Chaotic Recurrence Analysis of Oil-Gas-Water Three-phase Flow in Vertical Upward Pipe

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Abstract: In this study, the conductance fluctuating signals of oil/gas/water were analyzed using nonlinear chaotic recurrence, were obtained four kinds of recurrence quantification indicator, recurrence rate, determinism, average length of diagonal segment, entropy which got the transition law of the oil/gas/water three-phase flow patterns in vertical upward pipe. Specific process was: Firstly the C-C algorithm was used to determine phase space embedding dimension and time delay of nonlinear time series, then the method of using the time series data generated by the Lorenz equation to verify the sensitivity of recurrence quantification analysis was presented, finally the chaotic recurrence analysis method was used to identify three-phase flow patterns. The results show that: the texture of chaotic recurrence plot can reflect the oil/gas/water three-phase flow pattern evolution, the recurrence quantification indicator with the phase flow are more sensitive.

Key words: Chaotic recurrence plot, chaotic recurrence quantification, oil-gas-water three-phase flow, flow pattern

INTRODUCTION

The extensive existence of oil-gas-water three-phase flow is obvious in the chemical, nuclear, oil, metallurgical industry. And because of the complex interaction and the relative motion property among the different phases, it is difficult to acquaint flow pattern transition nonlinear dynamic mechanism clearly. What is more, response of the well logging sensor and explanation of the observational material severely depend on the understanding of the flow property. Different flow pattern has a completely different flow law. Phase resistance characteristics, heat transferring law and wall resistance characteristics are different in different flow pattern. Therefore, it has a very important significance to research on flow pattern transition nonlinear dynamic mechanism (Ningde and Wanpeng, 2006).

Researcher discovered and researched the chaotic recursive of status in the nature long time ago but they lack appropriate measures of analyzing the dynamics model of recurrence in high dimensional space (Bochuan et al., 2002; Zhong et al., 2009). Eckman (Eckman et al., 1987) etc., did not present a new graphical tool for measuring the time constancy of dynamical systems until nineteen eighty seven, this tool is recurrence plot. After that Zhilut and Webber (1992) presented a method of nonlinear data analysis for the investigation of dynamical systems. And this method is recurrence quantification analysis. It quantifies the number and duration of recurrences of a dynamical system presented by its phase space trajectory. And in the recent years this recurrence quantification analysis is widely applied in the physical and biomedical field (JiKang and Zhihuai, 2002; Yang et al., 2009) and it becomes a fashionable tool to analyze the Chaotic Time Series.

This study researches the recurrence structure and recurrence characteristic quantity of the oil-gas-water three-phase flow in vertical upward pipe under different flows and investigates the low of the recurrence structure and recurrence characteristic quantity fluctuate with different flow regimes. Compare and analysis indicate that the texture of recurrence plot can quickly identify the flow pattern, recurrence quantitative indicators can be better quantitative analysis for the change of phase flow, chaotic recurrence characteristics analysis have the fast and intuitive characteristics, so a kind of efficacious auxiliary tool for oil-gas-water three-phase flow regime identification was provided.

RECURRENT PLOT AND RECURRENT QUANTIFICATION ANALYSIS

Recurrence plot: Recurrence Plot (RP) analysis is an effective tool to judge whether there is some deterministic laws in data, is consisted of some of the points scattered in the square (Jin et al., 2007). In the chaotic time series analyses, phase space reconstruction is absolutely

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necessary. In this study, the near-optimum parameters of embedding dimension $m$ and delay time $\tau$ are estimated by means of C-C algorithm (Jin-Hu et al., 2001). The recurrence plot algorithm is presented as followed:

