Performance Evaluation of M-APSK Modulation in the Nonlinearly Distorted LTE Uplink

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ABSTRACT: Single Carrier Frequency Division Multiple Access (SC-FDMA) scheme has been considered to be promising technology for the uplink of the 4G mobile communication systems. In spite of the preceding application, SC-FDMA signal using the standard M-QAM base-band modulation scheme still suffers from its high envelope fluctuation resulting in a nonlinear distortion of the transmitted signals due to the transmitter High Power Amplifier (HPA). In order to improve the SC-FDMA performance, the M-ary Amplitude Phase Shift Keying (M-APSK) is proposed as a base-band modulation scheme for SC-FDMA in this study. The application of M-APSK for SC-FDMA has been motivated by its high spectrum efficiency and at the same time smaller signal fluctuation of the M-APSK signals than that of M-QAM. The performance properties of M-APSK and M-QAM based SC-FDMA transmission scheme will be illustrated using numerical results. The obtained results will show very clearly that the M-APSK based SC-FDMA transmission can provide the preferable performance to M-QAM SC-FDMA if the nonlinear operation of HPA has to be assumed.

KEY WORDS: APSK, LTE, nonlinear amplification, QAM

INTRODUCTION

The Third Generation Partnership Project (3GPP) is the standard-developing body that specifies the third generation of mobile communication systems and evolutions therefore, in Release 8 an evolved universal terrestrial radio access network termed as Long Term Evolution (LTE) was considered. It is assumed that LTE and its modifications will form the standard of mobile communication systems that can be exploited by the telecommunication industry up to 2020s (Dahlman and Parkvall, 2007). In LTE no backward compatibility with WCDMA and HSPA is required and therefore, it exploits the new technologies that were not previously considered in mobile systems.

LTE and LTE ADVANCED (Dahlman and Parkvall, 2007) utilize in the downlink the Orthogonal Frequency Division Multiplexing (OFDM) scheme, which is in general extremely robust against multipath propagation and allows using very simple one-tap channel equalization (Arioua et al., 2012; Zahrani, 2010). On the other hand, OFDM transmitted signal is complex Gaussian distributed and thus, peak-to-average power ratio (PAPR) of OFDM signal is notably high (Al-Kebisi, 2008; Latif and Godar, 2008; Seddiki et al., 2008). This fact in combination with highly nonlinear characteristics of the Power Amplifiers (PA) requires the additional hardware and computational requirements of the signal processing/hardware considered therein.

Therefore, in order to reduce the effect of nonlinear amplification, 3GPP introduced Single Carrier Frequency Division Multiple Access (SC-FDMA) scheme in the LTE uplink (Myung and Goodman, 2008). SC-FDMA is characterized by the significant lower PAPR compared to that of OFDM, nevertheless, it is still higher compared to the conventional single carrier systems (Priyanto and Codina, 2007).

The analysis of nonlinearities effects in OFDM has been addressed in the literature. Moreover, many transmitter and receiver strategies to improve the OFDM performance have been proposed. However, the impact of the nonlinear amplification on the SC-FDMA transmitted signal requires further research and there exist still a lot of open issues. In this study, we will briefly address the effect of oversampling and the frequency resources selection on the overall performance of SC-FDMA systems inflicted by the nonlinearities. We will show that the PAPR characteristic of two users transmitting at different frequency positions using the same number of subcarriers, will be the same. Moreover, from the
presented analysis we will point out the strategies to be possibly considered to increase the robustness of LTE uplink towards nonlinear effects.

Up to this date, several strategies capable of mitigating the nonlinear effects have been proposed. Frequently used solution in the transmitter include application of the companding technique on the transmitted SC-FDMA signal (El-Samie et al., 2010), Wavelet Transformation application (Al-Kamali et al., 2010), or alternatively using efficient Repeated Clipping and Filtering (RCF) algorithm (Alfishaidy et al., 2011). Another possible solution leading to PAPR reduction is adopting conventional OFDM PAPR reduction technique (e.g., partial transmit sequences, subblock mapping, etc.) (Suzuki et al., 2008). Very promising solution for the reduction of the nonlinearity effect due to HPA is to use the nonlinear detection technique in the receiver. Gazda and Deunal (2011) and Gazda and Drotar (2009) proposed nonlinear SC-FDMA maximum likelihood detector with reduced complexity has been proposed. The main drawback of all these techniques is the computational complexity increase, which might not be feasible in some scenarios.

