Motion Modeling and Simulation of Biped Robot

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Abstract: Present study is combining the legs’ architecture characteristics of human beings to build the lower limbs mathematical model with 12 degree of freedom. Describe space relations among each joint and integrate ZMP (Zero Moment Point) as the movement stability criterion. Moreover, do the single analysis of mathematic model, discuss and solve the inverse of motion matrix in details then to work out angle speed relation among each leg joint of the robot. Take the advantage of ADAMS (Automatic Dynamic Analysis of Mechanical Systems) to stimulate dimensional virtual prototype. The stimulation can successfully achieve the stability of biped robot movement without sideslip and moves forward a single step to prove the exactness and rationality of mathematic model.

Key words: Biped robot, D-H method, ZMP, ADMAS, virtual prototype

INTRODUCTION

Biped robot is the machine recreation of biped biology motion. Compare to the other mobile robots (such as wheel type, crawler type, creep type and so on) that biped robot has the higher suitability and flexibility as the hot point of present robot research. However, the traditional physical prototype will be the bottleneck of machine design technology with huge cost and cycle length to research the biped robot. Through virtual prototype, the potential problems will be detected before create the physical prototype machine while testing in the stimulate environment. That will short the production development cycle, decrease the research cost and improve the design quality by a large margin (Tao et al., 2002).

Before robot gait design and programming, we need to build the mathematic model and kinematical equation in the first place. Moreover, lead in the constraint condition to obtain the conditional value of each rotor angle and the value in different times. Then ensure the leg position of each moment. In theory, biped robot can walk stable need the regular reciprocate movement of legs. This article simplify the planning process that only design and programming the leg structure of robot to aiming at the purposeful analysis and research.

Present study takes example by the leg design of humanoid robot in bilioloid company to build the mathematic model and biped robot virtual prototype by ADAMS, programming the complete movement and apply it into the virtual prototype to simulate kinesiology.

Establishment of mathematic model: Figure 1 is the leg coordinate frame of robot. There have 12 degree of freedom with completely rotational joints. The coordinate frame 0 and 13 means the reference frame of left feet and right feet. They are just the calculate conference that more intuitive in calculation without relying on the degree of freedom. The coordinate 1-12 is the appendage coordinates of 12 joints.

The neighboring position matrix is the homogeneous coordinate matrix between neighboring coordinates. It describes the position and attitude relation between two rod pieces (Zou et al., 2007). That can be the following matrix:

\[
M_{0-13} = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & \cos \alpha_i & \sin \theta_i & \sin \alpha_i & 1 & \cos \theta_i \\
\sin \theta_i & \cos \theta_i & \cos \alpha_i & -\cos \theta_i & \sin \alpha_i & 1 & \sin \theta_i \\
0 & 0 & \cos \alpha_i & 0 & 0 & 1 & d_i
\end{bmatrix}
\]

where, \(d_i\) is the length of connecting rod. That means the shortest distance between two joint axes as the common perpendicular length. \(\alpha_i\) is the torsion angle of connecting rod. The included angle project by the extension of both sides joint axes. \(d_i\) is the translation capacity of joint \(i\). It is the discrepant distance of two joint-connected rod piece one the axes. \(\theta_i\) is the joint corner. Through the two rod piece with connected joint that along with its axes will project to one joint then will be the rolling joint. Only \(\theta_i\) is the system variable.

Then will get the right answer of kinematical equation.

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Fig. 1: Dimensional model and mathematic model

\[ M_{6,6} = M_{6,4} \begin{bmatrix} M_{3,3} & M_{3,4} \end{bmatrix} M_{4,5} \begin{bmatrix} M_{5,4} & M_{5,5} \end{bmatrix} = \begin{bmatrix} n_1 & O_1 & a_1 & p_1 \\ n_2 & O_2 & a_2 & p_2 \\ n_3 & O_3 & a_3 & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

(2)

**Monopod inverse analysis of single leg position about biped robot:** The solving that aiming at inverse kinematics problem of robot is called kinematical equation inverse (Chevallereau et al., 2008). When provide the position and gesture of robot movement in the coordinate system, it means preset each element of \( M_{6,6} \) in the position matrix of kinematical equation. Now we analyze the single leg of robot that combines with 6 matrix from \( M_{i,6} \) to \( M_{i,6} \) will get following inverse matrix:

