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# An Intuitionistic Fuzzy AHP Based on Synthesis of Eigenvectors and its Application

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**Abstract:** With regard to multi Criteria Decision Making (MCDM) problems, performances of the Analytical Hierarchy Process (AHP) are prominent. Fuzzy AHP, an extension of AHP, serves as a grateful approach due to its outstanding advantage when dealing with uncertainties. Based on advantages of Intuitionistic Fuzzy Sets (IFSs) in expressing information of preferences, this study presents an intuitionistic fuzzy AHP (IF-AHP) approach. The proposed IF-AHP synthesizes eigenvectors of Intuitionistic Fuzzy Comparison Matrix (IFCM) in which all the information for decision are represented by Intuitionistic Fuzzy Values (IFVs). The IF-AHP approach enables to handle MCDM problems without loss of information or defuzzification and represent arbitrary hesitation in interval [0, 1]. Firstly, Intuitionistic Fuzzy (IF) matrix and IFCM associated with its consistency and satisfactory consistency are defined after some relative basal knowledge are introduced. Secondly, the eigenvector and eigenvalue of IFCM is defined and a linear program model is presented to obtain it as the priority of relative criteria. Furthermore, methods for comparisons of IFVs are proposed in order to rank alternatives utilizing eigenvectors. And then, a integrate procedure of IF-AHP involving comparison and rating is presented and illustrated by two applied examples cited from literatures. An involved decision support system can be setup according to the procedure. Comparing with some existing methods, the proposed approach gives both rational global priorities and robust final decision.

**Key words:** Fuzzy AHP, intuitionistic fuzzy set, eigenvector; comparison matrix, multi criteria decision making

## INTRODUCTION

The Analytical Hierarchy Process (AHP) approach which was developed by Saaty (1980), Saaty (1977) possesses distinct advantage of dealing with subjective information of Decision Makers (DMs). As one of the most outstanding Multi Criteria Decision Making (MCDM) approach (Saaty *et al*., 2007; Saaty, 2008) or a weight estimation technique (Vaidya and Kumar, 2006; Mahdavi *et al*., 2008a), AHP can be applied in many areas such as selection (Parsakhoo and Lotfalian, 2009; Mohammaditabar and Teimoury, 2008), evaluation (Huang *et al*., 2011; Wu *et al*., 2010; Abdullah *et al*., 2009), planning and development (Maskani-Jifroudi *et al*., 2009), decision making (Mahdavi *et al*., 2008b), forecasting and so on. Pair-wise comparison makes it simple to express DM's preferences and meanwhile the consistent ratio insures the validity of judgments. Interestingly, the pair-wise comparison methods are based on crisp real number (Saaty, 2006) while DMs' assessments in pair-wise comparison always include uncertainty in reality so that DMs' may sometimes feel more confident to provide fuzzy judgment than crisp comparisons (Wang *et al*., 2008).

In order to deal with its inability in handling the uncertain and imprecise decision-making problems, recent years, scholars extended the real comparison matrix to fuzzy comparison matrix and then fuzzy-AHP was proposed on the basis of the concepts of the fuzzy set theory (Zadeh, 1965). An example of application of fuzzy logic can be seen by Binwahlan *et al*. (2009). Although, Saaty and Tran (2007) argued the invalidity of using fuzzy number to improve the outcome from judgments, the convenience of express uncertainty of both vagueness and ignorance is apparent. Existing versions of fuzzy AHP usually focus on the comparison matrices and resultant priorities.

Atanassov (1986) and Atanassov (1999) extended the concept of Zadeh's fuzzy sets (Zadeh, 1965) and introduced Intuitionistic Fuzzy Sets (IFSs), whose prominent characteristic is that it assigns to each element a membership degree and a non-membership degree. So it

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gives a powerful tool to deal with uncertainty in real applications especially when to express a pair-wise comparison. Amer *et al.* (2010) utilized IFSs to analyze the reliability of a large system. In present study, therefore, IFS is introduced to pair-wise comparison matrix and refer to it as Intuitionistic Fuzzy Comparison Matrix (IFCM) or Intuitionistic Fuzzy Preference Relation (IFPR) (Xu, 2007; Qian and Feng, 2008). And then a new approach named IF-AHP is proposed, in which the eigenvector of IFCM is introduced to represent the priority of the compared elements with respect to a criterion in the upper level of the hierarchy. The outcome of a hierarchy, in other words, the priority of the alternatives with respect to the object in the top level can be derived by synthesizing or multiplying all the intuitionistic fuzzy matrices composed by the eigenvectors in the same level orderly. Besides, its applications in decision support are also discussed for prospective users or DMs.

Thus, the aim of this study was to develop a novel fuzzy AHP approach as a solution of MCDM problems in intuitionistic fuzzy setting. Preliminaries such as definition, product, eigenvectors and consistency of IFCMs are presented hereinafter. The effectiveness of proposed methodology is requisite to clarify as well, comparing to existing methods. That will be illustrated by two realistic examples.

## RECENT ADVANCES OF FUZZY AHP

Over the last twenty years several authors commented on shortcomings of pair-wise judgments and comparison scale of traditional AHP. Spontaneously, improved approaches were worked out to focus on the denotative form of pair-wise judgment matrix and its resultant priority. To utilize fuzzy logic conveniently, the mutually complement comparison matrix was used in fuzzy AHP (Kang and Lee, 2007). Other more complex fuzzy theories were used and the comparison matrix was extended. Interval-value fuzzy comparison matrix was used in (Chamodrakas *et al.*, 2010; Wang and Chin, 2006; Wang and Chen, 2008). Triangular fuzzy comparison matrix was introduced and used by Amiri (2010), Cakir (2008), Chen and Wang (2010), Kaya and Kahraman (2010) Sen and Cinar (2010) and Tang (2009). And trapezoidal fuzzy comparison matrix emerged by Wang and Chin (2006), Wang and Chin (2008), Fu *et al.* (2008) and Huang *et al.* (2008). Qian and Feng (2008) developed the concept of IFCM. Meanwhile the methods of deriving the priority of a comparison matrix are mainly listed as follow. The lambda-max method or fuzzy eigenvector method emerged by Wang and Chin (2006), Chang *et al.* (2009) Chang *et al.* (2008), Duran and Aguilo (2008), Nepal *et al.* (2010), Huang *et al.* (2008) and Csutora and Buckley (2001). The geometric/arithmetic mean method can be found by Wang and Chin (2008), Amiri (2010), Che *et al.* (2010) Chen *et al.* (2008), Chen and Wang (2010) and Kahraman *et al.* (2009). And the extent analysis method was used by Bozbura and Beskese (2007), Ertugrul and Karakasoglu (2009), Heo *et al.* (2010), Kreng and Wu (2007) and Secme *et al.* (2009) (which is criticized by Wang *et al.* (2008)). Wang *et al.* (2006) and Kang and Lee (2007) introduced the least squares method and the entropy weight method, respectively. Further, fuzzy preference programming method was used by Qian and Feng (2008), Chamodrakas *et al.* (2010), Cakir and Canbolat (2008) and Wang and Chin (2008).

In the aspect of theory and technique of fuzzy AHP, it is focused on utilizing fuzzy theory to overcome disadvantages of traditional AHP and new issues along with it. Wang and Chin (2006) extend the lambda-max method and proposed an eigenvector method to generate normalized interval, triangular and trapezoidal fuzzy priority of comparison matrix. Situations where the eigenvector method is inapplicable are also analyzed. A modified fuzzy logarithmic least squares method which is formulated as a constrained nonlinear optimization model, is suggested by Wang *et al.* (2006). Wang *et al.* (2008) argued the shortcoming of the extent analysis method, pointed out that untrue weight and wrong decision making may take place by the extent analysis method. Furthermore, Wang and Chin (2008) proposes a sound yet simple priority method for fuzzy AHP which utilizes a linear goal programming model to derive normalized fuzzy weights for fuzzy pair-wise comparison matrices. Cakir (2008) focused on the issue of rank reversal and suggested a fuzzy preference programming methodology to prevent the order of preference. Wang and Chen (2008) presented fuzzy linguistic preference relations method which is an easy and practical way to provide a mechanism for improving consistency in fuzzy AHP method.

