A DS-AHP Approach for Multi-attribute Decision Making Problem with Intuitionistic Fuzzy Information

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Abstract: A new method to solve multi-attribute decision making problem with Intuitionistic Fuzzy (IF) information is given here. This method is based on the Dempster Shafer-Analytical Hierarchy Process (DS-AHP) theory. The DS-AHP theory can solve a problem directly based on the decision matrix which is different from most of current methods. But in reality, the decision maker may provide several types of uncertain information such as fuzzy or interval values. So, the original DS-AHP method is combined with the IF information here. The expected utility is used to transform decision matrix with intuitionistic fuzzy information which is obtained from assessment. And then a non-linear optimization model is used to combine all the attributes. Using the combined results, the rank of the alternatives can be finally got. At the end of the paper a numerical example is studied to illustrate the details.

Key words: Dempster Shafer-analytical hierarchy process theory, intuitionistic fuzzy information, multi-attribute decision making problem, non-linear optimization model

INTRODUCTION

Multi-attribute Decision Making (MADM) is concerned with the elucidation of the levels of preference of decision alternatives, through judgments made over a number of criteria. Many complex MADM problems are characterized with both quantitative and qualitative-of-preference of decision alternatives, through judgments made over a number of criteria. Many complex MADM problems are characterized with both quantitative and qualitative-attributes. For instance, the design evaluation of an engineering project may require the simultaneous consideration of several attributes such as cost, quality, safety, reliability, maintainability and environmental impact; in selection of its suppliers, an organization needs to take into account such attributes as quality, technical capability, supply chain management, financial soundness, environmental and so on. Most of such attributes are qualitative and could only be properly assessed using human judgments which are subjective in nature and are inevitably associated with uncertainties due to the human being's inability to provide complete judgment, or the lack of information, or the vagueness of the meanings about attributes and their assessments. For decades, many MADM methods have been developed, such as Analytical Hierarchy Process (AHP) (Saaty, 1980), TOPSIS (Lan, 2009), ELECTRE, PROMETHEE, LINMAP (Jiang and Fan, 2005) etc. for certain MADA and fuzzy multi-attribute decision making under uncertainty (Wang et al., 2011). AHP has been widely used in many areas such as accounting (Webber et al., 1997), assessment (Wu et al., 2010), programming (Yang and Kuo, 2003), research and development management (Liu and Tsai, 2007) and information management (Liu and Shih, 2005). AHP can be applied under the precondition that the decision maker can make pairwise comparison between decision alternatives. This prerequisite, however, may not be satisfied in practice. For a practical MADM problem, information about decision alternatives may be incomplete because of time pressure, lack of data, intangible of some attributes (Kim and Ahn, 1997, 1999), limitation of attention, or limitations on information processing capabilities (Kahneman et al., 1982), etc. The Dempster-Shafer (DS) theory of evidence (Dempster, 1967; Shafer, 1976) models have both quantitative and qualitative attributes with an appropriate framework. The power of the DS theory in handling uncertainties has found wide applications in many areas such as expert systems (Beynon et al., 2001), diagnosis and reasoning (Jones et al., 2002), pattern classification (Deneux and Zoughal, 2001), information fusion (Telmoudi and Chakhar, 2004), sort (Xu, 2012). In recent years, there have been several attempts to use the DS theory of evidence for MADA (Yang et al., 2006; Liu et al., 2011; Zhang et al.,

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In many cases, the DS theory has been used as an alternative approach to Bayes decision theory (Beynon et al., 2000; Yager et al., 1994) incorporated the DS theory with the AHP process. The method can not only model both quantitative and qualitative attributes but also take advantage of the AHP to lower the number of alternatives that fit the limited number of opinions given so far, with only a few opinions stated.

