An Improved Particle Localization Algorithm for Mobile Robot in Indoor Environment

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Abstract: A novel extended particle algorithm is proposed aiming at solving the unmodeled motion problem, which is inextricable merely using conventional MCL. KLD sampling is utilized to measure the approximation error by Kullback-Leibler distance between the true distribution and its sampled representation by generating a dynamically sized set of samples according to the exact posterior being estimated. At the same time, the over-convergence and uniformity validations are introduced to verify correspondence between sample distribution and sensor information for timely re-sampling which highly saves computational resource and enhances localization efficiency. A further experiment, obtained with a real robot in an indoor environment, illustrates the favorable performance of this approach.

Key words: Mobile robot, localization, Monte Carlo algorithm, KLD sampling, over-convergence validation, uniformity validation

INTRODUCTION

The localization process is an essential component of any complex task and autonomous capabilities of mobile robots. It endows the robot with the ability to determine its correct pose as it navigates in the environment. This process can be divided into local localization and global one. Local localization aims to track the robot pose \((x, y, \theta)\) from its known initial pose correcting the small errors that might arise in systematic or non-systematic way. If the initial position is unknown or the robot is kidnapped to somewhere, the problem is one of global localization, i.e., the mobile robot has to estimate its global position through a sequence of sensing actions. It is particularly important during the exploration and building of environment maps (Montemerlo et al., 2002; Begum et al., 2005; Stachniss et al., 2005; Leung et al., 2006).

The classic optimal solution for linear system models under Gaussian noises is the well known Kalman Filter (Grewal and Andrews, 2001; Ristic et al., 2004). However, most real-world problems involve elements of non-Gaussianity and nonlinearity. Consequently, it is usually impossible to derive a solution based on the Kalman Filter. A suboptimal method, the so called Extended Kalman Filter, has been quite popular in dealing with nonlinear stochastic and measurement models. However, it has a drawback in that this approximation method does not take into account all statistical characteristics of the processes and hence leads to poor results (Arulampalan et al., 2002).

Grid-based localization was introduced by (Lim and Kang, 2002). Different successful applications of grid-based Markov localization can be found. Simmons et al. (2000) used this method to localize a robot in an office environment and Thrun et al. (2001) applied the algorithm to estimate the position of a robot operating in museums. Gutmann and Fox (2002) compared the grid-approaches and Kalman filtering. Fox et al. (2000) and Howard et al. (2003) extended this approach to the multi-robot localization problem. But for large environments, these approaches often need big memory and high performance computing when using high resolution grid map to avoid collisions.

The most common approach to handle both global and local localization is the particle filter (Thrun et al., 2005). The algorithms based on particles belong to a class of algorithms called Monte Carlo Localization (MCL). MCL uses fast sampling techniques to represent the robots posterior belief. When the robot moves or senses, importance re-sampling is applied to estimate the posterior distribution. Instead of an analytic approximation of the Probability Density Function (PDF) or a discretization of the whole state space only a set of weighted samples is needed in MCL. Under the right conditions these samples can then represent any given PDF and thus can do global and kidnapped localization assuming that enough samples are used. In recent years many studies to improve MCL
are employed. For example, Fox (2003) brought forward KLD-sampling algorithm, which adapts the number of particles over time. The idea behind it is to determine the number of particles based on a statistical bound on the sample-based approximation quality. However, the sample size problem is still a bottleneck problem in real-time global localization.

In this study, we derive the algorithm for generating a dynamically sized set of samples according to the exact posterior being estimated. As a result, the situation that sampling sets don’t contain the real robot pose particle is avoided and the global localization and kidnapped problems are figured out efficiently. KLD sampling is utilized to measure the approximation error by Kullback-Leibler distance between the true distribution and its sampled representation. In contrast to conventional MCL, two extended validation procedure as the over-convergence and uniformity validations are introduced, thus highly saving computational resource and enhancing localization efficiency. The real experiments in an indoor environment illustrate that the proposed approach can reduce the uncertainty in localization significantly.

**NEW IMPROVED MCL ALGORITHM**

Specific improvements are made in this study by the optimal proposal distribution selection and the introduced KLD sampling with two extra checkup procedures in re-sampling, which greatly improve the localization ability of the robots.

