \( \alpha_1 \) may be bigger than 0 in a zone around the equilibrium of system in Eq 3 and 4. In such a case, we have the following corollaries on the local stability of system and system.

**Corollary 1:** Assume that \( h_i \) satisfies \( 0 < \alpha_i \leq h_i \leq \beta_i \leq 1 \) in a region \( \Omega \) that contains the equilibrium \( x = 0 \). System in Eq. 4 is locally asymptotically stable at the equilibrium \( x = 0 \) if there exist matrices \( P \geq 0 \) and \( Y_i \geq 0, i = 1, \ldots, s \) satisfying Eq. 7 and 8. In such a case, the set of the initial state \( \Omega^* = \{x \in \mathbb{R}^n \mid x^TP x \leq V_0 \} \) is a domain of attraction, where,

\[ V_0 = \min \{x^TP x \mid x \in \partial \Omega \} \]

with \( \partial \Omega \) denoting the boundary of \( \Omega \).

**Proof:** It is easy to see that if \( x(0) \in \Omega^* \), then \( x(t) \in \Omega \). From the proof of Theorem 1 one has that:

\[ V(x) \leq x^T \left( \sum_{i=1}^{s} \beta_i Y_i + \sum_{i=1}^{s} \alpha_i (A_i^TP + PA_i - Y_i) \right) x \quad (14) \]

for any \( x \in \Omega^* \). Suppose that Eq. 7 and 8 hold, one has that \( V(x) < 0 \) for any \( x \in \Omega \) and \( V(x) = 0 \) only when \( x = 0 \). Therefore, \( \Omega^* \) is an invariant set. LaSalle's theorem ensures that every solution of system in Eq. 4 starting in \( \Omega^* \) will approach \( x = 0 \). And the proof is completed.

**Corollary 2:** Assume that \( h_i \) satisfies \( 0 < \alpha_i \leq h_i \leq \beta_i \leq 1 \) in a region \( \Omega \) that contains \( x = 0 \). System in Eq. 3 locally asymptotically stable at the equilibrium \( x = 0 \) if there exist matrices \( X \geq 0, Y_i \geq 0 \) and \( M_i, i, j = 1, \ldots, s \) satisfying Eq. 9 and 10. In such a case, the controller gain matrix can be given by \( K_i = M_iX^{-1} \) and \( \Omega^* = \{x \mid x^TP x \leq V_0 \} \) is a domain of attraction, where, \( V_0 = \min \{x^TP x \mid x \in \partial \Omega \} \) with \( \partial \Omega \) denoting the boundary of \( \Omega \).

**Fig. 1:** Basin of attraction in example 3

**Proof:** Corollary 2 can be derived easily based on Theorem 2, which is similar to the derivation of Corollary 1.

To illustrate the effectiveness of Corollary 1, we give the following example. The application of Corollary 2 is similar to that of Corollary 1. Hence, it is omitted here.

**Example 3:** Consider the system investigated in example 1. Suppose that:

\[ h_i = \frac{1 + \sin x_i(t)}{2}, \quad h_i = \frac{1 - \sin x_i(t)}{2} \]

then \( h_i \) hand \( h_i \) satisfy \( 0.01 \leq h_i \leq 0.09 \) and \( 0.01 \leq h_i \leq 0.09 \)

when \( \arcsin 0.98 \leq x_i \leq \arcsin 0.98 \). Solving Eq. 7 and 8 with \( \alpha_i = \alpha_2 = 0.01 \) and \( \beta_i = \beta_2 = 0.99 \), one has that:

\[
\begin{bmatrix}
P & -5.9513 \\
-5.9513 & 24.0501
\end{bmatrix}
\]

\[
N_i = \begin{bmatrix} 54.2115 & -1.8999 \\ -1.8999 & 52.9116 \end{bmatrix}
\]

\[
N_i = \begin{bmatrix} 59.7889 & 59.5099 \\ 59.5099 & 70.5902 \end{bmatrix}
\]

So, the system is locally stable at the equilibrium \( x = 0 \).

Let,

\[ V_M = \min \{x^TP x \mid x_i = \pm \arcsin 0.98 \} \]

According to Corollary 1, one has that the basin of attraction of \( x = 0 \) is \( \Omega^* = \{x \mid x^TP x \leq V_M \} \), which is shown in Fig. 1.

**CONCLUSION**

Some membership-dependent LMI conditions have been derived on the stability analysis and stabilization for
continuous T-S fuzzy systems. It has been shown that if
the lower bound and the upper bound of the membership
functions are known, some new stability conditions can
be obtained. Furthermore, when the bound of the
membership functions in a region of \( x = 0 \) is known,
relaxed conditions can be obtained on the local stability
of the fuzzy systems. In such a case, a basin of
attraction of \( x = 0 \) is also provided. The given numerical
examples have shown the effectiveness of the proposed
approaches.

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