Simultaneously, application of fuzzy AHP with different improvement of the details is paid more attention. Kang and Lee (2007) constructed an analytical approach under a fuzzy subjective judgment environment to deal with uncertainty and to generate performance ranking of different priority mixes for semiconductor fabrication. Kreng and Wu (2007) evaluated three knowledge portal system development tools taking use of fuzzy AHP in group decision setting. Similarly, Fu *et al.* (2008) assessed the impact of market freedom on the adoption of third-party electronic marketplace, while Chamodrakas *et al.* (2010) discussed supplier selection in the same background using satisfying. Kahraman *et al.* (2009), Huang *et al.* (2008) and Gungor *et al.* (2009) also talked over the multi criteria problem of selection from alternatives. Duran and Aguilo (2008) proposed a method to computer-aided investment evaluation and justification of an advanced manufacturing system. Nepal *et al.* (2010) utilized fuzzy AHP to prioritize customer satisfaction attributes for automotive product development, considered a broad range of strategic and tactical factors for determining the weights which were then incorporated into target planning. Heo *et al.* (2010) established the criteria and factors and assessed for renewable energy dissemination program evaluation the importance of each factor using fuzzy AHP. While Lin (2010) evaluated course website quality with a fuzzy AHP approach to determine the relative weights linking the criteria between high and low online learning experience groups.

Furthermore, scholars studied to combine fuzzy AHP with other decision making methods based on advantages of both. The most familiar hybrid form is methods aggregating fuzzy AHP and TOPSIS (Amiri, 2010; Dagdeviren *et al.*, 2009; Ertugrul and Karakasoglu, 2009; On't and Soner, 2008; Secme *et al.*, 2009; Torfi *et al.*, 2010). The framework of the aggregation could be come down to a pattern. Fuzzy AHP is used to determine the relative weights of evaluation criteria and then TOPSIS is used to rank the alternatives. Stirn (2006) integrated fuzzy AHP with discrete dynamic programming approach to evaluate the conflicting objectives to determine the optimal forest management decisions. Tang (2009) budgeted allocation for an aerospace company with fuzzy AHP and artificial neural network, respectively and then compared the two outcomes to help decision-making. Chang *et al.* (2008) and Chen and Wang (2010) proposed hybrid approaches combining fuzzy AHP with modified Delphi approach. The former select unstable slicing machine to control wafer slicing quality. While the later develop global business intelligence for information service firms, respectively. The same idea for selection can be found by Hsu *et al.* (2010). Besides, balanced scorecard method was associated with fuzzy AHP to support decision making (Cebeci, 2009; Lee *et al.*, 2008), as well as data envelopment analysis (Che *et al.*, 2010), multidimensional scaling analysis (Chen *et al.*, 2008) and Dempster-Shafer theory of evidence (Hua *et al.*, 2008). Moreover, Gumus (2009) presented a method which congregates fuzzy AHP, TOPSIS and Delphi approach to evaluate hazardous waste transportation firms.

The aforementioned literatures in aspect of both theories and applications expand the practicable scope of fuzzy AHP to a large extent. However, there are still facets to be improved. Firstly, fuzzy number including interval, triangular and trapezoidal fuzzy number consider merely degree of membership to a fuzzy concept. For example, a triangular fuzzy number (7, 8, 9) can express a concept "about 8", while the degree of membership is default. By contraries, theory of IFS consider degree of non-membership besides membership with respect to a fuzzy concept for each judgment, hence preference of judgments depicted by IFS could include more information from DMs which may lead to more precise decision making. That's why we choose IFS in present study. In the proposed approach, two problems caused by introducing IFS to comparison matrices are consistency and priority of IFPR. Secondly, defuzzification in the process of fuzzy AHP (Chang *et al.*, 2009; Che *et al.*, 2010; Ertugrul and Karakasoglu, 2009; Heo *et al.*, 2010; Lin, 2010; Fu *et al.*, 2008) would cause loss of information or even result in an untrue decision making. IFVs are used to express information without defuzzification in the entire process of the proposed approach. Furthermore, it is rational that if preference information takes the form of fuzzy number, priorities of criteria or alternatives should take the form of fuzzy number as well. Yet most of the existing fuzzy AHP approaches take the form of real numbers to represent priorities. We propose an approach in which all information for decision making take the form of IFVs. Lastly, fundamentals or illustrated examples in some literatures mentioned above do spell out vividly the proposed ways and means, whereas the lack of the intact and operable methodology may limit the application for the perspective users.

## PRELIMINARIES

**IFS and its algorithm:** Intuitionistic Fuzzy Sets (IFS) introduced by Atanassov have been proven to be highly useful to deal with uncertainty and vagueness which was characterized by a membership function and a non-membership function. Recently, many authors have applied the intuitionistic fuzzy set theory to the field of decision making. Chen and Tan (1994) and Hong and Choi (2000) presented some techniques for handling multi-criteria fuzzy decision-making problems based on vague set theory Bustince and Burillo (1996) pointed out that vague sets were IFS). They provided some functions to measure the degree of suitability of each alternative with respect to a set of criteria presented by vague values. Lin *et al.* (2007) proposed new method that allowed the decision-maker to assign the degrees of membership and non-membership of the criteria to the fuzzy concept "importance". The concept of IFS is as follows.

- **Definition 1:** (Atanassov, 1986). Let X be an ordinary finite non-empty set. An IFS in X is an expression A given by:

$$A = \{\langle x, u_A(x), v_A(x)\rangle | x \in X\} \quad (1)$$

where, $u_A$: X→[0, 1], $v_A$: X→[0, 1] with the condition: $0 \le u_A(x) + v_A(x) \le 1$, for all x in X.

The numbers $u_A(x)$ and $c_A(x)$ denote, respectively, the degree of the membership and the degree of the non-membership of the element x in the set A. Denotation $\pi_A(x) = 1 - u_A(x) - v_A(x)$ represents the degree of hesitation or intuitionistic index or non-determinacy of x to A. Therefore, for ordinary fuzzy sets the degree of hesitation $\pi_A(x) = 0$.

The ordered pair $\alpha_A(x) = (u_\alpha(x), v_\alpha(x))$ is referred to as an IFV (Xu, 2007), where, $u_\alpha(x), v_\alpha(x) \in [0, 1]$, $\pi_\alpha(x)$ and $u_\alpha(x), v_\alpha(x) \le 1$. Associated with the degree of hesitation, an IFV can also be equivalently denoted by $\alpha(x) = (u_\alpha(x), v_\alpha(x), \pi_\alpha(x))$ where $u_\alpha(x), v_\alpha(x), \pi_\alpha(x) \in [0, 1]$ and $u_\alpha(x), v_\alpha(x), \pi_\alpha(x) = 1$. In the rest of this study, IFV $\alpha = (u, v, \pi)$ is abbreviated as $\pi = (u, v)$ when no misunderstanding raises. Two useful operations on IFVs are as follows:

- **Definition 2:** (Xu, 2007). Let $a = (ú_a, v_a)$ and $b = (u_b, v_b)$ be two IFVs, then:

$$(1)\ a \oplus b = (u_a + u_b - u_a u_b, v_a v_b) \quad (2)$$

$$(1)\ a \otimes b = (u_a u_b + v_b - v_a v_b) \quad (3)$$

It is apparent that the results are also IFVs and both of the operations are commutative and associative as the following properties. If $a = (u_a, u_a)$, $b = u_b u_b$ and $c = u_c u_n$ are three IFVs, then:

$$(1)\ a \oplus b = b \oplus a \quad (4)$$

$$(2)\ a \otimes b = b \otimes a \quad (5)$$

$$(3)\ (a \oplus b) \oplus c = a \oplus (b \oplus c) \quad (6)$$

- **Definition 3:** (Qian and Feng, 2008). $X = (x_1, ..., {}_n)^T$ with $x_I = (u_I, v_I)$ is said to be an IF vector if for all I = 1,..., n satisfies:

$$\sum_{i=1}^{n} u_i \le 1, \ \sum_{i=1}^{n} v_i \le n-1 \quad (7)$$

Especially, if an IF vector satisfies $\pi_i = 1 - u_i - v_i = 0$ for all I = 1,..., n, then it is called certain IF vector.