In the core part of this study, based on results of the presented analysis, we propose to use the baseband signal constellation optimization to further reduce the signal envelope fluctuation. Here, the signal constellation optimization reflects using the M-ary Amplitude Phase Shift Keying (M-APSK) instead of conventional originally considered M-QAM modulation scheme. Basically, the idea presented in this paper is inspired by the recent progress in the satellite communications. DVB-S2 represents the new standard, which is designed as a successor for the popular DVB-S digital television broadcast standard (De Gaudenzio et al., 2006). The recent technical enhancements in satellite broadcasting require the exploitation of highly efficient power and spectrally modulation schemes designed to operate over the nonlinear satellite channel environment. In this regard, APSK represents an attractive modulation scheme for digital transmission over nonlinear satellite channels. Qatawneh and Rida (2003) and Qatawneh (2005a, b) also indicate that other feasible properties of APSK modulated signals might be found and used with advantage.

It will be showed that APSK SC-FDMA transmission scheme performs well and overcome the performance of the conventional and standardized QAM SC-FDMA system in the both investigated criteria (Bit Error Rate (BER) and out-of-band radiation), especially when strong nonlinear distortion due to PA is present. Moreover, envelope fluctuation of {QAM, APSK} SC-FDMA signal will also be later statistically investigated by using well-known indicators, PAPR and Cubic Metric (CM).

It should be noted that the performance improvement here is achieved with no further computation complexity in the transmitter/receiver, since {QAM, APSK} signal detection requirements are roughly the same. This fact is of special relevance especially in the LTE uplink, in order to keep the computational complexity in the mobile terminal at the tolerant level.

**THE SC-FDMA SIGNAL**

In SC-FDMA, a block of N data symbols from some modulation alphabet, such as QPSK or 16-QAM, is first applied to a size-N Discrete Fourier Transform (DFT). The output of the DFT is then applied to consecutive inputs of a size-M inverse DFT (IDFT), where M = N and the unused inputs of the IDFT are set to zero. Note that the SC-FDMA signal is generated by first precoding the data symbols by means of the DFT operation and then applying it to consecutive subcarriers of an OFDM system. Since the DFT precoding can alternatively be seen as spreading in the frequency domain, the SC-FDMA transmission scheme is also known as DFT-spread OFDM (DFTS-OFDM).

Let \( a_k, k = 0, \ldots, N-1 \) be the complex data symbols, then the signal at the output of DFT precoder can be expressed as:

\[
S_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{-j2\pi nk/N}, \quad k = 0, \ldots, N-1
\]  

(1)

In the OFDM block the N pre-coded data symbols are transmitted over N consecutive subcarriers. Consider a baseband OFDM symbol \( s(t) \) defined over the time interval \( t \in [0; T_s] \):

\[
s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi nkT_s/2^N}, \quad k_0 \text{ is the position of the first assigned subcarrier.}
\]  

(2)

where, \( k_0 \) is the position of the first assigned subcarrier. For the sake of brevity and without loss of generality we assume \( k_0 = 0 \). If \( s(t) \) is sampled at a frequency \( L/\Delta T \), where, \( L - M/N \) is the oversampling factor and \( N/T_s \) is the Nyquist rate, the signal at the output of the SC-FDMA modulator is:

\[
\tilde{s}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi knM}, \quad n = 0, \ldots, M-1
\]  

(3)

The expression above can be computed by means of a length-M scaled IDFT. Therefore, from (1) and (3) the SC-FDMA signal can be expressed as:
\[ s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left( \sum_{m=0}^{\frac{N-1}{2}} h_k e^{j2\pi k \frac{m}{N}} \right) e^{j2\pi n k \frac{m}{N}} \]
\[ = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left( \sum_{m=0}^{\frac{N-1}{2}} h_k e^{j2\pi k \frac{m}{N}} \right) e^{j2\pi n k \frac{m}{N}} \]
\[ = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{m=0}^{\frac{N-1}{2}} h_k e^{j2\pi k \frac{m}{N}} e^{j2\pi n k \frac{m}{N}} \]

Note that for the mapping schemes used in LTE uplink (-1/√d, is always a point of the constellation, therefore, we denote it as d').

Now, let us analyze the SC-FDMA signal at sample position multiple of the spreading factor. If \( n = Lr \), the time domain signal in (4) reduces to:

\[ s_{Lr} = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{m=0}^{\frac{N-1}{2}} e^{j2\pi k \frac{m}{N}} a_m = \frac{1}{N} \left( \sum_{m=0}^{\frac{N-1}{2}} a_m e^{j2\pi k \frac{m}{N}} + \sum_{m=0}^{\frac{N-1}{2}} a_m e^{j2\pi r k \frac{m}{N}} \right) = s_r \]

which means that the \( Lr \)-th sample of the time domain SC-FDMA signal is equal to the data symbol \( a_r \). The samples at positions \( n = Lr \) describe the transition of the time-domain signal between the values \( a_m \) and \( a_{m+1} \). The above formulation was done assuming that the spreading factor is an integer value. If \( L \neq Z \) then the SC-FDMA signal will take values equal to \( d' \) at time instants different than the sampling instants and therefore, it will only occur in the analog signal.