**Step 1:** The phase space vector $X_i$ of conductance signal time series $X = (x_0, x_1, \ldots, x_n)$ is reconstructed:

$$X_i = (x_{i}, x_{i+\tau}, \ldots, x_{i+(m-1)\tau})$$

**Step 2:** The distance between any two points in the phase space is calculated:

$$d_i = |X_i - X_j|, i, j = 1, \ldots, M$$

where, $M$ means the amount of the phase point in the reconstruction phase space:

$$M = n-(m-1)\tau$$

**Step 3:** The recurrence value of time series is calculated:

$$R_{ij} = Q(e-d_{ij}), i, j = 1, \ldots, M$$

where, $e = \alpha \text{ std}(X)$ means the threshold. The std $(X)$ means the standard deviation of original time series, $\alpha$ means threshold coefficient. At present, it is popular to choose $\alpha$ by means of empirical method. As usual, $\alpha = 0.1 - 0.25$. $Q(X)$ means the Heaviside function:

$$Q(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

**Step 4:** Paint the recurrence plot. Threshold can define a ball by $X_i$ as the center, if $X_j$ fall into the ball, indicating that the state is close to $X_i$, that is $R_{ij} = 1$. This study delimits the position of recurrence matrix $R_{ij} = 1$ as the original recurrence point and draws point at the corresponding position of coordinate $M \times M$ in the flat, that means the phase space of time series has been reconstructed, so we obtain the recurrence plot. Because $|X_i - X_j| = 0$, there is always a main diagonal in RP

**Recurrence quantification analysis:** Zbilut and Webber (1992) proposed the Recurrence Quantification Analysis (RQA) based on the main diagonal structures which analyzed the graphic details structure based on the recurrence plot and extracted structural features, thus achieved the purpose of qualitative analysis for the time series. This study investigates the following four kinds of recursive quantitative indicators:

**Recurrence rate (RR):** The Recurrence Rate means the percentage of all the recurrence points among covering points of the plot and it characterizes the proportion of congenial phase space points between the congenial track in $m$-dimensional space:

$$RR = \frac{1}{N^2} \sum_{i \neq j} R_{ij}$$

**Determinism (DET):** The determinism means the percentage of recurrence points which composition of main diagonal segment among total recurrence points and it characterizes proportion of deterministic signal among total recurrence points:

$$DET = \frac{\sum_{i \neq j} \frac{p(i)}{2}}{\sum_{i \neq j} R_{ij}}$$

where, $P(i)$ means amount of the segment which length is $i$ and only if $i$ is bigger than the pre-set lower limit $I_{\text{min}}$ it begins to count, at the same time, the recurrence point of main diagonal segment does not need to be counted. Generally $I_{\text{min}}$ choose to not less than 2 integer, the overlarge $I_{\text{min}}$ make expression of DET become worse. The isolated recurrence points in recurrence plot and the recurrence points that are organized to form continuous diagonal line are distinguish from DET.

**Average length of diagonal segment ($L_{\text{mean}}$):** The $L_{\text{mean}}$ means the weighted average length of diagonal segment and it characterizes the average cycle time of this system, the main diagonal is not included:

$$L_{\text{mean}} = \frac{\sum_{i \geq 1} i \times \frac{p(i)}{\sum_{i \geq I_{\text{min}}} p(i)}}{\sum_{i \geq I_{\text{min}}} p(i)}$$

**Entropy (ENTR):** The Shannon entropy of the length of the main diagonal segment characterizes the complicated level of deterministic construct of this system:

$$\text{ENTR} = -\sum_{i \geq I_{\text{min}}} P_i \times \ln P_i$$

**RECURSIVE QUANTIFICATION ANALYSIS OF LORENZ CHAOTIC TIME SERIES**

Lorenz (1963) U.S. Weather meteorologist, found that the deterministic equations appeared the chaotic phenomenon while he studied the weather report. Lorenz equations:
Equation, $a$, $r$, $b$ are the variable parameters, when taken $a = 10$, $r = 28$, $b = 8/3$, the equation is a typical chaotic strange attractors (Fig. 1).

\[
\begin{align*}
\dot{x} &= -a(x-y) \\
\dot{y} &= -xz + rx - y \\
\dot{z} &= xy - bz 
\end{align*}
\] (10)