**The proposal distribution:** The optimal proposal distribution that minimizes the variance of the next weights for any generic PF (Doucet et al., 2000) is as following:

\[
x_i^{t} \sim q(x_i^{t} | x_{i}^{t-1}, z_i, u_i) = \frac{p(x_i^{t} | x_{i}^{t-1}, z_i, u_i)}{p(x_i^{t-1} | x_{i}^{t-1}, z_i, u_i)}
\]

For mobile robots this proposal requires drawing samples from the robot motion model and observation models, which are the terms that appear in the numerator of Eq. 1. Since the system state for the last time step does not appear in the denominator, this is a constant value \( \lambda \) for each particle \( i \). Therefore, drawing samples from the optimal proposal is equivalent to drawing from:

\[
x_i^{t} \sim \frac{1}{\lambda} p(z_i | x_i^{t-1}, z_{i-1}, u_i) p(x_i^{t-1} | x_{i}^{t-1}, z_i, u_i)
\]

By replacing this optimal proposal in the general equation for the weight update in a SIS filter, we obtain:

\[
\pi_i^{t \rightarrow (t+1)} p(z_i | x_i^{t+1}, z_{i+1}, u_i)
\]

The algorithm is to generate samples exactly distributed according to the density in Eq. 1, while dynamically adapting the number of samples to assure a good representation of the true posterior at each moment.

**Adaptive particle filter:** In particle filtering, a large number of samples are required to achieve a certain level of accuracy. However, the larger the size of the sample set, the larger the computational cost. Therefore, an adaptive particle filter is proposed in order to reduce the computational cost. In this study, Kullback-Leibler distance sampling is used, which measures the approximation error by Kullback-Leibler distance between the true distribution and its sampled representation. The KLD is a measure of the difference between two probability \( p_i(x) \) distributions \( p_i(x) \) and is defined as:

\[
D(p_i, p_j) = \int p_i(x) \log \frac{p_i(x)}{p_j(x)} dx
\]

As that work shows, the minimum number of particles \( N_k \) to ensure that the KLD between the estimated and the real distributions is kept below a certain threshold \( \epsilon \) with 1-\( \delta \) a probability is given by:

\[
N_k = \frac{1}{2\epsilon} x_{\text{ch}}^2
\]

where, \( x_{\text{ch}}^2 \) denotes the \( q \)th quantile of the chi-squared distribution with \( d \) degrees of freedom. In order to determine \( N_k \) we need to compute the values of the chi-square distribution. The best solution is given by the Wilson-Hilberty transformation (Johnson et al., 1994), which yields:

\[
N_k = \frac{l-1}{2\epsilon} [1 + \frac{2}{9(l-1)} + \frac{2}{9(l-1)^2}]^{1/2}
\]

where, \( z_{\text{ch}} \) is the upper 1-\( \delta \) value of the standard normal distribution. This method shows excellent performance in practice.

In this study a new APF that can lead to effective sampling by adjusting the variance size is utilized (Park et al., 2010). The basic idea is to increase variance inversely proportional to the likelihood and generate
samples. Such likelihood is correlated to the KLD. The lower the likelihood, the further away the KLD is. If the KLD is large, it means that the true distribution and the sample are far apart from each other. So the variance size has to be increased by a large quantity in order to generate samples near the true distribution. Using the Eq. 6 the number of required samples can be calculated. The adjusted variance is calculated by using the relationship between the maximum number of samples and the number of required samples. The adjusted variance is given by:

$$\sigma - \sigma' + \epsilon \cdot \frac{N_x}{N_{\text{req}}}$$

(7)

where, $\sigma$ denotes the adjusted variance, $\sigma'$ denotes the low bound variance satisfying $\sigma' \leq \epsilon$, $\epsilon$ denotes a maximum value of KLD, $N_x$ denotes the number of required samples and $N_{\text{req}}$ denotes the number of total samples. If the KLD is small, $N_x$ tends to decrease. For example, $N_x/N_{\text{req}}$ becomes very small and $\sigma$ will be similar to $\sigma$. On the contrary, if the KLD is large, $N_x/N_{\text{req}}$ will become closer to 1 and $\sigma$ will be similar to $\Sigma$.