As an example, Fig. 1 shows the instantaneous power of an oversampled SC-FDMA signal with \( N = 64 \) and QPSK mapping. The envelope at multiples of \( T_s/N \) is marked with the dots. Since, QPSK mapping is used, the envelope at these normalized times is always equal to 1. However, between those samples the signal fluctuates. To understand why this happens, we must consider the way oversampling is implemented. In SC-FDMA, oversampling is done by zero padding the complex pre-coded baseband signal before applying IFFT. Notice that this means that no discrete frequency outside the transmission bandwidth is allowed. Therefore, if an abrupt change in the value of contiguous data symbols occurs, since a fast transition in the time domain samples is not possible because of the spectral properties imposed by frequency-domain oversampling, it can only be solved by showing a smooth transition at expense of the peak values. This is similar to the effect observed when a rectangular pulse is low-pass filtered. On the other hand, if we are allowed to perform some spectral outgrowth we will be able to reduce the fluctuations of the transmitted signal since faster transitions will be possible. This suggests that a good strategy to reduce the envelope fluctuation of the SC-FDMA signal is by using some out-of-band subcarriers with appropriate complex amplitude. This is somehow the principle of the spectrum shaping technique discussed by Hanzo (2003), where the signal is periodically expanded in the frequency domain and then multiplied with some spectrum shaping function, such as root raised cosine function.

**EFFECTS OF OVERSAMPLING AND FREQUENCY RESOURCES USAGE**

To be able to further investigate the effects of the baseband modulation scheme selection on transmitted analog SC-FDMA signal, there is a need to clarify the effects of oversampling and subcarrier selection on the overall SC-FDMA system performance. Let us first consider the effect of oversampling in the SC-FDMA signal. In Fig. 1, we observed that the envelope of an oversampled SC-FDMA signal differs from that of a non-oversampled, thus, affecting the PAPR and Cubic Metric (CM) properties of the SC-FDMA signal. We should take into account that, in practice, the signal undergoes nonlinear distortion in analog domain. Therefore, in order to better approximate PAPR and CM of the analog signal, they must be computed by using oversampling. Extensive simulations of the probabilistic distribution of the PAPR and CM of each SC-FDMA symbol using the different configurations and oversampling factors were done. It was observed that, as we expected there is a difference between oversampled and non-oversampled signals. However, the oversampling factor \( L = 2 \) is enough to compute the analog signal metrics with sufficient accuracy. Note that from the above discussion it follows that, as long as \( L=2 \), PAPR and CM of SC-FDMA signal employing a pre-coder size \( N \) is same regardless of the size of the IFFT.

The next issue we investigate is the effect of the frequency resources allocation. The expression of the SC-FDMA signal in (4) was derived assuming that the subcarriers were allocated at consecutive positions starting from 0. In general, the first subcarrier is placed at

![Fig. 1: Envelope of an oversampled signal with N = 64, QPSK](image-url)
position $k_0 \neq 0$. In such case, the expression of the SC-FDMA signal is found to be $s_n(k_0) = s_{0k}^{exp} \cos(\phi_n)$ for $n = 0, ..., M-1$. Thus, it can be easily shown that PAPR ($s^{(k_0)} = \text{PAPR}(s^{(0)})$ and that CM ($s^{(k_0)} = \text{CM}(s^{(0)})$). As a result, we can affirm that in LTE uplink the PAPR and CM characteristic of two users using the same number of resource blocks and modulation scheme but transmitting at different frequency positions, will be the same. For the sake of clarity, in the rest parts of the paper, we will assume the user transmitting at consecutive positions starting from 0.

The given discussion analyzes the SC-FDMA signal structure, the effects of oversampling and frequency resource usage. The results of the analysis will be used with advantage in the following section where we demonstrate the positive effects of the baseband modulation scheme selection on the system performance inflicted by the nonlinearities.