- **Definition 4:** (Qian and Feng, 2008). If an IF vector $x = (x_1, ..., x_n)^T = ((u_1, v_1, \pi_1), ..., (u_n, v_n, \pi_n))^T$ satisfies:

$$\begin{cases} \sum_{i=1}^{n} u_i + \pi_j \le 1 \\ \sum_{i=1}^{n} v_i + \pi_j \le n-1 \end{cases} \quad j = 1, \cdots, n \quad (8)$$

then it is normalized; otherwise it is not normalized.

A normalized IF vector can represent the priority or relative weights of n alternatives or criteria. In other words, if an IF vector $x = (x_1, ..., x_n)^T$ is normalized, then there exist a vector $\dot{x} = (\dot{x}_1, \cdots, \dot{x}_n)^T$ such that

$$\sum_{i=1}^{n} \dot{x}_i = 1, \ u_i \le \dot{x}_i \le 1 - v_i$$

If $\pi_j = 0$ for all j = 1,..., n, then the normalized certain IF vector is a weight vector satisfying

$$\sum_{j=1}^{n} u_j = \sum_{j=1}^{n} v_j = 1$$

**IF matrix and its synthesis:** The concept of matrix is extended to intuitionistic fuzzy setting for the purpose of representing decision information with intuitionistic fuzzy forms.

- **Definition 5:** An IF matrix is a matrix denoted by $M = (M_{IJ})_{m \times n} = ((u_{IJ}, v_{IJ}))_{m \times n}$, $M_{IJ}$ in which is IFV, where, I = 1,..., m, j = 1,...., n.

Notice that the notion of IF matrix is just an extension of traditional real matrix. In present study, it is employed to represent priorities in a level of a hierarchical structure with respect to all elements in the above level. Thus columns of an IF matrix should be an IF vector so that it can represent priorities. On the other hand, a special IF matrix could be used to reflect preference relations as follow:

- **Definition 6:** An square IF matrix $M_{IJ}$ is called an Intuitionistic Fuzzy Comparison Matrix (IFCM) or Intuitionistic Fuzzy Preference Relation (IFPR) if ∀I, j = 1,..., n, such that:

$$(1)\ M_{IJ} = (0.5, 0.5) \tag{9}$$

$$(2)\ u_{IJ} = v_{JI},\ v_{IJ} = u_{JI} \tag{10}$$

This kind of comparison matrix is called complementary matrix. If $M_{IJ}$ = (0.6, 0.35), for example, it could be judged that 0.6 and 0.35 represent the certainty degree of which criterion I is preferred than criterion j and the certainty degree of that criterion j is important than criterion I, respectively, while 0.05 is interpreted as the uncertainty degree of which criterion I is preferred than criterion j, according to DMs. Especially, if an IFCM M satisfies $\pi_{IJ} = 0$ for all I, j = 1,..., n, then M is reduced to two fuzzy comparison matrix $M_1 = (u_{IJ})$ and $M_2 = (v_{IJ})$ which satisfy $(M_1)^T = M_2$. Note that $M_{IJ}$ and $M_{JI}$ have the same hesitation.

After defining basic concepts, an arithmetical algorithm is presented.

- **Definition 7:** Let M, N be two IF matrices denoted by:

$$M = (M_{it})_{m\times s} = \left(\left(u_{it}^M, v_{it}^M\right)\right)_{m\times s},\ N = (N_{tj})_{s\times n} = \left(\left(u_{tj}^N, v_{tj}^N\right)\right)_{s\times n} \tag{11}$$

a matrix

$$C = (C_{ij})_{m\times n} = \left(\left(u_{ij}^C, v_{ij}^C\right)\right)_{m\times n}$$

is entitled the product or synthesis of M multiply by N, denote by C = M⊗N, where:

$$C_{ij} = \overset{s}{\underset{t=1}{\oplus}}\left(M_{it} \otimes N_{tj}\right) \tag{12}$$

The arithmetical operation is selected here instead of fuzzy operations i.e., max and min as which leads to a loss of information obviously. It can be easily proved that the outcome of multiply is an IF matrix as well.

- **Theorem 1:** If $M = (M_{IJ})_{m\times n}$, $N = (N_{IJ})_{m\times n}$ are two IF matrices, matrix $C = (C_{IJ})_{m\times n}$ with:

$$C_{ij} = \sum_{t=1}^{s} M_{it} \otimes N_{tj}$$

then Cis an IF matrix.

The multiply of IF matrices will be used to both synthesize decision information in each level of a hierarchical structure (represented by an IF matrix) and derive priorities from IFCMs.

**Consistency of IFCM:** Consistency is essential in human thinking because it enables ordering the world according to dominance. It is a necessary condition for thinking about the world in a scientific way (Saaty, 1980). Once judgments among a set of criteria or alternatives with respect to a certain criteria or object are given, it is essential to make sure those judgment are rational logically. In the following, the notion of consistent IFCM is presented.

- **Definition 8:** (Qian and Feng, 2008). An IFCM $M = (M_{IJ})_{m\times n} = ((u_{IJ}, v_{IJ}))_{m\times n}$ is consistent, if there exists a certain IF vector $x = (x_1,..., x_n)^T$ with $x_1 = (u_i, v_i)$ such that:

$$\begin{cases} u_i - u_j + 1 \geq 2u_{ij} \\ v_i - v_j + 1 \geq 2v_{ij} \end{cases},\ \text{for all } i = 1,\cdots n-1;\ j = i+1,\cdots,n \tag{13}$$

Based on Definition 4 and Definition 8, an approach is presented to check if an IFCM is consistent conveniently.

- **Theorem 2:** $M = (M_{IJ})_{m\times n} = ((u_{IJ}, v_{IJ}))_{m\times n}$ is a consistent IFCM, if and only if it satisfies the following inequality constraints:

$$\max_k\left(u_{ik} + u_{kj} - 1\right) \leq -\max_k\left(v_{ik} + v_{kj} - 1\right)\ \text{for all } i, j = 1,\cdots,n \tag{14}$$

**Proof:** If M is a consistent IFCM, then there exists a certain IF vector $x = (x_1,..., x_n)^T$ with $x_1 = (u_i, v_i)$ such that:

$$u_i - u_j + 1 \geq 2u_{ij},\ v_i - v_j + 1 \geq 2v_{ij},\ i, j = 1,\cdots,n \tag{15}$$

which means the vector $x = (x_1,..., x_n)^T$ satisfies:

$$u_{ik} \leq 0.5\left(u_i - u_k + 1\right) \leq 1 - v_{ik},\ i, k = 1,\cdots,n \tag{16}$$

$$u_{kj} \leq 0.5\left(u_k - u_j + 1\right) \leq 1 - v_{kj},\ k, j = 1,\cdots,n \tag{17}$$

Adding (16) by (17) leads to the following implied indirect inequalities:

$$u_{ik} + u_{kj} - 1 \leq 0.5\left(u_i - u_j\right) \leq 1 - \left(v_{ik} + v_{kj}\right),\ i, j, k = 1,\cdots,n \tag{18}$$

Since (18) holds for any k = 1,..., n, it follows that:

$$\max_k\left(u_{ik}+u_{kj}-1\right)\le-\max_k\left(v_{ik}+v_{kj}-1\right),\ \text{for all } i,j=1,\cdots,n \quad (19)$$

Conversely, if (14) holds for ∀I, j, k, then:

$$\max_k\left(u_{ik}+u_{kj}-0.5\right)\le\min_k\left(1-v_{ik}+1-v_{kj}-0.5\right),\ i,j,k=1,\cdots,n \quad (20)$$

By the condition of consistency of interval complementary comparison, there exists $\bar{x}=(\bar{x}_1,\cdots,\bar{x}_n)^T$, where:

$$\sum_{i=1}^{n}\bar{x}_i=1,\bar{x}_i\ge 0,i=1,\cdots,n$$

such that:

$$u_{ij}\le 0.5\left(\bar{x}_i-\bar{x}_j+1\right)\le 1-v_{ij} \quad (21)$$

Suppose that $\bar{x}_i=u_i=1-v_i$, $\bar{x}_j=u_j=1-v_j$ so $u_i$-$u_u$+1 ≥2$u_{ij}$ and $v_i$-$v_u$+1 ≥2$v_{ij}$. By Definition 8, M is a consistent IFCM.