\[
M_{6,4} = \begin{bmatrix} 
\cos \theta_3 & \sin \theta_3 & 0 & -75 \sin \theta_3 \\
-\sin \theta_3 & \cos \theta_3 & 0 & -75 \cos \theta_3 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 
\end{bmatrix} \\
M_{6,6} = \begin{bmatrix} 
\cos \theta & \sin \theta & 0 & 32 \sin \theta \\
-\sin \theta & \cos \theta & 0 & 32 \cos \theta \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 
\end{bmatrix} \\
M_{6,4} = \begin{bmatrix} 
-\cos \theta & 0 & \sin \theta & -32 \sin \theta \\
\sin \theta & 0 & \cos \theta & -32 \cos \theta \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 
\end{bmatrix} \\
M_{6,6} = \begin{bmatrix} 
0 & \sin \theta & \cos \theta & 0 \\
0 & \cos \theta & -\sin \theta & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 
\end{bmatrix} \\
M_{6,4} = \begin{bmatrix} 
0 & \sin \theta & \cos \theta & 0 \\
0 & \cos \theta & -\sin \theta & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 
\end{bmatrix} \\
M_{6,6} = \begin{bmatrix} 
-\cos \theta & 0 & \sin \theta & 0 \\
\sin \theta & 0 & \cos \theta & 0 \\
0 & 0 & 0 & 30 \\
0 & 0 & 0 & 1 
\end{bmatrix} \\
(3)

Premultiplication of formula (2) by \( M_{6,6}^{-1} \) will get

\[ M_{6,6ut}^{-1} = M_{6,6}^{-1} \]

Where:

\[
M_{6,6ut}^{-1} = \begin{bmatrix} 
-\cos \theta & 0 & \sin \theta & -32 \sin \theta \\
\sin \theta & 0 & \cos \theta & -32 \cos \theta \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 
\end{bmatrix} \\
M_{6,6}^{-1} = \begin{bmatrix} 
\cos \theta & 0 & \sin \theta & 0 \\
\sin \theta & 0 & \cos \theta & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 
\end{bmatrix} \\
M_{6,6ut}^{-1} = \begin{bmatrix} 
-\cos \theta & 0 & \sin \theta & -32 \sin \theta \\
\sin \theta & 0 & \cos \theta & -32 \cos \theta \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 
\end{bmatrix} \\
M_{6,6}^{-1} = \begin{bmatrix} 
0 & \sin \theta & \cos \theta & 0 \\
0 & \cos \theta & -\sin \theta & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 
\end{bmatrix} \\
(4)

Then get the result:

\[ \theta_1 = -\arctan \left( \frac{30a_1 p_1}{(30a_1 p_1 + 32)} \right) \]

From the similar method will get:

\[ \theta_2 = \arctan \left( \frac{30a_2 + c \cos \phi + 32 \cos \phi}{(p_2 + 30a_2)} \right) \]

\[ \theta_3 = \arctan \left( \frac{30a_3 c \sin \phi + 32 \sin \phi}{(p_3 + 30a_3)} \right) \]

\[ \theta_4 = \arctan \left( \frac{-a_4}{s_4 + c \sin \phi} \right) - (\theta_2 + \theta_3) \]

\[ \theta_5 = \arctan \left( \frac{-a_5}{s_5 + c \sin \phi} \right) \]

\[ \theta_6 = \arctan \left( \frac{-a_6}{s_6 + c \sin \phi} \right) \]

where, \( A - s \cos (2+3+4) - a_1 c (2+3+4) - a_2 c (2+3+4) - a_3 c (2+3+4) - a_4 c (2+3+4) - a_5 c (2+3+4) - a_6 c (2+3+4) \).
\[ B = c_1 n_x + s_1 n_z \]
\[ \theta_\theta = \arctan(C/D) \]

where, \( C = s_1 s (2+3+4) n_x + c (2+3+4) n_z + c s (2+3+4) n_z \)
\( D = s_1 s (2+3+4) n_z + c (2+3+4) o_z + c s (2+3+4) o_z \)

The jacobian matrix calculation of biped robot: The feet movement speed and joint speed linear transformation is define as the Jacobi Matrix of robot legs (Zhijiang et al., 2007). It is the transmission ratio of movement speed from joint space to the operation space. The kinematic equation that can make robot move is \( X = x(q) \). The \( x \) is stand for locational space, \( q \) stand for joint space. The above formula represents the relation between space and replacement of joint. In the formula, do the derivations of \( t \) (time) in the both sides will get the differential relation of \( q \) and \( x \). That is:
\[ dX = J(q) \quad dq \quad (5) \]

In the formula, \( x \) is the generalized speed of terminal in the operation space with abbreviation of operation speed. The \( q \) is the joint variable. When the joint is the rotational joint, \( q_i = \theta_i \). And the joint is the movement joint, \( q_i = \bar{d} \). The little movement of joint space can reflect in \( dq \). The position and direction of robot terminal in the operation space can show as \( X = X(q) \). \( X = X(q) \) is the function of joint variable. \( J(q) \) is the partial derivative matrix of \( 6 \times n \) dimension which also means the Jacobi Matrix about robot \( n \) degree of freedom.