Two validation procedure: To solve the computing problem brought by re-sampling procedure at every cycle, the timing of re-sampling according to the distribution information after perception updating is a prerequisite. With regard to the pose-tracking problem, most samples gather at the real pose of the robot and the localization error becomes smaller after importance re-sampling. However, when the sample distribution does not match with perceptual information, there are two situations: One is the over-convergence in which some sample weights are too high because of the interfering signal; the other one is that all the samples are uniformly distributed and the weights are all very small. The two situations are corresponding to initial localization or kidnapped procedure in which sample sets at present are not good approximation of the real robot pose distribution.

In order to solve problems above, two validations processes for over-convergence and uniformity are introduced into the algorithm other than the conventional three-recursive procedures.

Over-convergence validation process is implemented with entropy of normalized sample distribution and effective sample size. Entropy is a metric of uncertainty in information theory and also can denote uncertainty of probability with the from, $H = -\sum p_i \cdot \log p_i$. $p_i$ is probability of events. In MCL, pose distribution is described with probability distribution of sample set, so uncertainty of pose distribution can also be described by entropy. And effective sample size of sample sets is obtained through the following equations:

$$\text{ESS} = \frac{N}{1 + \epsilon}$$

(8)

$$\epsilon = \frac{\text{var}(\pi(0))}{E^2(\pi(0))} - \frac{1}{N} \sum_{i=1}^{N} (\pi_i - \bar{\pi})^2$$

(9)

When effective sample size is lower than the given threshold (as percentage of sample size), no matter however entropy changes or, over-convergence occurs, new samples are introduced with number of $n_s = c$ (N-ESS) with $c$ as a constant. Otherwise, over-convergence is tested through the difference of entropy of sample distribution before and after update, the larger the difference is the more likely over-convergence occurs. The method is:

$$\frac{|H_x - H_y|}{H_y} \geq \lambda$$

(10)

where, $H_y$ and $H_x$ are, respectively entropy of sample distribution before and after sensor update, $\lambda$ ranges in interval $[0, 1)$ and decreases with $H_x$ increasing. The number of new sample size is:

$$n_s = (1 - \lambda)(N - \text{ESS})$$

(11)

Uniformity validation is implemented through summation of sample weight of non-normalized sample distribution. As sample distribution is uniform after motion update and sensor update only re-weights the sample set, with summation as:

$$\pi = \frac{1}{N} \sum_{i=1}^{N} p(Z_t | X_i(t))$$

(12)

If $\pi$ is less than threshold $\pi_{\text{th}}$, difference of sample set with sensor information is greater, so re-sampling from vision information is applied with the whole size as the new sample size, otherwise, importance re-sampling is applied. Threshold $\pi_{\text{th}}$ is set according to sensor model.

IMPLEMENTATION

Robot motion: Let $x_{k-1} = [x_{k-1}, y_{k-1}, \theta_{k-1}]$ denote the initial pose of the robot (Jensfelt, 2001) and suppose we keep the robot running for some time $\Delta t$. After $\Delta t$ time of motion, the robot will be at $(x_k, y_k, \theta_k)^T$ and the following model is used:
Fig. 1: Sampling-based approximation of robots position belief

\[
(x_k, y_k, \theta_k)^T = f(x_{k-1}, u_{k-1}, w_{k-1}, x_{k-1})
\]
\[
(x_{k-1} + D_z(0 + w_{z_{k-1}}^x)\cos(\theta_{k-1} + \Delta \phi), x_{k-1} + D_z(0 + w_{z_{k-1}}^y)\sin(\theta_{k-1} + \Delta \phi), \\
\theta_{k-1} + D_\theta w_{\theta_{k-1}})
\]

\[
(\Delta x_k, \Delta y_k)^T, D_z = \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2}, \Delta \phi = \arctan(y_{z_{k-1}} (\Delta x_k) - x_{z_{k-1}})^{-1}
\]

\[
w_k = (w_1^x, w_1^y)^T, u_k = (\Delta x_k, \Delta y_k)^T, \Delta \phi = \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2}, \Delta \phi = \arctan(y_{z_{k-1}} (\Delta x_k) - x_{z_{k-1}})^{-1}, \Delta \phi = \Delta \phi_k
\]

where,

\[
x_{z_{k-1}} = \text{argmax}_{x_{z_{k-1}}} (x_{z_{k-1}} | Z_{k-1})
\]

In the detailed implementation, grid is used to find an estimate of the robot pose. The grid is composed of 0.5x0.5 m² cells covering the entire known environment and each grid cell is given the total weight of all samples that fall within this cell. The estimate is then found by searching the cell with the highest weight.