The characteristics of the equation can be changed by changing the parameters of Lorenz equation, so we can use $x$-data of Lorenz under solutions of different $r$ values to draw recurrence plots (Fig. 2a-d). Recurrence plot in Fig. 2 shows that following the change with $r$, the difference of the recurrence plot is very obvious. When $r = 23$, the solution of the equations is convergent, with recurrence plot showing long lines parallel to the main diagonal line. When $r \geq 28$, the solutions of the equations appear significant chaotic, with recurrence plot showing that the lines parallel to the main diagonal line have no regulation and become short, sparse following $r$ increases.

From Fig. 3, respectively RR (a), DET (b), L (c) and ENTR (d), can be seen that the four RQA features are presented downward tendency along with the increase of $r$ which is consistent with the variation tendency of the solution of Loren equations in recurrence plot, indicating that the chaotic fluctuations brought the changes of $r$ values are also able to reflect by recurrence quantification analysis method and performing the sensitivity of the recurrence quantification.

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**Fig. 1:** Lorenz chaotic strange attractor

**Fig. 2 (a-d):** Recurrence plots of $x$-data of lorenz
Chaotic recurrence analysis of oil-gas-water three phase flow regime

Experimental system and data acquisition: The experiments were performed in oil-gas-water three-phase flow loop which consists of a plexiglass test pipe with inside diameter is 125 mm, an oil-water flow stabilizing tower with height is 45 m, an oil tank and a water tank, the oil water separating tanks, the flowrate adjusting instruments on the ground and these can be shown schematically in Fig. 4.

For the water volume fraction measurement, the testing flow rate $Q$ of oil-water mixtures were 20–60 m$^3$ d$^{-1}$ with 10 m$^3$ d$^{-1}$ increment, the testing flow rate $Q$ of gas was 0–50 m$^3$ d$^{-1}$ with 10 m$^3$ d$^{-1}$ increment and the standard water volume fraction was keeping 75%.

Recurrence quantification analysis of flow regime: The method of flow regime identification of oil-gas-water three-phase flow in vertical upward pipe can be seen in the literature (Li and Guo, 2010). In this study, by means of C–C algorithm, the near-optimum parameters of embedding dimension $m$ and delay time $\tau$ are estimated. Compare and analysis indicate that $m$ is 2–4 and $\tau$ is 1–14. After a lot of experiments, this paper discovers that the magnitude of $m$ has something to do with rarefaction of recurrence points but have nothing to do with topology structure of recurrence plot. So in order to observer the recurrence property of power system in the same dimensional space, in this paper $m = 3$ and $\tau$ can be changed and it sets $\alpha = 0.25$, the length of time series is 1024.

Texture of recurrence plot: The inertial power of gas and gravity of liquid are the main control factors of flow pattern transition and the viscosity of oil-water mixture can also affect flow pattern transition. With the flow rate of oil increased, the apparent viscosity of liquid is increased and also the friction drag coefficient of the gas-liquid phase interface is increased and that would change the absolute roughness of gas-liquid phase interface and at last the flow rate of liquid which needed by flow pattern transition would increase with the flow rate of oil increased. From Fig. 5, when the gas flow rate is constant, with the gradual increase in the total flow of liquid, the decrease in gas-liquid flow ratio, the texture of
Fig. 4: The schematic diagram of oil-gas-water flow loop

Fig. 5 (a-d): Texture of recurrence plot for three-phase flow pattern

the lines along the diagonal direction become gradually shorter in recurrence plot, the degree of aggregation of the recurrence point gradually become sparse. It’s also consistent with the flow patterns of the bubbly flow, bubble flow-slug flow and slug flow.