Robot speed analysis: Analyze the robot speed through Jacobi Matrix. Both sides of formula (2) divide \( dt \) will get:
\[ \frac{dx}{dt} = J(q) \frac{dq}{dt} \quad \text{As well as} \quad v = \dot{X} = J(q)q \quad (6) \]

In the formula:
\[ V = \text{The generalized speed of robot terminal in the operation space} \]
\[ q = \text{The joint speed of robot joint in the joint space} \]
\[ J(q) = \text{The Jacobi Matrix that determines the relation between the speed of joint space and the operation space} \]

The element \( T_j(q) \) in the line \( i \) will decide by element \( iM_6 \). For the rotational joint from the result \( \theta_\theta, \theta_{\theta} \) there has:

\[ \text{From the comprehension, we will get: } J = [J_1(q) \quad J_2(q)] \]
\[ J_1(q) = [J_1, \cdots, J_1] \quad J_2(q) = [J_2, \cdots, J_2] \]

Gait analysis and programming based on ZMP: From the mechanical theory, we will know when the objects are in the static condition, the necessary and sufficient condition of balance need the shape about the center of gravity to place in the bearing surface. Generalized speaking, when the objects are moving, the necessary and sufficient condition of balance need the extended line of resultant force that combine with gravity and inertia force should through the bearing surface. This cross point of extended line and bearing surface is ZMP. When the object is in the static condition, there will have no effect of inertia force with the unification of both. Therefore, the former is one specialization of the latter. The pressure center of ZMP and the counter-force of feet from the ground will be coincident when the biped robot is in the movement balance. The static area is the projection on the plane that convex area combines with feet support. During the single feet support, the static area projection is the biggest area which constitute by touch point of frame feet (Hidezi, 2007).

The necessary and sufficient condition for humanoid robot to keep balance is the ZMP need to fall in the static area throughout the walking. The represent formula is:

\[ X_{ZMP} = \sum_{i=1}^{n} m_i \left( z_i + g - \frac{m_i z_i}{V_{z}} \right) \]
\[ Y_{ZMP} = \sum_{i=1}^{n} m_i \left( y_i + g - \frac{m_i y_i}{V_{y}} \right) \]

\[ x, y, z = \text{Barycentric coordinates in each part of robot body} \]
\[ m_i = \text{weight of each part about the robot body} \]
\[ V_{X}, V_{Y} = \text{joint thrust} \]

Formula (8) is the moment zero point formula. If the practical ground reaction point that act from robot frame feet can coincide with ideal ZMP and fall in the bearing surface, the robot will has no turning torque. After that,
we can deduce the robot is in the static walking. Analyze formula (8) we will know, if the robot is in the static condition during the walking, it is in the static balance at every moment (Nishiwaki et al., 2002). Static walking is the basic simply walking condition with low speed. When the robot is in the low speed and static walking, we can ignore the inertia force. At this time, the ZMP calculation formula can simplify to the following:

\[
\begin{align*}
X_{2,0} &= \frac{\sum m_i g_i x_i}{\sum m_i g} \\
Y_{2,0} &= \frac{\sum m_i g_i y_i}{\sum m_i g}
\end{align*}
\]  

(9)

From the analysis, the static walking will be stable if the center of gravity can fall over the stability area during the robot walking.

In the search of this article, the robot walking is through ankle deflection of the joint then to achieve the shuffling about center of gravity. The ideal ZMP curve of robot during the entire walking period is Fig. 2. The ZMP curve is similar with the conversion of S.

**Simulation experiment and resolve analysis:** ADAMS is the present perfect software in dynamical simulation. It is combines with modeling function, simulation solving function and animation that will do mechanical system operation, dynamics analysis and simulation as the common design software of visual prototype.

From the above biped robot dynamics and dynamical model, the humanoid robot walking model will be created through the simulation of ADAMS software. Figure 3 is the presentation.

We can see from the walking movement simulation through the ADAMS, biped robot can walk stable. Moreover, compare with Fig. 4 and 2, the simulation is very close to the ideal ZMP curve that there only has some inertia impact when alter the left feet and right feet. From the simulation of Fig. 5 and 6, move forward a single step analysis of the angular speed among each joints. There has not much great impact to the electrical machine during the walking period of biped robot. The curve is tactful and gentle without tip or wave means the situation almost fit with the walking characteristics of human beings with stable simulation.

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**Fig. 2: Ideal ZMP Curve**

**Fig. 3: Simulation of Biped Robot Walking**

**Fig. 4: Practical Center of Gravity about ZMP Curve**

**Fig. 5: Angular Speed of Right Joints** (Motion 1 is right ankle joint rolling, Motion 2 is right ankle joint pitching, Motion 3 is right knee joint pitching, Motion 4 is right hip joint pitching, Motion 5 is right hip joint rolling, Motion 6 is right hip joint deflection)
CONCLUSION

Present study takes the advantage of D-H method to build mathematic model of biped robot movement. From the joint inverse then get Jacobi Matrix. Moreover, ensure the robot stable walking model from ZMP rule and build virtual prototype through ADAMS to simulate. The simulation achieves robot movement and the ZMP curve is very close to the ideal ZMP curve. Angular speed curve transition is very tactful and gently of each joint. That verifies the effectiveness and exactness of modeling.

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REFERENCES