In some uncertain decision problems with qualitative attributes, however, it may be difficult to define assessment grades as independent crisp sets. It would be more natural to define assessment grades using subjective and vague linguistic terms which may overlap in their meanings. While intuitionistic fuzzy set has been proven to be highly useful in dealing with uncertainty and vagueness, accordingly, intuitionistic fuzzy set is a very suitable tool to be used to describe the imprecise uncertain decision information (Zhang et al., 2012; Amer et al., 2010). Nevertheless, the current DS-AHP approach does not take into account vagueness or fuzzy uncertainty. As such, there is a clear need to combine the DS-AHP theory for handling both types of uncertainties. The purpose of this paper is to investigate how to incorporate the intuitionistic fuzzy information with the DS-AHP method.

PRELIMINARY

The existing DS-AHP approach: On the base of Dempster-Shafer theory of evidence, the method first identifies all possible focal elements from the decision matrix and then it calculates the basic probability assignment of each focal element and the belief interval of each decision alternative. The AHP approach is used to describe the MADM problem. Using AHP approach, the MADM is decomposed into three levels. The first level is the MADM problem with incomplete information, the second level is the decision attributes of the MADM problem and the third level is focal elements identified from the decision matrix. Next, we will describe the main steps of the DS-AHP approach (Gong, 2007; Hua et al., 2008).

First, we identify focal elements from an incomplete decision matrix. Let \( \Theta = \{a_1, \ldots, a_n\} \) be a collectively exhaustive and mutually exclusive set of decision alternatives, called the frame of discernment.

Given a decision matrix \( V = \{f(a_i, C_j)\} \) where \( f(a_i, C_j) \) is the evaluation of the decision alternative \( a_i (i = 1, 2, \ldots, N) \) under the \( j^{th} \) attribute \( C_j (j = 1, 2, \ldots, M) \). The decision matrix \( V \) implies the body of evidence of the MADM and a focal element can be defined from the decision matrix as follows:

\[
\text{For } \forall a_i, a_k \in \Theta, \text{ and } a_i \neq a_k \text{ if } (a_i, C_j) = f(ak, C_j)
\]

then \( a_i \) and \( a_k \) belong to the same focal element.

According to above definition, we can obtain the focal elements of each attribute. Then, the hierarchical structure of DS-AHP can be constructed. And similar to AHP, the weights of importance of decision attributes \( w_j, j = 1, 2, \ldots, M \) in DS-AHP can be determined through pairwise comparison.

Second, construct belief interval of each decision alternative. Denote by \( A_i \) (\( j = 1, 2, \ldots, M, k = 1, 2, \ldots, t; t < 2^n \) the set that consists of all focal elements under decision attribute \( C_j \) and if \( a_i \in A_i \), we can view \( \omega f(a_i, C_j) \) as the decision maker's preference on the focal element \( A_i \), where \( \omega \) is the importance weight of decision attribute \( C_j \). Denote by \( F(A_i) \) the decision maker's preference on the focal element \( A_i \), then \( p(A_i) = \omega f(a_i, C_j) \). Because \( \Theta \) is the frame of discernment which consists of all decision alternatives, we let the \( p(\Theta) \). Then we can define the basic probability assignment (BPA) of each focal element as follows:

\[
m_i(A_i) = \frac{p(A_i)}{\sum_{A_{j}} p(A_{j})}
\]

Applying the operator of combination in the DS theory, the BPA of each focal element considering all decision attributes can be obtained. Suppose \( A_{i1} \) and \( A_{i2} \) are two focal elements under decision attribute \( C_i \) and \( C_i \), respectively, \( i, j \in \{1, 2, \ldots, M\}, i \neq j \). Denote the intersection of \( A_{i1} \) and \( A_{i2} \) as \( E \), then according to DS rule of combination, the BPA of \( E \) is defined as follows:

\[
[m_i \oplus m_j](E) = \frac{m_i(A_{i1})m_j(A_{i2})}{1 - \sum_{A_{k} \neq E} m_k(A_{i1})m_k(A_{i2})}, \quad E = \Phi,
\]