The above Theorem 2 can be used to test whether or not an IFCM is consistent without solving any mathematical programming mode. It only requires simple algebraic operations. Because an IFCM is reciprocal in nature, only its lower or upper triangular need to be checked.

Rationally, it is not necessary to keep consistent everywhere. By contraries, inconsistency is usually unavoidable, yet it is not useless indeed. Saaty (1980) argued that inconsistency must be precisely one order of magnitude less important than consistency, or simply 10% of the total concern with consistent measurement. If it were larger it would disrupt consistent measurement and if it were smaller it would make insignificant contribution to change in measurement. We infer that satisfactory consistency is acceptable.

A programming model is proposed here to verify whether an IFCM is of satisfactory consistency or not. First the Geometric Consistency Index (GCI) proposed by Aguaron and Mereno (2003) which is similar to Consistency Ratio (CR) proposed by Saaty, is introduced here.

- **Definition 9:** Aguaron and Mereno (2003). Given a pairwise comparison matrix A = $(a_{ij})_{n\times n}$ and the vector of priorities, ω = $(\omega_1,..., \omega_n)^T$, obtained by the Row Geometric Mean Method, the GCI can be defined as:

$$GCI=\frac{2}{(n-1)(n-2)}\sum_{i<j}\log^2 e_{ij} \quad (22)$$

where, $e_{ij}$ = $a_{ij}$ $\omega_j/\omega_i$ is the error obtained when the ratio $\omega_j/\omega_i$ is approximated by $a_{ij}$ and I j = 1, 2,..., n

In addition, approximated thresholds for the GCI are proposed in (Aguaron and Mereno, 2003). Qian *et al.* (2009), afterwards, switch those thresholds to be available in the case of complementary comparison matrix, namely, Complementary Geometric Consistency Index (CGCI) as shown in Table 1 and the CGCI of a real complementary comparison matrix could be defined as follows.

- **Definition 10:** Given a complementary pairwise comparison matrix A = $(a_{ij})_{n\times n}$ and the vector of priorities, ω = $(\omega_1,..., \omega_n)^T$, the CGCI can be defined as:

$$CGCI=\frac{2}{(n-1)(n-2)}\sum_{i<j}\left(2a_{ij}-\omega_i+\omega_j-1\right)^2 \quad (23)$$

where, I j = 1, 2,..., n

- **Definition 11:** An IFCM M = $(M_{ij})_{n\times n}$ is of satisfactory consistency, if there exists a complementary pairwise comparison matrix A = $(a_{ij})_{n\times n}$ with $CGCI_A$≤δ, where δ is the approximated threshold in Table 1 and $u_{ij}\le a_{ij}\le 1$-$v_{ij}$

Based on Definition 8 and 11, the relationship between consistency and satisfactory consistency of an IFCM is concluded as following theorem.

- **Theorem 3:** If an IFCM M = $(M_{ij})_{n\times n}$ is consistent, then it is of satisfactory consistency

Further, Definition 11 gives a straightforward way to check if an IFCM is of satisfactory consistency.

- **Theorem 4:** Let M = $(M_{ij})_{n\times n}$ = $((u_{ij}, v_{ij}))_{n\times n}$ be an IFCM, M is of satisfactory consistency if and only if there exist a vector of priorities ω = $(\omega_1,..., \omega_n)^T$ such that:

$$\frac{2}{(n-1)(n-2)}\sum_{i<j}\left(2a_{ij}-\omega_i+\omega_j-1\right)^2\le\delta$$

where, $u_{ij}\le a_{ij}\le 1$-$v_{ij}$ and δ is the approximated threshold in Table 1.

Then a programming model is established to verify the existence of vector ω:

$$\min CGCI=\frac{2}{(n-1)(n-2)}\sum_{i<j}\left(2a_{ij}-\omega_i+\omega_j-1\right)^2$$

$$\text{s.t.}\quad \begin{aligned} & u_{ij} \le a_{ij} \le 1 - v_{ij}; \\ & \sum_{i=1}^{n} \omega_i = 1; \\ & \omega_i \ge 0, i = 1, 2, \cdots, n \end{aligned} \tag{24}$$

Then the rule to make the judgment whether an IFCM is of satisfactory consistency could be come down to the following theorem.

- **Theorem 5:** An IFCM $M = (M_{ij})_{n\times n}$ is of satisfactory consistency if $CGCI_A^* \le \delta$, where, CGCI* represent the optimal solution of programming model (24)

Obviously, if $\delta = 0$, then satisfactory consistency reduces to consistency. Thus consistency can be seen as a special case of satisfactory consistency. In addition, the vector $\omega = (\omega_1, \ldots, \omega_n)^T$ derived by (24) is not appropriate to the priority reflected by an IFCM directly although the existence of $\omega$ ensures the satisfactory consistency of the IFCM. Because the optimal solution of (24) discards hesitation and uncertainty of the original IFCM, and which may lead to a loss of information. Similar to relative literatures, we used right eigenvector of IFCM to represent the priority instead.

**Eigenvector of IFCM:** If an IFCM is consistent or at least satisfactory consistency, then it implies priority of DM in a level of hierarchical structure with respect to a criterion of upper level. Motivated by the classical AHP (Saaty, 1980), the idea of right eigenvector is used to depict it. As hesitations and uncertainties exist in an IFCM, they should be retained by the vector of priority. Thus the eigenvector of an IFCM is defined by IF vector. First, the definition is present as follows:

- **Definition 12:** Assume M is an n×n IFCM. If there exists a n×1 IF vector:

$$x = \left(\left(u_i^x, v_i^x\right)\right)_{n\times 1}$$

and an IFV $\lambda = (u_\lambda, u_\lambda)$ which satisfying:

$$M \otimes x = \lambda \otimes x \tag{25}$$

then $\lambda$ is named the eigenvalue of M, and x is the eigenvector of M correspondingly.

Each IF matrix M can be split into two nonnegative matrices:

$$u_M = \left(u_{ij}^M\right)_{m\times n}$$

referred to as membership matrix and:

$$v_M = \left(v_{ij}^M\right)_{m\times n}$$

referred to as non-membership matrix, and denote as $M = ((u_M, v_M))$ for convenience. It is transparent that $u_M$ and $v_M$ are real number matrices but no longer complement matrices.

- **Theorem 6:** Let M be an IFCM. If $u_\lambda$ $v_\lambda$ are the eigenvalue of $u_M$, $v_M$ respectively, $u_x$ and $v_x$ are the corresponding eigenvector of $u_M$, $v_M$ respectively, i.e., $u_M$, $u_x$ and $v_M$, $v_x$, then $\lambda = u_\lambda\ v_\lambda$ is the eigenvalue of M and the corresponding eigenvector of M can be denoted as $x = ku_x, 1\text{-}lv_x$, k and $l$ in which are real number such that $0 \le ku_x \le lv_x \le 1$. That is:

$$(u_M, v_M) \otimes (ku_x, 1 - lv_x) = (u_\lambda, v_\lambda) \otimes (ku_x, 1 - lv_x) \tag{26}$$

**Proof:** Since $0 \le ku_x \le lv_x \le 1$, therefore we have $0 \le ku_x \le lv_x \le 1$, then $x = (ku_x, 1\text{-}lv_x)$ is IF vector. On the one hand:

$$\begin{aligned} &(u_M, v_M) \otimes (ku_x, 1 - lv_x) = \left(u_M ku_x, v_M + 1 - lv_x - v_M(1 - lv_x)\right) \\ &= \left(ku_M u_x, 1 - lv_x + lv_M x_x\right) = \left(ku_\lambda u_x, 1 - lv_x + lv_\lambda v_x\right) \end{aligned}$$

On the other hand:

$$\begin{aligned} &(u_\lambda, v_\lambda) \otimes (ku_x, 1 - lv_x) = \left(u_\lambda ku_x, u_\lambda + 1 - lv_x - u_\lambda(1 - lv_x)\right) \\ &= \left(ku_\lambda u_x, 1 - lv_x + lv_\lambda v_x\right) \end{aligned}$$

So, (26) is proofed. By Definition 12, the theorem is thus proofed completely.