**EXPERIMENTAL RESULTS**

The following localization experiment consists of tracking the pose of a mobile robot equipped with a laser range finder while it is manually guided through an office environment. Concretely, the path described by the robot and the (already built) map of the environment about 12x7.5 m is shown in Fig. 2a. We compare the localization accuracy between the SIR and our optimal PF. The resolution of the occupancy grid is 0.04 m. In the whole experiments, the threshold \(\lambda\) of the ESS is 0.1, constant \(c\) is 0.8, \(\sigma\) is 0.5 and \(\tau\) is 0.5.

Firstly a whole extended MCL process is illustrated with sample distributions from initial global localization to kidnapped, entropy and effective sample size with time. Figure 2 shows the setup of our experiments along with a part of the occupancy grid map used for position estimation. Figure 2a also shows the path from A to D taken by Pioneer3 DX with laser range finder, which was in the process of global localization. At first, the robot dealt with global localization from A to B. Then, it was kidnapped to C on arriving at B and resolved the kidnapped problem during the process of moving from C to D. Fig. 2b represents the uncertain belief of the robot at point A from scratch. The sample distribution at position
Fig. 2: Robot localization procedure using improved MCL

Fig. 3: (a) Effective sample size in localization process and (b) Entropy before and after re-sampling

Fig. 4: Total samples using the proposed method

Fig. 5: Localization error of extended MCL and conventional MCL
B is shown in Fig. 2c. At the position C, robot is kidnapped to D with new heading, thus causing great decrease of effective sample size and entropy. After importance re-sampling, until the end point D with sample distribution in Fig. 2d, robot re-localized itself successfully. Figure 3 shows the plots of entropy and effective sample size varying with time. In this procedure, robot is kidnapped to D from C and effective sample size and entropy decreased greatly because of the enormous error between the estimation according to sampling distribution and the perception of itself. Figure 4 shows the results of the proposed method with adjusted variance in terms of the total number of samples. From the figure we can see that the total sample number is at maximum at the point of A and D when robot knows little about its pose information.

Ten experiments of this kind were conducted and the performance to conventional MCL for robot was compared. To evaluate the localization performance the true locations of the robot were determined by performing position tracking and measuring the position on each second. For each second, the estimation error was then computed at the reference positions. The average distance of all samples from the reference position measured the estimation error. The results are summarized in Fig. 5. The graph plots the distance error (y-axis) as a function of iteration (x-axis), averaged over the ten experiments. As can be seen in the Figures, the quality of position estimation increases much faster when using this improved MCL than the conventional MCL. The robot resolves its global localization completely about at 15th iteration, which is much faster than the conventional MCL.

To sum up, the experiments are specifically well suited to demonstrate the advantage of our approach in solving the robot global localization and kidnapped problems. Of course, the performance of our methodology in more complex situations, especially indoor environments, is more attractive to solve robot's global localization.

**CONCLUSIONS**

New extended Monte Carlo localization algorithm that uses a sample-based representation of the state space of a robot is presented in this study. The algorithm generates a dynamically sized set of samples according to the exact posterior being estimated, thus making global localization more efficient and kidnapped problem tractable. KLD sampling is utilized to measure the approximation error by Kullback-Leibler distance between the true distribution and its sampled representation, along with two extended validation procedure as over-convergence and uniformity validations to verify correspondence between sample distribution and sensor information for timely re-sampling. Experimental results demonstrate that our approach yields significantly better localization results than conventional MCL localization.

**ACKNOWLEDGMENT**

This work was supported by the National Nature Science Foundation of China (Grant No. 60805032, 61075090).

**REFERENCES**