After obtaining BPA of each focal element considering all decision attributes, we can define its belief measure (Bel) and plausibility measure (Pls). Denote by \( \text{Bel}(\{a_i\}) \) and \( \text{Pls}(\{a_i\}) \) the exact support to a decision alternative \( a_i (i = 1, 2, \ldots, N) \) and the possible support to \( a_i \), respectively. The two values can be got as follows:

\[
\text{Bel}(\{a_i\}) = \sum_{E \subseteq \Theta} m(E) \quad \text{and} \quad \text{Pls}(\{a_i\}) = \sum_{E \subseteq \Theta} m(E) \quad \forall i \in \{1, 2, \ldots, N\}
\]

Using \( \text{Bel}(\{a_i\}) \) and \( \text{Pls}(\{a_i\}) \), we obtain the belief interval [\( \text{Bel}(\{a_i\}), \text{Pls}(\{a_i\}) \) for all decision alternatives of the MADA problem with incomplete information. Then in
order to obtain the preference relations among all decision alternatives, we need a mechanism to generate the rank of decision alternatives based in their belief intervals. There are many available methods to rank the alternatives. Here we cite the method described as follows.

We define the degree of preference of \( a_i \) over \( a_k \), denoted by \( P(a_i > a_k) \) as follows:

\[
P(a_i > a_k) = \max \left( \frac{\text{Bel}(\{a_i\}) - \text{Bel}(\{a_k\})}{\text{Bel}(\{a_i\}) + \text{Bel}(\{a_k\})} \right)
\]

with \( P(a_i > a_k) \in [0, 1] \).

By applying the above definition, we define the preference relation between decision alternative as follows.

- Decision alternative \( a_i \) is said to be superior to \( a_k \) (denoted by \( a_i > a_k \)) if \( P(a_i > a_k) > 0.5 \)
- Decision alternative \( a_i \) is said to be inferior to \( a_k \) (denoted by \( a_i < a_k \)) if \( P(a_i < a_k) < 0.5 \)
- Decision alternative \( a_i \) is said to be indifferent to \( a_k \) (denoted by \( a_i = a_k \)) if \( P(a_i = a_k) = 0.5 \)

The above formulas consist of the main process of the DS-AHP approach. After these steps, we can obtain the rank of all the decision alternatives.

The intuitionistic fuzzy set (IFS): An intuitionistic fuzzy set \( A \) on a universe \( U \) is defined as an object of the following form (Hua et al., 2008):

\[
A = \{(u, \mu_A(u), v_A(u)) \mid u \in U\}
\]

where, the functions \( \mu_A : U \to [0, 1] \) and \( v_A : U \to [0, 1] \) define the degree of membership and the degree of non-membership of the element \( u \in U \) in \( A \), respectively and for \( \forall u \in U, 0 \leq \mu_A(u) + v_A(u) \leq 1 \).

For convenience, we call \( u = (u, v) \) an intuitionistic fuzzy number.

THE COMBINATION OF THE DS-AHP METHOD WITH INTUITIONISTIC FUZZY INFORMATION

Just as the existing DS-AHP approach, we first need to obtain a decision matrix. Here we assume there are \( n \) evaluation grades to which all alternatives can be assessed, denoted by \( H = \{h_1, h_2, \ldots, h_n\} \) are mutually exclusive and collectively exhaustive. And we also need to define the utility of each grade as \( u(h_i) \) with \( u(h_{m(i)}) < u(h_i) \) if it is assumed that the grade \( h_m \) is preferred to \( h_i \). And the assessment value is provided with intuitionistic fuzzy information. Then we have that: if an alternative \( a_i \) is assessed on an attribute \( C_j \) to a grade \( h_i \) with an intuitionistic fuzzy data, we denote this by:

\[
S(C_j(a_i)) = (h_i, (\mu_j(a_i), v_j(a_i))), i = 1, 2, \ldots, n
\]

where, \( \mu_j(a_i) \) is the degree of membership of an alternative \( a_i \) on the attribute \( C_j \) associated with the grade \( h_i \) and \( v_j(a_i) \) is the degree of non-membership. For convenience, we use \( (\mu_j, v_j) \) to represent the assessment \( (\mu_j(a_i), v_j(a_i)) \) in the following paragraphs. After \( N \) alternatives are all assessed on \( M \) attributes, we obtain the following decision matrix: \( D = (S(C_j(a_i))) \).