Assume that $\hat{u}_x = (\hat{u}_1, \cdots, \hat{u}_n)^T$, $\hat{v}_x = (\hat{v}_1, \cdots, \hat{v}_n)^T$ are the normalized principal eigenvectors of $u_M$, $v_M$, respectively, which satisfying:

$$\sum_{i=1}^{n} \hat{u}_i = 1, \sum_{i=1}^{n} \hat{v}_i = 1$$

Then eigenvector of M can be denoted as $x = (k\hat{u}_x, 1 - l\hat{v}_x)$ according to theorem 6. Substituting it to (8), we get:

$$\begin{cases} k\left(1 - \hat{u}_j\right) + l\hat{v}_j \le 1 \\ l\left(1 - \hat{v}_j\right) + k\hat{u}_j \ge 1 \end{cases} \quad j = 1, \cdots, n \tag{27}$$

In order to determine k and $l$, the following Linear Programming (LP) model is constructed:

$$\text{Max } J = k + \delta$$

$$\text{s.t.} \begin{cases} k\left(1 - \hat{u}_i\right) + l\hat{v}_i \le 1 \\ l\left(1 - \hat{v}_i\right) + k\hat{u}_i \ge 1 \\ l\hat{v}_i - k\hat{u}_i - \delta \ge 0 \\ k, \delta \ge 0 \end{cases} \quad i = 1, \cdots, n \tag{28}$$

The meaning of maximizing δ is to make each weight interval as wide as possible, while the implication of maximizing k is to avoid k = 0. Note that not all IF comparison matrices can generate normalized eigenvector weights by LP model (28).

- **Example 1:** Let M be the following IF comparison matrix:

$$M=\begin{bmatrix}(0.5,0.5) & (0.6,0.2) & (0.5,0.4) & (0.6,0.2) & (0.4,0.5)\\ (0.2,0.6) & (0.5,0.5) & (0.4,0.3) & (0.5,0.2) & (0.3,0.6)\\ (0.4,0.5) & (0.3,0.4) & (0.5,0.5) & (0.6,0.2) & (0.4,0.5)\\ (0.2,0.6) & (0.2,0.5) & (0.2,0.6) & (0.5,0.5) & (0.3,0.6)\\ (0.5,0.4) & (0.6,0.3) & (0.5,0.4) & (0.6,0.3) & (0.5,0.5)\end{bmatrix}$$

First, the consistency is checked by Theorem 2 and as a result, the matrix is consistent. Then, we solve the LP models (28) after $\hat{u}_x$ and $\hat{v}_x$ are derived and get IF priority vector $x = (x_1,..., x_n)^T$, where:

$$x_1=(0.1867,0.7986),\ x_2=(0.1317,0.7544),\ x_3=(0.1587,0.7637)$$
$$x_4=(0.0975,0.6733),\ x_5=(0.1961,0.7808)$$

The vector x is normalized according to Definition 4.

**Comparisons of IFVs:** In this study, all kinds of information are depicted by IFVs. For example, priorities or weights are expressed by IF vectors in which each entry is an IFV. IFVs need to be compared and further ranked in order to rank alternatives or criteria. Following the idea of Wang *et al.* (2005), the degree of preference of two IFVs based on probabilities is defined as:

- **Definition 13:** Let $a = (u_a, v_a)$ and $b = u_b, v_b$ be two IFVs, the degree of preference of a over b (a>b) is derived by:

$$P(a>b)=\frac{\max\{0,1-v_a-u_b\}-\max\{0,u_a-(1-v_b)\}}{\pi_a+\pi_b} \quad (29)$$

where, $\pi_a = 1-u_a-v_a$, $\pi_b = 1-u_b-v_b$.

The degree of preference of b>a can be defined in the same way. Then two IFVs can be ranked by the following rules.

- **Definition 14:** If P (a>b)>P (b>a), then a is said to be superior to b to the degree of P (a>b), denoted by:

$$a \overset{P(a>b)}{\succ} b$$

if P (a>b)>P (b>a) = 0.5, then a is indifferent to b, denoted by b~a; else if P (a>b)>P (b>a), then a is inferior to b to the degree of P (b>a), denoted by

$$a \overset{P(b>a)}{\prec} b$$

- **Example 2:** Let a = (0.65, 0.3), b = (0.55, 0.25), then P (a>b) = 0.6 and P (b>a) = 0.4, so according to Definition 14, a is superior to b to the degree of 0.6, denoted by:

$$a \overset{0.6}{\succ} b$$

For the purpose of the necessity of comparison of a serial IFVs of an IF vector hereinabove of this paper, a method based on Definitions 13 and 14 is introduced to achieve it. Let $x_i = (u_i, v_i)$ $\pi_i = 1-u_i-v_i$, I = 1,...,n, where $x_i$ is IFV defined by Definition 1. A ranking process is outlined below:

- **Step 1:** Calculate the matrix of the degrees of preference:

$$P=\begin{array}{c} \\ x_1\\ x_2\\ \vdots\\ x_n\end{array}\begin{array}{c}\begin{array}{cccc}x_1 & x_2 & \cdots & x_n\end{array}\\ \begin{bmatrix} - & P_{12} & \cdots & P_{1n}\\ P_{12} & - & \cdots & P_{12}\\ \vdots & \vdots & \ddots & \vdots\\ P_{12} & P_{12} & \cdots & -\end{bmatrix}\end{array} \quad (30)$$

Where:

$$P_{ij}=P(x_i>x_j)=\frac{\max\{0,1-v_i-u_j\}-\max\{0,u_i-(1-v_j)\}}{\pi_i+\pi_j},\ i,j=1,\cdots,n;\ i\neq j \quad (31)$$

Symbol '-' in P means that the value does not have to be calculated.

- **Step 2:** Calculate the matrix of preference relation:

$$R_P=\begin{array}{c} \\ x_1\\ x_2\\ \vdots\\ x_n\end{array}\begin{array}{c}\begin{array}{cccc}x_1 & x_2 & \cdots & x_n\end{array}\\ \begin{bmatrix} - & R_{12} & \cdots & R_{1n}\\ R_{12} & - & \cdots & R_{12}\\ \vdots & \vdots & \ddots & \vdots\\ R_{12} & R_{12} & \cdots & -\end{bmatrix}\end{array} \quad (32)$$

Where:

$$R_{ij}=\begin{cases}1 & \text{if } P_{ij}>0.5\\ 0 & \text{if } P_{ij}\leq 0.5\end{cases}\quad i,j=1,\cdots,n;\ i\neq j \quad (33)$$

- **Step 3:** Calculate the sum of the elements of each row in the above matrix of preference relation and generate the final aggregated ranking $R = (r_1,..., r_n)$. $X_i$ is ranked higher than $x_j$ if and only if the sum for the ith row is larger than that for the jth row

It is worthy to note that this ranking method presents not only the order of n IFVs but also the degree of possibility of one IFV is superior to another.

- **Example 3:** Ranking the order of normalized IF vector derived by Example 1

When utilizing the process above, the resultant preference order is:

$$x_4 \overset{0.5177}{\succ} x_5 \overset{0.6605}{\succ} x_3 \overset{0.5741}{\succ} x_1 \overset{0.5417}{\succ} x_2 \tag{34}$$

and the matrix of the degrees of preference P is:

$$P = \begin{bmatrix} - & 0.5417 & 0.4621 & 0.4258 & 0.1395 \\ 0.4583 & - & 0.4538 & 0.4317 & 0.3615 \\ 0.5379 & 0.5462 & - & 0.4524 & 0.3995 \\ 0.5724 & 0.5683 & 0.5476 & - & 0.5177 \\ 0.8605 & 0.6385 & 0.6005 & 0.4823 & - \end{bmatrix} \tag{35}$$

Notice that the probabilities of $x_4$ preferring to $x_i$ (I = 1, 2, 3, 5) are scarcely more than 0.5, so it isn't very sure about the judgment $x_4$ is superior to $x_i$ (I = 1, 2, 3, 5) in fact. Therefore, the preference order isn't sustained by merely probabilities, a modified process is suggested in this study to ranking the order of entries in an IF vector considering both probabilities and hesitations of them. The modified ranking process is as follows:

- **Step 1:** The same as Step 1 in the above process
- **Step 2:** Calculate the sum of the elements of each row in the above matrix P, denote by:

$$SP = (SP_1,\cdots,SP_n)^T = \left(\sum_{j=1, j\neq i}^{n} P_{ij}\right)_{n\times 1}, \; i = 1,\cdots,n$$

and each entry in which divided by its hesitation $\pi_i$, respectively. The more superior $x_i$, the bigger value $Sp_i/\pi_i$. Denote the resultant index vector as $I = (I_1,..., I_n)^T$, where, $I_i = Sp_i/\pi_i$ with I = 1,..., n.