**APSK Modulation Scheme Impact on SC-FDMA PAPR Performance**

Let us note that due to the specific SC-FDMA signal generation, the data symbols at the input of DFT spreader periodically occur in the transmitted SC-FDMA signal. However, the signal between these samples fluctuates. The effect of the fluctuation due to the IFFT interpolation inherently increases PAPR. In general, PAPR in this case could be decreased by the reduction of the normalized power levels in between IFFT interpolates the resultant signal. The promising solution seems to be the application of APSK modulation, which is spectrally and power efficient baseband modulation scheme.

M-APSK constellations are composed of $n$ concentric rings. Each ring of constellation diagram has uniformly spaced phase shift points. The signal constellation points $\theta$ are complex numbers, drawn from the set (De Gaudenzi et al., 2006):

$$X = \begin{bmatrix}
\tau_1 \exp\left[j(2\pi/n_i)i + \theta_1\right] & i = 0, ..., n_1 - 1 \\
\tau_2 \exp\left[j(2\pi/n_2)i + \theta_2\right] & i = 0, ..., n_2 - 1 \\
... & ...
\end{bmatrix}$$

$$...$$

$$\tau_k \exp\left[j(2\pi/n_k)i + \theta_{nk}\right] & i = 0, ..., n_k - 1$$

where, $n_k$, $\tau$, and $\theta$, denote the number of constellation points, the radius and relative phase shift, respectively. In the case of the optimum M-APSK, its parameters $n_k$, $\tau$, and $\theta$, are designed using a suitable optimization criterion. Here we have selected the method introduced in (Liolis et al., 2009), where the optimization criteria is based on the minimum Euclidean maximization and the mutual information maximization for Additive White Gaussian Noise (AWGN) and nonlinear channels.

Typical examples of the 16-QAM (64-QAM) and 4+12 (referred as 16)-APSK (4+12=20+28, referred as 64)-APSK signal constellations are given in Fig. 2. The notable difference between these particular constellation schemes is that while in 16-QAM the constellation symbols occupy three constellation rings (i.e., $R = 3$), in the latter example for 16-APSK, $R = 2$. Note that this analysis might be easily extended to higher constellation examples (e.g., 64-QAM vs. 64-APSK) with the same result regarding the M-\{QAM, APSK\} constellation ring numbers.

Taking into account (4) and the particular characteristics of M-\{QAM, APSK\} signal constellations, one would expect that the PAPR characteristic of M-APSK SC-FDMA signal might be potentially lower than that of M-QAM SC-FDMA. To illustrate this reasonable assumption, Fig. 3 shows the instantaneous power of an oversampled SC-FDMA signals ($L = 8$), with $N = 12$ and 16-\{QAM, APSK\} mapping. The envelope at multiples of $T/N$ is again marked with dots. As it can be seen, in both investigated cases, the normalized time instances $T/N$ takes the values of normalized constellation rings. However, the signal in between again fluctuates. Due to the specific M-APSK signal constellation with lower number of magnitude power levels (2 in case 16-APSK vs. 3 in case of 16-QAM etc.), PAPR of 16-APSK SC-FDMA is substantially lower compared to the latter case.

To verify the positive effects of APSK application on the PAPR reduction, extensive simulations with various occupied bandwidth have been performed, however, since the same behavior has been observed, for the sake of clarity only the resource block of size $N = 12$ is plotted. Note that resource block of size $N = 12$ is used in the LTE uplink and thus, is of special relevance here. Fig. 4 shows the distribution of PAPR represented by Complementary Distribution Function (CCDF) of SC-FDMA signals using \{16, 64\} \{QAM, APSK\} modulation scheme. As it can be appreciated in both investigated cases, the PAPR reduction achieved by using APSK modulation scheme is notable, nevertheless for 64 modulation level case, the PAPR reduction is reduced. To provide more general analysis from nonlinear distortion point of view, the CM distribution of \{16, 64\} \{QAM, APSK\} SC-FDMA signal is given in Fig. 5. Here, we can again observe the same conclusion, CM improvement when using APSK baseband modulation scheme is observable.
Fig. 2: QAM, APSK signal constellation diagram; (a) 16-APSK, (b) 16-AM, (c) 64-APSK and (d) 64-QAM

Fig. 3: Envelope of an oversampled signal with $N = 64$; (a) 16-APSK and (b) 16-QAM

However, note that CCDF of the PAPR (CM) distribution is not the relevant performance figure of merit, it just shows the distribution of PAPR (CM) varying in time. BER performance, Error Vector Magnitude (EVM) and out-of-band radiation are commonly used indicators of the system performance and therefore, we do analyze them in the following section devoted to the comparison of {QAM, APSK} SC-FDMA system performance.
NUMERICAL EVALUATION