First, to every alternative assessed on each attribute, combine the assessment given to the different grades. With the utility of each grade, a comprehensive assessment value of alternative \( a_i \) on the attribute \( C_j \) can be got as follows:

\[
u(a_i, C_j) = \sum_{i=1}^{n} u(h_i)(\mu_j, v_j)
\]

The operational rules of intuitionistic fuzzy set were defined by Atanassov (1986). Obviously, \( u(a_i, C_j) \) is still an intuitionistic fuzzy data. As the process of assessment is often accompanied with incomplete information, so we need to revise the results obtained using the expected utilities.

We denote \( \delta_i \) as the weight assigned to alternative \( a_i \) on the attribute \( C_j \), where:

\[
\delta_i = \frac{\sum_{i=1}^{n} u(h_i)}{\sum_{i=1}^{n} u(h_j)}
\]

And \( \delta_i \) represents the counts of grades an alternative is assigned to \( \delta_i \) is about the following formula:

\[
de_i = \begin{cases} 1, & \text{alternative } a_i \text{ is assigned to } h_i, \\ 0, & \text{others.} \end{cases}
\]

Using the weight above, we can revise \( u(a_i, C_j) \). We denote \( (\mu_j, v_j) = \delta_i u(a_i, C_j) \). From now onwards, we can finally obtain the transformed decision matrix \( V = (\mu_j, v_j) N \times M \).

Second, identify the focal element of each attribute. On the base of the transformed decision matrix \( V = (\mu_j, v_j) N \times M \), we can define the following method. The decision alternatives which have the same assessment value or the same degree of influence on an attribute belong to the same focal element of the attribute. A focal element can be defined as follows:
Definition 1: For $\forall a_i, a_j \in \Theta$ and $a_i \neq a_j$, if $\mu_{ij} = \mu_{ji}$ then $a_i$ and $a_j$ belong to the same focal element.

The next step, we will define the interval probability masses of each focal element. Denote by $A^i_j (j = 1, 2, ..., M; k = 1, 2, ..., t, t < 2^k)$ the set that consists of all focal elements under decision attribute $C_i$. We can obtain the interval probability masses of $A^i_j$.

Definition 2: Considering the weight of $C_i$ for $\forall a_i \in \Theta$, $\forall A^i_j \in 2^\Theta$ if $a_i \in A^i_j$, then we define the interval probability masses as:

$$m_i(A^i_j) = [\omega_i \mu_{ij}, \omega_i (1-\min v_{ij})]$$

We denote it as:

$$m_i(A^i_j) = [m_i(A^i_j)^- , m_i(A^i_j)^+]$$

As to the whole set $\Theta = \{a_1, a_2, ..., a_n\}$, the followings are used to obtain its interval probability masses:

$$m_i(T) = \left[ \max(0, 1 - \sum_{j \in T} m_i(A^i_j)^+), 1 - \sum_{j \in T} m_i(A^i_j)^+ \right]$$

where, $t < 2^n$.

Third, to obtain the interval probability masses of each focal element considering all decision attributes, we need to define the combination rule.