- **Example 4:** Ranking the order of normalized IF vector derived by Example 1 by the modified ranking process

The hesitations of $x_i$ is presented in Example 1 and outcome of the modified process is (106.8181, 14.9740, 24.9558, 9.6312, 11.8220). Thus, the priority of IFCM in Example 1 is $x_5 \succ x_1 \succ x_3 \succ x_2 \succ x_4$.

The modified ranking process considers probabilities and hesitations at the same time, it is clear that outcomes derived by which is more reasonable. However, it is nonsensical if $\pi_i = 0$. Note that this modified process is suitable when P ($x_i \succ x_j$) verges on 0.5 and hesitations are slightly bigger than 0, for other cases, the original ranking process is suitable. According to the fact that hesitations will magnify gradually along with the process of synthesis of IF matrices, the modified process is more adaptive to the proposed method in this study.

## IF-AHP METHODOLOGY

A hierarchy is a powerful manner of classification used to order information gained either from experience or from our own thinking. Thus, the complexity of the world around us could be understood according to the order and distribution of influences which make certain outcomes happen (Saaty and Shih, 2009). Due to the confinement of the ability of expressing a judgment accurately and the advantage of IFS in considering both degree of membership and degree of non-membership at the same time, IFS is introduced to the traditional AHP.

**Structuring hierarchies:** Saaty and Shih (2009) defined hierarchical structures by the notions of ordered sets and finite partially ordered sets and suggested the following procedure for structuring hierarchies.

- **Step 1:** Define the goal or focus of the decision problem at the top level. For instance, it could be a mission statement of an organization
- **Step 2:** Break down the purpose into some supportive elements in the first level below the goal. The elements on the first level should be comparable and homogeneous or close in their possession of a common attribute. For instance, a system can be broken down physically into sub-systems, units, sub-units, components, etc
- **Step 3:** Insert actors into a suitable level. The function of the actors is similar to a filter that screens out some influences at the upper levels. It might be more than one level of actors depending on the requirements
- **Step 4:** Establish the bottom level for choice. The bottom level of the hierarchy could be alternatives, actions, consequences, scenarios or policies to be chosen

- **Step 5:** Examine the hierarchic levels forward and backward. One usually needs to check and revise the elements and even the levels, backward and forward iteratively to ensure the consistency of the structure
- **Step 6:** Check the validation of structures. Two guidelines to check the structure are: 1) is the structure logical? and 2) is the structure complete?

**The procedure of IF-AHP:** In present study, a panoramic methodology and an operable procedure for the general and curbstone DMs is mainly focused on. For the purpose, the proposed IF-AHP approach is depicted step by step as follows.

- **Step 1:** Structuring hierarchies. DMs could structure hierarchies following advices in section 4.1. Suppose there are n levels in the hierarchical structure, in which the top level is named as the first level and the last level or the alternatives level as the nth level, there are $n_i$ elements or criteria in the ith level, where the elements are denoted by $\{e_1^i, \cdots, e_{n_i}^i\}, i = 1, \cdots, n$ . Note that $n_1 = 1$
- **Step 2:** Comparison. In order to give expression to a comparison precisely, it is proposed in this paper to employ IFS to express all the pair-wise comparison in the hierarchies. Assume that comparison in level I with respect to the jth criteria $e_j^{i-1}$ in level I-1, where I = 1,..., n, j = 1,..., $n_{i-1}$ is formed to an IFCM $M_j^{(i)}$ . Note that each entry (u, v) in $M_j^{(i)}$ consists of two parts i.e., the certainty degree u of which a criterion is preferred than another and the certainty degree v of that the latter is important than the former, respectively, by two separate judgments such that $u+v \le 1$. In practice, DMs may not sure about translate judgments into IFVs straightway. Therefore, two methods to accomplish successful and effectual transformation of judgments are proposed when comparing $e_{j1}^i$ with $e_{j2}^i$ with respect to $e_h^{i-1}$ . First, if a decision group is formed and x percents of the group prefer $e_{j1}^i$ while y percents of the group prefer $e_{j2}^i$ , then the corresponding IFV of the judgments can be set as (u, v, π) = (x/100, y/100, 1-x/100-y/100) and synthesize weights of members of the group if needed. Second, if there is only one DM, he or she could estimate the lower bound x and upper bound y of the degree or probability of $e_{j1}^i$ preferred to $e_{j2}^i$ , then the corresponding IFV of the judgments can be derived as (u, v) = (x, 1-y)
- **Step 3:** Calculating eigenvectors. For each $M_j^{(i)}$ , if it is of satisfactory consistency, its weighting vector or eigenvectors $x_j^i = (x_{1j}^i, \cdots, x_{n_i j}^i)^T$ can be derived by the LP model (28) if it is of satisfactory consistent according to Theorem 5. The eigenvectors of comparison in ith level with respect to each $e_{i-1}^j$ , j = 1,..., $n_{i-1}$, form a IF matrix as:

$$M^{(i)} = \begin{pmatrix} x_{11}^{(i)} & \cdots & x_{1,n_{i-1}}^{(i)} \\ \vdots & \ddots & \vdots \\ x_{n_i,1}^{(i)} & \cdots & x_{n_i,n_{i-1}}^{(i)} \end{pmatrix} \quad (36)$$

- **Step 4:** Synthesis. The priority of elements in the last level or alternatives with respect to general object is, immediately, derived by $x = M^{(n)} \otimes \ldots \otimes M^{(2)}$. Obviously, x is an IF vector and, usually, has been normalized (see example 5 and example 6)
- **Step 5:** Ranking. Alternatives could be ranked according to entries which are IFVS in x by methods in section 3.5

Actually, the above process can deal with the case with incomplete information. If DMs have absolutely no idea about with one or some judgments of comparisons, just set the corresponding locations of the IFCM by IFV (0, 0) with 1 as its hesitation. The flow chart correspond to the above procedure is shown in Fig. 1.

**Rating:** There are two ways to create priorities of alternatives i.e., rank from comparisons which give relative measurements and from ratings which give absolute measurements. When rating alternatives, they must be assumed to be independent and rank should be preserved. While alternatives are usually dependent with each other and rank may not always be preserved when comparing (Saaty, 2006; Saaty and Sagir, 2009). Due to the excellent performance of rating in aspect of rank preservation, rating process in IF environments is addressed separately. The last level of a hierarchy is always minute criteria rather than alternatives when rating, therefore priorities of these criteria with respect to the goal of the hierarchy is represented by x derived by Step 5 in Section 4.2.

Only one alternative is considered here with the hypothetic condition of that $e_1$,..., $e_n$ are n criteria, entries in the normalized IF vector $x = (x_1, \ldots, x_n)^T = ((u_1, v_1), \ldots, (u_n, v_n))^T$ represent the priority of all criteria. If the score of performance of the alternative with respect to the ith criteria is denoted by an IFV $s_i = (u_i^{(s)}, v_i^{(s)})$ , where I = 1,..., n, Then the alternative's score is computed by:

$$s = \bigoplus_{i=1}^{n} (s_i \otimes x_i)$$

Using the absolute or ratings method of the AHP, categories or standards are usually established for each

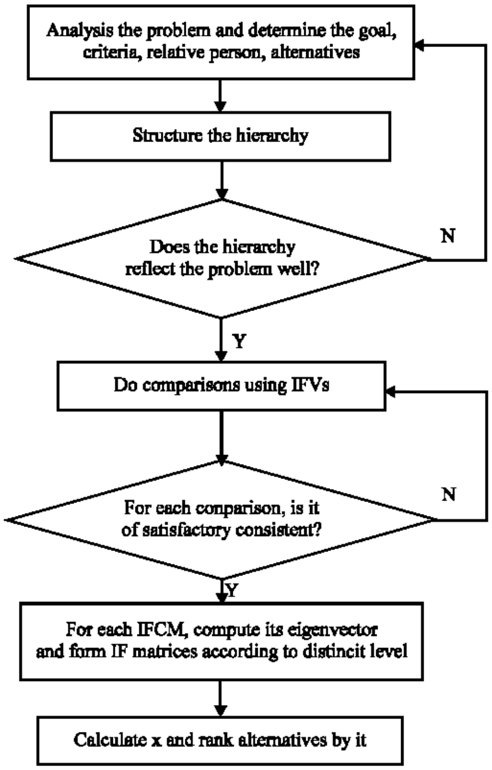


Fig. 1: The flow chart of IF-AHP

criterion. Then alternatives are rated one at a time by selecting the appropriate category under each criterion rather than compared against other alternatives. The standards are prioritized for each criterion by making pair-wise comparisons. The alternative's score is then computed by weighting the priority of the selected category by the priority of the criterion and summing for all the criteria.

