Control for Track of Swarm Systems

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Abstract: In this paper, we develop an algorithm for tracking the center of swarm systems to a desired trajectory by using the sliding-mode control method. The algorithm is robust with respect to system perturbations and external disturbance. Simulation further shows the effectiveness very well.

Key words: Swarm systems, center of swarm, sliding-mode control, dynamic trajectory

INTRODUCTION

Collective behavior in swarms of biological entities has long been observed in nature. Examples of swarms include flocks of birds, schools of fish, herds of animals and colonies of bacteria. Such a collective behavior has certain advantages such as threatening predators and increasing the chance of finding food.


Sliding-mode control is easy to come true. Sliding-mode manifold has full robustness for systemic perturbation and external disturbance. Gazi (2005) use sliding mode control theory to force vehicle dynamics motion to obey the dynamics of the swarm. Yao et al. (2007) presented a stable and decentralized control strategy for multiagent systems to capture a moving target in a specific formation. Mao et al. (2010) used sliding-mode control method to control the swarm agents to an expectant trajectory.

In this article we consider a swarm model with system parameter perturbations or external disturbance and develop an algorithm for tracking the center of swarm systems to a desired dynamic trajectory by controlling H agents using the method of sliding-mode control. Simulation further shows the effectiveness very well.

MODEL OF SWARM SYSTEMS

We consider a swarm of M+H individuals (members) in an n-dimensional Euclidean space and model the individuals as points and ignore their dimensions. The position of member i of the swarm is described by \( x_i \in \mathbb{R}^n \). We assume synchronous motion and no time delays, following the model of Gazi and Pasinno (2003) the equation of motion of individual i is given by:

\[
x'(t) = \sum_{j \in \mathcal{N}} g(x_i - x_j), i = 1, 2, ..., M + H
\]

We consider \( g(\cdot) \) as:

\[
g(y) = -y \left( a + b \exp(-\frac{||y||^2}{c}) \right)
\]

where \( a, b \) and \( c \) are positive constants such that \( b > a \) and \( ||y|| \) is the Euclidean norm given by:

\[
||y|| = \sqrt{y^Ty}
\]

Suppose that \( f'(t) (i = 1, 2, \ldots, M+H) \) represents the sum of system parameter perturbations and external disturbance of agent i and

\[
||f'(t)|| \leq \bar{f}, \text{ where } \bar{f} > 0
\]

is positive constant. \( x_i = [x_i(t), x_i(t), \ldots, x_i(t)]^T \) represents the track of the center aim and there are positive constant Q such that for any time t, we have:

\[
||x_i(t)|| \leq Q
\]

Suppose the agent M+1, M+2, \ldots, M+H as controlled agents, then the controlled systems can be expressed as:

\[
\begin{align*}
x'(t) &= \sum_{j \in \mathcal{N}} g(x_i - x_j) + f'(t), \quad i = 1, 2, \ldots, M \\
x_{M+i}'(t) &= \sum_{k \in \mathcal{N}} g(x_{M+i} - x_k) + f_{M+i} + f_k, \\
&\quad h = 1, 2, \ldots, H
\end{align*}
\]
DESIGN FOR CONTROL LAW

Now, we would like to design each of the control inputs $u_i$ such that the center of swarm will be transferred effectively to $x_d$.

First, we define the $n$-dimensional sliding manifold for agent $\text{M}+\text{h}$ as:

$$
\dot{s}^{\text{Mh}} = x^{\text{Mh}} - \frac{1}{H} \left[ (M+H)x_d - \sum_{i=1}^{M} x_i \right] - \left( \frac{H+1}{2} - h \right) d = 0, h=1,2,\ldots,\text{H}
$$

where, $d = [d_1, d_2, d_3, \ldots, d_n]$ and $d_1, d_2, \ldots, d_n$ is constant.

Then, we design the control inputs $u_i$ such as to enforce the occurrence of sliding mode. A sufficient condition for sliding mode to occur is given by (Decarlo et al., 1988):

$$
\langle s^{\text{Mh}} \rangle s^{\text{Mh}} < 0
$$

which also guarantees that the sliding manifold is asymptotically reached. Later we will choose $u_i$ which will actually guarantee finite time reaching of the sliding manifold. Differentiating the sliding manifold equation we obtain:

$$
\dot{s}^{\text{Mh}} - x^{\text{Mh}} - \frac{1}{H} \left[ (M+H)x_d - \sum_{i=1}^{M} x_i \right] - \left( \frac{H+1}{2} - h \right) d = 0, h=1,2,\ldots,\text{H}
$$

$$
\dot{s}^{\text{Mh}} = \sum_{j=1}^{\text{Mh}} g(x^{\text{Mh}} - x^{'}) + f^{\text{Mh}}(t) + u_i - \left( \frac{H+1}{2} - h \right) d = 0, h=1,2,\ldots,\text{H}
$$

where, $\text{sgn}(s^{\text{Mh}}) = (\text{sgn}(s_1^{\text{Mh}}), \text{sgn}(s_2^{\text{Mh}}), \ldots, \text{sgn}(s_n^{\text{Mh}}))^\top$ and $\text{sgn}(y)$ is sign function:

$$
\text{sgn}(y) = \begin{cases} 
1, & y > 0 \\
-1, & y < 0 \quad (y \in \mathbb{R})
\end{cases}
$$

we obtain:

$$
\langle s^{\text{Mh}} \rangle s^{\text{Mh}} = \langle s^{\text{Mh}} \rangle [-u_i \text{sgn}(s^{\text{Mh}}) + f^{\text{Mh}}(t) + \frac{1}{H} \sum_{j=1}^{\text{Mh}} f^j(0)]
$$

and note also that the above inequality implies that:

$$
\dot{s}^{\text{Mh}} = \frac{1}{2} \langle s^{\text{Mh}} \rangle s^{\text{Mh}} < 0
$$

Solve it, we can obtain that the sliding manifold of agent $\text{M}+\text{h}$ is reached at time:

$$
T_{\text{Mh}} = \frac{2 \sqrt{V^{\text{Mh}}(0)}}{\varepsilon} = \frac{2}{\varepsilon} \frac{\| s^{\text{Mh}}(0) \|}{\varepsilon}
$$

Theorem: Adopt the control inputs Eq. 7 to agents $\text{M}+1$, $\text{M}+2$, $\ldots$, $\text{M}+\text{H}$, the center of members will transfer to $x_d$ in finite time:

$$
t_0 = \max(T_{\text{Mh}})
$$

Proof: Once all the agents $\text{M}+1$, $\text{M}+2$, $\ldots$, $\text{M}+\text{H}$ reach their sliding manifolds $s^{\text{Mh}} = 0$ we have:

$$
x^{\text{Mh}} = \frac{1}{H} \left[ (M+H)x_d - \sum_{i=1}^{M} x_i \right] + \left( \frac{H+1}{2} - h \right) d, h=1,2,\ldots,\text{H}
$$
Figure 1 shows the trajectories of the agents with external disturbance and without control. Figure 2 shows the trajectories of the agents with three controlled agents. The red broken line represents $x_d$, the other broken lines represent the trajectories of controlled agents and full lines represent the trajectories of no control members. It shows that the center of swarm systems will transfer effectively to $x_d$ by controlling three agents using the control inputs (Eq. 7).

**CONCLUSION**

In this paper, we present a procedure based on sliding mode control theory, which can control the center of swarm systems to a desired dynamic trajectory by controlling a few agents. The algorithm is robust with respect to system perturbations and external disturbance. It solves well the stabilization problem of motion tracking for complex systems. Simulation further shows the effectiveness very well.

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**REFERENCES**