Definition 3: Let $m_1(A^i_k), m_2(A^i_l)$ be two interval belief structure with interval probability masses $m_k(A^i_k) = m_l(A^i_l)$ for $k = 1, 2, ..., t$, and $m_i(A^i_k) = m_i(A^i_l) = m_2(A^i_k)$ for $k = 1, 2, ..., t$. We define $E$ as the intersection of two focal elements $A^i_k$ and $A^i_l$. Their combination, denoted by $m_1 \otimes m_2$, is also an interval belief structure defined by:

$$[m_1 \otimes m_2](E) = \begin{cases} 0, & E = \emptyset, \\ \left( \left[ m_1 \otimes m_2 \right](E), (m_1 \otimes m_2)^-(E) \right), & E \neq \emptyset. \end{cases}$$

where, $(m_1 \otimes m_2)^-(E)$ and $(m_1 \otimes m_2)^+(E)$ are, respectively, the minimum and the maximum of the following optimization model:

$$\max/\min \left[ m_1 \otimes m_2 \right](E) = \frac{1}{1 - \sum_{E_2} m_1(A^i_k) m_2(A^i_l)} \sum_{E_2} m_1(A^i_k) m_2(A^i_l)

s.t. \frac{1}{1} \sum_{E_2} m_1(A^i_k) = 1,

\sum_{E_2} m_1(A^i_k) = 1,

m_1(A^i_k) \leq m_2(A^i_k), k = 1, 2, ..., t,

m_1(A^i_k) \leq m_2(A^i_k), k = 1, 2, ..., t.$$

When there are three or more attributes to be combined, we can use the combination rule recursively to obtain the final combined results. It should be pointed out that when using the combination rule recursively, we should do follow the next processes. We can first do the combination between two attributes and then combine the intersections of the two with the third attribute. This process will be stopped when all decision attributes are considered. These non-linear programming models can be solved by some mathematical software such as LINGO. Similar models can also be constructed for $m_i(\Theta)$.

The last step is to construct belief interval of each decision alternative.

Definition 4: The belief measure (Bel) and the plausibility measure (Pls) of alternatives $a_i$ are defined, respectively by:

$$\text{Bel}(a_i) = \frac{\text{Bel}_{\{a_i \}}}{\text{Bel}_{\{a_i \}}}, \text{Pls}_{\{a_i \}} = \frac{\text{Pls}_{\{a_i \}}}{\text{Pls}_{\{a_i \}}}$$

where:

$$\text{Bel}(a_i) = \min \sum_{E \in \Theta} m_i(E) = \max \sum_{E \in \Theta} m_i(E),\text{Bel}_{\{a_i \}} = \sum_{E \in \Theta} m_i(E),\text{Pls}_{\{a_i \}} = \sum_{E \in \Theta} m_i(E),\text{Pls}_{\{a_i \}} = \sum_{E \in \Theta} m_i(E)$$

Finally, we obtain the belief interval of all the decision alternatives which can be used to rank the alternatives. As for the ranking problem, there are a lot of methods provided by many scholars. Here we use the following formula:

$$\text{P}(a_i > a_k) = \max \left[ \text{Pls}_{\{a_i \}} - \text{Bel}_{\{a_i \}} - \text{Pls}_{\{a_k \}} + \text{Bel}_{\{a_k \}} \right]$$

(1)

If $\text{P}(a_i > a_k) > 0.5$ then $a_i \succ a_k$, else if $\text{P}(a_i > a_k) < 0.5$, else $a_i \sim a_k$.

Numerical Example

Here, we give an example on how to select a good project when the investment is carried on. The problem is described as follows: there are four available projects which consist of the decision alternative set $\Theta = \{a_1, a_2, a_3, a_4\}$ and there are four attributes related to the investment problem: marketing opportunity ($C_1$), support degree of policy ($C_2$), economy ($C_3$), technical capability ($C_4$) and the