If there is only one alternative to be considered in this kind of rating problem and in which denotations of criteria and their priorities are the same as above. Suppose $c_i$ categories are established for the criteria $e_i$, the utility of the category $c_{ik}$ for criteria $e_i$ is:

$$utl_{i_k} = \left(u_{i_k}^{(utl)}, v_{i_k}^{(utl)}\right)$$

where, k = 1,..., $c_i$ and I = 1,..., n. The alternative's score is then calculated by:

$$s = \bigoplus_{i=1}^{n}\left(utl_{i_k} \otimes x_i\right)$$

## EXAMPLES OF APPLICATION

To illuminate the process and the validity of the proposed IF-AHP in present study convectively, two examples are cited from literatures and data are used straightforwardly without any modification. Thus, it is easy to compare rank of alternatives with different versions of AHP. In order to give facilities for application of proposed method, the original data are necessary to transform into IFVs in both examples.

**Two examples:**

- **Example 5:** Supplier selection is a critical and demanding task for companies that participate in electronic marketplaces to find suppliers and to execute electronically their transactions. Chamodrakas *et al*. (2010) aimed to suggest a fresh approach for decision support enabling effective supplier selection processes in electronic marketplaces. An evaluation method with two stages is consequently introduced: initial screening of the suppliers and final supplier evaluation. Initial screening is conducted through the enforcement of hard constraints on the selection criteria. And final supplier evaluation is implemented through the application of a modified variant of the fuzzy preference programming method. The approach was demonstrated with the example of a hypothetical metal manufacturing company that finds and selects suppliers in the environment of an electronic marketplace. Here this example is figured out again with IF-AHP. Its hierarchy is presented in Fig. 2. Three main criteria are quality, cost, delivery which form the second level of the hierarchy. The third level of the hierarchy occupies the sub-criteria defining the three criteria of the second level. Only one sub-criterion related to cost has been chosen, namely cost reduction, two sub-criteria related to quality, namely rejection rate from quality control and supplier remedy of quality problems and two sub-criteria related to delivery, namely compliance with due date and compliance with quantity. After initial screening, Supplier 3, Supplier 5 and Supplier 7 were moved out from original set of alternatives

Since, the original data were presented by interval numbers with 1-9 scales which is suggested by Saaty, it is necessary to transform them to IFVs with a value range of 0-1. First, for an original interval comparison matrix:

$$C = \left(c_{ij}\right)_{n\times n} = \left(\left[c_{ij}^L, c_{ij}^U\right]\right)_{n\times n}$$

it could be transformed into an interval comparison matrix:

$$T = \left(T_{ij}\right)_{n\times n} = \left(\left[T_{ij}^L, T_{ij}^U\right]\right)_{n\times n} = \left(\left[1/\left(c_{ji}^U+1\right), 1/\left(c_{ji}^L+1\right)\right]\right)_{n\times n}$$

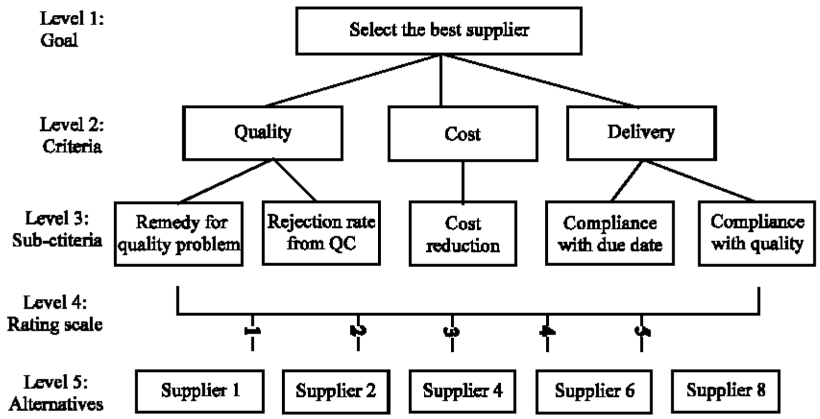


Fig. 2: Hierarchical structure of example 5

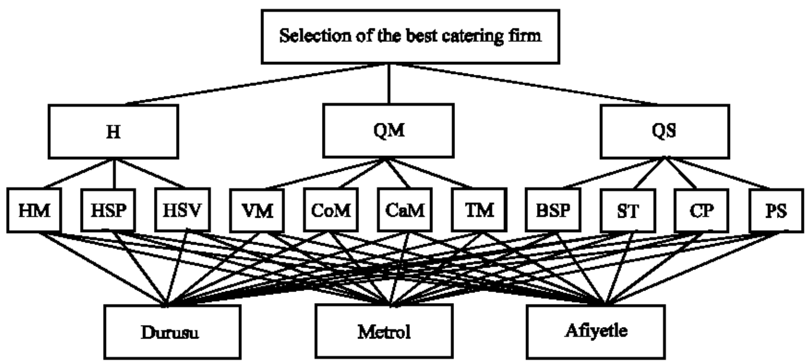


Fig. 3: Hierarchy of catering firm selection problem of example 6

Xu (2002). Notice that ranges of each entry of T are [0.1, 0.9]. Then these entries are transformed it to [0, 1] uniformly and still denote as T. And then a corresponding IFCM is derived by $M = \left(\left(T_{ij}^{L}, 1 - T_{ij}^{U}\right)\right)_{n\times n}$ using the relationship between interval numbers and IFVs (Xu, 2007).

Follow the Steps 3-5 in Section 4.2, the IF priority and the index vector of 5 Suppliers derived by Matlab are, respectively, $x = (x_1,..., x_n)^T$ with $x_1 = (0.0187, 0.1792)$, $x_2 = (0.0196, 0.1848)$, $x_3 = (0.0234, 0.1936)$, $x_4 = (0.0159, 0.1793)$, $x_5 = (0.0159, 0.1793)$.

Apparently, x is normalized according to Definition 4 and the index vector:

$$I = (2.5013, 2.5124, 2.5430, 2.4871, 2.4871)^T \quad (37)$$

Hence, Supplier 4 is the best alternative followed by Supplier 2, 1, 6 and 8, while Supplier 6 (and 8) is superior that Supplier 1 in original literature.