Water-based dispersed bubble flow is shown in Fig. 5a, it can be seen that there is much small bubble distributed in the pipe, at the same time the forming and disappearing of bubbles is appeared. M means the amount of the phase point in the reconstruction phase space. As a result of the minor power of the small bubble, the trajectory of the bubble group is random variable, overall the signal generated by dispersed bubble flow resembling random noise, as a result, the texture of
recurrence plot for dispersed bubble flow resembling uniform, dispersed and isolated dot structure characteristic.

Water-based churn flow is shown as Fig. 5b and d, with proportion of the gas increased, the agitation between gas and liquid leads to the flow pattern transition. The texture of recurrence plot is conspicuous along with the diagonal and that resembling moving characteristic of slug flow to some extent, the black rectangle churn texture is dimly visible.

Water-based slug flow is shown as Fig. 5c, it can be seen that the cyclical alternate diversification of gas bound and liquid bound existed obviously. Some small bubble accumulate churn big bubble, its diameter is closed to the pipe's internal diameter and its length is closed to the pipe's diameter, the big bubble is separated by liquid, there is no aggregation dispersed bubble flow between gas bound, oil droplet exists in the continuous water phase in the form of bubble. As a result, the conductance noisy signals of slug flow have cyclical characteristic, the recurrence plot is conspicuous along with the diagonal and the black rectangle churn texture is dimly visible.

**Recurrence quantification indicator:** This study analyzed the recurrence quantification indicators of the oil-gas-water three-phase flow noisy signals which collected by conductance sensor under different flow and try to find its variation. As shown in Table 1, take signal '20753' for an example, '20' means the flow rate of oil-water two-phase, '75' means the standard water volume fraction and '3' means the flow rate of gas, it can be seen that, the recurrence characteristic variable under high flow rate is bigger than the variable under low flow rate and there exists obvious differences of recurrence characteristic variable under three kinds of flow pattern.

Specifically speaking, as to the recurrence rate, under water-based dispersed bubble flow it is 0.0171-0.0312, under water-based dispersed churn flow it is 0.0383-0.0492, under water-based dispersed slug flow it is 0.0746-0.1281. In the process of Water-based dispersed bubble flow transform to Water-based dispersed slug flow, no matter recurrence rate or the deterministic, the average length of diagonal segment or the entropy, they all tend to increase totally. And this shows that the complexity of the dynamics characteristic of the water-based dispersed slug flow is consistent with the analysis mentioned above.

When the flowrate of gas is fixed, the entropy decreased continuously as the flow rate of liquid increased. That means the rate of system information loss is lower, the degree of chaotic is lower and it also means the dynamics characteristic of the flow is changing relatively simple. But, when the flowrate of liquid is fixed, the entropy increased continuously as the flow rate of liquid increased. That means the dynamics characteristic of the flow is changing more complex. And this conclusion is consistent with the analysis mentioned above.

Overall, the chaotic recurrence structure could preferably characterize flow pattern transition and the recurrence quantification provides a new indicator for analyzing oil-gas-water three-phase flow dynamic transition mechanism.

**CONCLUSIONS**

The method of using the time series data generated by the Lorenz equation to verify the sensitivity of recurrence quantification analysis was presented and this method was used to identify oil/gas/water three-phase flow patterns, the following conclusions:

- The texture of recurrence plot for oil-gas-water three-phase flow water-based dispersed bubble flow resembling uniform, dispersed and isolated dot structure characteristic; the texture of recurrence plot is conspicuous along with the diagonal and that resembling moving characteristic of slug flow to some extent, the black rectangle churn texture is dimly visible; the recurrence plot is conspicuous along with the diagonal and the black rectangle churn texture is dimly visible. The chaotic recurrence plot has the fast and intuitionial characteristics and it provides a kind of efficacious auxiliary tool for oil-gas-water three-phase flow regime identification.
In conclusion, the four recurrence quantification indicators including recurrence, deterministic, average length of diagonal segment and entropy, they all fluctuate with the flowrate changing. And also they all obey the objective law, so the conclusions can be drawn that recurrence quantification provides efficacious judgment basis for three-phase flow regime identification.

REFERENCES