Table 1: Decision matrix of the project selection problem

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>h₁ (0.6, 0.2)</td>
<td>h₂ (0.3, 0.4)</td>
<td>h₃ (0.8, 0.1)</td>
<td>h₄ (0.5, 0.4)</td>
</tr>
<tr>
<td>a₂</td>
<td>h₁ (0.5, 0.3)</td>
<td>h₂ (0.8, 0.1)</td>
<td>h₃ (0.5, 0.3)</td>
<td>h₄ (0.3, 0.5)</td>
</tr>
<tr>
<td>a₃</td>
<td>h₁ (0.2, 0.5)</td>
<td>h₂ (0.1, 0.7)</td>
<td>h₃ (0.1, 0.5)</td>
<td>h₄ (0.1, 0.8)</td>
</tr>
<tr>
<td>a₄</td>
<td>h₁ (0.5, 0.4)</td>
<td>h₂ (0.3, 0.6)</td>
<td>h₃ (0.3, 0.4)</td>
<td>h₄ (0.5, 0.3)</td>
</tr>
<tr>
<td>a₅</td>
<td>h₁ (0.4, 0.5)</td>
<td>h₂ (0.1, 0.7)</td>
<td>h₃ (0.4, 0.4)</td>
<td>h₄ (0.2, 0.6)</td>
</tr>
<tr>
<td>a₆</td>
<td>h₁ (0.8, 0.1)</td>
<td>h₂ (0.5, 0.4)</td>
<td>h₃ (0.6, 0.2)</td>
<td>h₄ (0.4, 0.5)</td>
</tr>
<tr>
<td>a₇</td>
<td>h₁ (0.6, 0.2)</td>
<td>h₂ (0.2, 0.6)</td>
<td>h₃ (0.5, 0.4)</td>
<td>h₄ (0.8, 0.1)</td>
</tr>
<tr>
<td>a₈</td>
<td>h₁ (0.1, 0.6)</td>
<td>h₂ (0.1, 0.8)</td>
<td>h₃ (0.1, 0.6)</td>
<td>h₄ (0.2, 0.6)</td>
</tr>
<tr>
<td>a₉</td>
<td>h₁ (0.5, 0.4)</td>
<td>h₂ (0.6, 0.2)</td>
<td>h₃ (0.1, 0.7)</td>
<td>h₄ (0.8, 0.1)</td>
</tr>
<tr>
<td>a₁₀</td>
<td>h₁ (0.6, 0.3)</td>
<td>h₂ (0.8, 0.1)</td>
<td>h₃ (0.6, 0.3)</td>
<td>h₄ (0.5, 0.4)</td>
</tr>
<tr>
<td>a₁₁</td>
<td>h₁ (0.2, 0.7)</td>
<td>h₂ (0.1, 0.7)</td>
<td>h₃ (0.2, 0.5)</td>
<td>h₄ (0.3, 0.5)</td>
</tr>
</tbody>
</table>

According to definition 3 and 4, we can obtain the belief measure and the plausible measure of alternative aᵢ, as follows:

Bel(aᵢ) = [0.1029, 0.15]  Pb(aᵢ) = [0.1029, 0.1627]
Bel(aᵢ) = [0.1964, 0.2726]  Pb(aᵢ) = [0.1964, 0.2843]
Bel(aᵢ) = [0.2007, 0.2742]  Pb(aᵢ) = [0.2007, 0.2859]
Bel(aᵢ) = [0.3866, 0.4647]  Pb(aᵢ) = [0.3866, 0.4764]
Bel(aᵢ) = [0.0000, 0.0117]  Pb(aᵢ) = 1

Then according to Eq. 1, we have the final rank of the four alternatives: a₇ < a₁ < a₆ < a₅, that is to say a₇ is the best project to invest.

CONCLUSION

The DS-AHP approach is a novel, flexible and systematic method. It can solve the problems directly based on its decision matrix. In this paper, we introduced something about intuitionistic fuzzy data and gave the basic steps of the DS-AHP method. We used the expected utilities to transform the original decision matrix. And then we defined interval probability masses which is different from the original basic probability assignment. In fact, it is an interval BPA. With the interval probability masses, we used a non-linear model to combine the focal element. In the end, we obtain the belief interval, where belief measure and plausible measure are both intervals. As we know the intuitionistic fuzzy information is used commonly in real assessment, so this combination is meaningful.

Further extension about DS-AHP approach includes developing methods with information expressed in interval intuitionistic fuzzy values or other uncertain values.

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REFERENCES