- **Example 6:** A big Turkish textile company wishes to make a contract with one catering firm. Alternative Turkish catering firms are Durusu, Mertol and Afiyetle. The goal is to select the best among the three alternatives. The criteria to be considered are Hygiene (H), Quality of meal (QM) and Quality of Service (QS) which involve 11 sub-criteria, i.e., Hygiene of Meal (HM), Hygiene of Service Personnel (HSP), Hygiene of Service Vehicles (HSV), Variety of Meal (VM), Complementary meals in a day (CoM), Calorie of meal (CaM), Taste of Meal (TM), Behaviour of Service Personnel (BSP), Service Time (ST), Communication on Phone (CP) and Problem Solving (PS) ability. Figure 3 shows the hierarchical structure of the problem. A decision-making group consisting of the customers of the catering firms and five experts is responsible for making comparisons and constructing fuzzy comparison matrices. The information of judgments is firstly shown by Kahraman *et al.* (2004) and then by Wang *et al.* (2008) with a slight change. The problem is computed with both methods and data are adopted from the latter at first

A prepared work needs to be completed due to the form of original data are presented by triangular fuzzy number. Let a = (*l*, m, r) be a triangular fuzzy number and α = (u, v) is a corresponding IFV calculated by u = *l* and v = 1-r. It seems as if some information for decision making is discarded during the transformation. But this example aims at comparing the rank of alternatives with distinct methods. In the framework of proposed method, only two pieces of data are needed: membership function and non-membership function. The effectiveness of IF-AHP will be compared with other methods using only part of the given information. Then all original triangular fuzzy judgments could be switched to IFCMs so as to apply the proposed IF-AHP method. Follow the steps in Section 4.2, IF priorities, namely, eigenvectors of three alternatives derived by Matlab are: $x = (x_1,..., x_n)^T$ with $x_1 = (0.0596, 0.4331)$, $x_2 = (0.0546, 0.39)$, $x_3 = (0.0668, 0.4854)$, which is already normalized according to Definition 4 and the matrix of the degrees of preference P is:

$$P = \begin{pmatrix} - & 0.4809 & 0.5222 \\ 0.5191 & - & 0.5414 \\ 0.4778 & 0.4585 & - \end{pmatrix} \quad (38)$$

Then final index of the IF priority vector x is derived by the modified ranking process mentioned in Section 3.5 is:

$$I = (1.9665, 1.9096, 2.0913)^T \quad (39)$$

All appearances, Afiyetle is the best alternatives followed by Durusu and Mertol which is absolutely the same as in Wang *et al.* (2008). Whereas, there is a rank reversal between Durusu and Mertol in Kahraman *et al.* (2004). Moreover, nearly the opposite result is offered by Wang *et al.* (2008) if the Extend Analysis Method (EAM) proposed by Kahraman *et al.* (2004) is used.

Meanwhile, the proposed IF-AHP is operated again with the original data presented in Kahraman *et al.* (2004) and the following result is obtained:

$$I = (2.1479, 2.0674, 2.3259)^T \quad (40)$$

This accorded with the result of Kahraman *et al.* (2004) nicely.

## DISCUSSION

Example 5 is presented to illustrate the validity and capability of IF-AHP to deal with MCDM problems, while Example 6 is presented to compare three outcomes derived by different approaches including IF-AHP, the extend analysis method and modified fuzzy logarithmic least squares method (MFLLSM) (Wang *et al.*, 2006). Comparing Example 5 with Chamodrakas *et al.* (2010), it can be concluded that the proposed IF-AHP could offer a satisfying assessment of a MCDM problem and furthermore provide reliable and believable decision making support for DMs. For the convenience of comparison three methods, the local weight of three alternatives with respect to four sub-criteria of quality of service using data of Wang *et al.* (2008) by different methods are shown in Table 1-4 for example. The following differences and indifference are summarized from these Tables.

- All judgments expressed by triangular fuzzy numbers or IFVs yet priorities derived by EAM are real

Table 1: Approximated thresholds for the CGCI

| CR | 0.01 | 0.05 | 0.1 | 0.15 |
|---|---|---|---|---|
| CGCI(n = 3) | 0.0345 | 0.1727 | 0.3486 | 0.5184 |
| CGCI(n = 4) | 0.0387 | 0.1936 | 0.3872 | 0.5808 |
| CGCI(n>5) | ~0.0345 | ~0.2032 | ~0.4063 | ~0.6095 |

Table 2: Local weights of three catering firms with respect to QS by EAM

| | BSP | ST | CP | PS | Local weights |
|---|---|---|---|---|---|
| Weight | 0 | 0.99 | 0.00 | 0.01 | |
| Durusu | 1 | 0.05 | 0.86 | 0.0 | 0.05 |
| Mertol | 0 | 0.64 | 0.00 | 0.0 | 0.634 |
| Afiyetle | 0 | 0.31 | 0.14 | 1.0 | 0.317 |

Table 3: Local weights of three catering firms with respect to QS by MFLLSM using IFVs

| | BSP | ST | CP | PS | Local weights |
|---|---|---|---|---|---|
| Weight | (0.1408,0.8403) | (0.4826,0.4528) | (0.0707,0.9198) | (0.2321,0.7134) | |
| Durusu | (0.6110,0.3511) | (0.2443,0.6709) | (0.4242,0.4427) | (0.1714,0.8231) | (0.2878,0.6369) |
| Mertol | (0.1705,0.8196) | (0.3729,0.5386) | (0.1782,0.7902) | (0.1782,0.7899) | (0.2709,0.6566) |
| Afiyetle | (0.1719,0.7914) | (0.2904,0.6982) | (0.2329,0.6024) | (0.6130,0.3495) | (0.3430,0.6037) |

Table 4: Local weights of three catering firms with respect to QS by IF-AHP using IFVs

| | BSP | ST | CP | PS | Local weights |
|---|---|---|---|---|---|
| Weight | (0.0955,0.5881) | (0.1836,0.7900) | (0.0667,0.4100) | (0.1309,0.6886) | |
| Durusu | (0.2566,0.7227) | (0.2584,0.5599) | (0.2194,0.6230) | (0.1431,0.4074) | (0.1015,0.5098) |
| Mertol | (0.1431,0.4074) | (0.3000,0.6483) | (0.1369,0.3959) | (0.1507,0.4204) | (0.0947,0.3692) |
| Afiyetle | (0.1507,0.4204) | (0.2599,0.6101) | (0.1765,0.5138) | (0.2566,0.7227) | (0.1036,0.4554) |

numbers which may loss some information of uncertainty and fuzziness. No defuzzification process exists in both MFLLSM and IF-AHP, so results by which are more reliable

- EAM may assign a zero weight to a decision criterion or alternative, leading to the criterion or alternative not to be considered in decision analysis as can be seen in Table 2. Approximated thresholds for the CGCI are presented in Table 1
- Comparing Table 3 with Table 4, it is found that hesitations of MFLLSM are smaller than that of IF-AHP on average which mainly because there is a most possible value in the middle of a triangular fuzzy number
- MFLLSM and IF-AHP both adopt optimization model to derive priorities, the former takes use of anonlinear optimization model yet the latter utilize a linear optimization model (28) along with smaller degree of complexity of calculation
- MFLLSM obtains global fuzzy weights by solving two LP models and an equation for each alternative, while IF-AHP obtains global fuzzy weights by product or synthesis of matrices constituting by priorities expressed by eigenvectors of IFCMs

Furthermore, the advantages and limitations of the proposed IF-AHP are concluded as follows:

- Each comparison in IF-AHP composes with two separate judgments with the point of view of degree of membership and non-membership which allows arbitrary hesitation in interval [0, 1] in nature. So the method could be applicable to different actual cases with different hesitations for diverse comparisons. Whereas some existing methods define fixed hesitation with a certain linguistic variable which may be only applicable for a certain case
- Prospective users and DMs could adopt the proposed method step by step to deal with MCDM problems because of the detailed procedures presented in this study. This enhances the scope of its application to some extent
- In the entire process of IF-AHP, all information such as comparisons, eigenvalues and eigenvectors or priorities are expressed by IFVs which guarantees no loss of information during procedures of calculation
- However, the modified ranking process for IF vectors proposed in this study is suitable for IF-AHP merely but not fit the case when a hesitation is convergent to zero. The original ranking process based only on the degree of probability is available for the special case; nevertheless its validity is not very satisfactory as shown in Example 3

## CONCLUSION

In view of the prominent advantage of IFS, a new IF-AHP based on eigenvectors of IFCMs and their synthesis is proposed to deal with fuzzy uncertainties in MCDM problems. For the purpose of practical applications of the proposed methodology, some step-by-step procedures are presented in this study to facilitate idiographic operation of the potential users or DMs. At last two numerical examples are calculated to illustrate validity and correctness of IF-AHP and a comparison with some existing approaches is further given. However, there are several further works worthy to study. First, the validity of IF-AHP is confirmed by examples, yet whether the problem of rank reversal when alternatives are ranked from comparisons is essential for extensive application. Second, a decision support system or simply a toolbox based on Matlab is needful to be developed for prospective users so that they need only to construct a hierarchical model and give judgments of preferences and leave the task of calculating to the software.

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