Here, PAPR reduction capabilities of M-APSK modulation will be illustrated. The subcarrier spacing, cyclic prefix and the length of the resource block (N = 12) are set up according to the 3GPP specification. To evaluate the nonlinear effects in SC-FDMA, soft limiter of PA is assumed here. Soft limiter model is defined by the following amplitude-to-amplitude modulation (am/am) and amplitude-to-phase modulation (am/pm) characteristics:

\[ G(u) = \begin{cases} 0 & \text{if } u \leq 1 \\ \Phi(u) = 0 & \text{otherwise} \end{cases} \]  

(7)

The soft limiter is used to model the case when pre-distortion is done at the transmitter. The operating point of the nonlinearity is defined by the so called input back-off (IBO) which corresponds to the ratio between the saturated and average input powers.

Figure 6 shows EVM of the constellation at the output of receiver. Let \( \alpha_i \) and \( \alpha_i \) denote the constellation point at the output of the receiver and the ideally received constellation point, respectively, EVM is computed as:

\[ \text{EVM} = 10 \log_{10} \left( \frac{E}{N_0} \right) \left( \frac{|\alpha_i - \alpha_i'|}{|\alpha_i|} \right) \]  

(8)

where, \( P_{\text{ref}} \) is the power of the outmost ideal constellation point. As it can be seen, a noticeable EVM reduction with respect to \{16, 64\}-APSK modulation scheme is achieved at low and moderate IBO.

In Fig. 7-8, BER performance of SC-FDMA system is showed. Both linear and nonlinear scenarios are considered. As can be seen from these figures, when no nonlinear source is present, \{16, 64\} QAM provides slightly better performance. This can be explained by the higher minimum Euclidean distance between the neighbouring symbols in rectangular QAM lattice specially compared to the minimum Euclidean distance of the outer M-APSK constellation points (Hanzo, 2003). However, when the strong source of the nonlinearity is

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Fig. 5: Analog CM of the SC-FDMA system, \{16, 64\}-\{QAM, APSK\}

Fig. 6: EVM reduction of \{16, 64\}-APSK with respect to the conventional \{16, 64\}-QAM SC-FDMA signal

Fig. 7: BER performance of SC-FDMA based transmission system, 16-\{QAM, APSK\}

Fig. 8: BER performance of SC-FDMA based transmission system, 64-\{QAM, APSK\}
present, effect of reduced PAPR dominates over AWGN contribution and M-APSK provides substantially better performance.

Finally, Fig. 9 shows the BER performance of the 64-\{QAM, APSK\} SC-FDMA system undergoing nonlinear distortion over a 6-tap ITU Pedestrian A frequency selective channel. The receiver applies conventional MMSE equalization technique to avoid the noise enhancement due to the de-spreading of the signal in the receiver. In order to increase the robustness against both nonlinear effects and multipath propagation, the transmitted bits are encoded using convolutional code of rate 1/2 and the polynomial generators \((91, 121)\). A soft limiter nonlinearity operating at IBO = \(\{5, 8\} \) dB is present. As can be seen, for both investigated cases, APSK based transmission system outperforms the standardized QAM SC-FDMA systems.

Figure 10 and 11 illustrate the out-of-band radiation of the M-\{QAM, APSK\} SC-FDMA system characterized by the parameters given above. PA operates over different values of IBO = \(\{2, 3, 4, 5\} \) dB. As it can be seen from this figures, the energy radiated to the adjacent channels is reduced using M-APSK modulation scheme in all cases and therefore, outperforms M-QAM.

**CONCLUSION**

In this study, we evaluated the performance of M-APSK baseband modulation for SC-FDMA. It was shown that in the particular scenarios, M-APSK shows the better performance in comparison with the standardized M-QAM. In general, it can be concluded that M-APSK provides high robustness against nonlinear amplification and its application is especially reasonable for highly nonlinearly disturbed scenarios. It could be also appreciated that the positive effects of M-APSK furnish in BER and out-of-band radiation and hence, a high flexibility in term of nonlinear detection is also sustained. The obtained results show very clearly, that if the user employs M-APSK, it will be able to use the larger transmission power than if M-QAM is employed, keeping PAPR, out-of-band radiation and BER at the required level. This fact is very positive, since it allows a larger transmitted power under conditions when the link quality is quite poor, e.g., at cellular cell edge. The APSK modulation technique might be further used together with some other PAPR reduction techniques to further alleviate the effects of the nonlinearities. Of the special relevance here is the application of the Active Constellation Extension (ACE) technique due to the larger number of outer constellation points in the signal constellation compared to QAM and thus, larger flexibility of this method.
REFERENCES


