An Adaptive Fuzzy Control Algorithm for Unknown Multivariable Unknown Non-affine Systems

Chunzhi Yang, Heng Liu and Ning Li
Department of Mathematics and Computational Science, Huainan Normal University, Huainan, 232038, China

Abstract: A new scheme was proposed for the design of fuzzy adaptive control for a class of non-affine systems with unknown control direction. To facilitate the controller design based on the affine-like equivalent model, a Taylor series expansion was used to transform unknown non-affine system into an affine system. The adaptive laws were derived in the Lyapunov sense to ensure the asymptotic stability and tracking of the controlled system. To prove the validity of the proposed method, the simulation results were illustrated in this study.

Key words: Non-affine system, fuzzy controller, adaptive control, fuzzy logic system

INTRODUCTION

The fuzzy controllers emerged to give a new technology to obtain a good performance without having accurate models of process to be controlled. Generally, the basic objective of adaptive control was consistent with the performance, to keep the system in the presence of these uncertainties (Wang, 1994). Direct adaptive fuzzy control was based on the deviation between the actual system performance and ideal performance, through a certain method to adjust the parameters of the controller (Chekireb et al., 2003). Adaptive laws to adjust the parameters, the artificial output tracked the reference mode output. Adaptive control was the social nature of the internal control system and improve the areas of research. In the literature (Labiod and Guerra, 2007; Chekireb et al., 2003, Wang, 1994), many of the results had been reported capable of preserving stability and robustness for some nonlinear systems.

In recent decades, uncertain nonlinear systems, adaptive fuzzy control theory is increasingly becoming a hot issue. The theory had made many achievements in the control field of the affine nonlinear system. But in practice, there were many non-affine structure of nonlinear systems, such as biochemical processes, so the study of non-affine system control has a practical significance (Labiod and Guerra, 2007; Liu et al., 2009).

A direct adaptive fuzzy controller based on any observers for a class of SISO (Single-Input-Single-Output) affine nonlinear system had been proposed by Boukroune et al. (2008). Three fuzzy adaptive controllers for a class of MIMO (Mult-Input-Mult-Output) uncertain nonlinear systems were introduced by Boukroune et al. (2010a), some similar class of MIMO non-affine systems had been considered by Park and Park (2003), Park and Kim (2004), Park et al. (2005) and Doudou and Khaber (2012). A class of MIMO uncertain nonlinear system was adaptive and fuzzy, its output tracking control be discussed by Liu et al. (2009). In order to estimate the true control direction, a unique Nussbaum-type function had been especially employed for dealing with the unknown control direction by Boukroune et al. (2012).

In this study, an adaptive fuzzy controller was proposed, this controller was a class of uncertain multivariable non-affine systems with unknown control direction. Taylor series expansion of higher order terms fuzzy approximation error to compensate for a dynamic and robust adaptive control. The fuzzy system was employed to approximate the unknown nonlinear function, the adaptive law updated the parameters of the Lyapunov stability analysis of the model so that finite-time convergence of tracking error and closed-loop stability can be guaranteed.

PROBLEM DESCRIPTION

Consider the following equation that can be used to describe a class of non-affine nonlinear uncertain systems:

\[
\begin{align*}
\dot{y}_1 &= f_1(x,u) + A_1(t) \\
\dot{y}_2 &= f_2(x,u) + A_2(t) \\
& \vdots \\
\dot{y}_n &= f_n(x,u) + A_n(t)
\end{align*}
\] (1)

Corresponding Author: Chunzhi Yang, Department of Mathematics and Computational Science, Huainan Normal University, Huainan, 232038, China
where, \( x = [x_1^T, ..., x_n^T]^T \in \mathbb{R}^n \) is the state vector which is assumed available for measurement. \( u = [u_1, ..., u_n]^T \in \mathbb{R}^n \) is the control input vector and \( f_i(x, u), i = 1, 2, ..., n \) are smooth unknown nonlinear functions, \( \Delta_i(t), i = 1, ..., n \) are unknown bounded disturbance.

Denote:

- \( y = [y_1, ..., y_n]^T \)
- \( F(x, u) = [f_1(x, u), ..., f_n(x, u)]^T \)
- \( \Delta(t) = [\Delta_1(t), ..., \Delta_n(t)]^T \)

The system (1) can be rearranged in the following compact form:

\[
y = F(x, u) + \Delta(t)
\]

(2)

The objective of this study is to design a control law \( u \), such that the output vector asymptotically tracks a time varying desired trajectory \( y_1 - [y_{d1}, ..., y_{dn}]^T \in \mathbb{R}^n \) with all signals in the closed-loop system remain bounded uncertainty and bounded external disturbances. Throughout this study the following assumptions and made:

**Assumption 1**: The desired trajectory \( x_d(t) = [y_{d1}, y_{d2}, ..., y_{dn}]^T \) is continuous, bounded and available for measurement, \( x_d \) is a known compact set.

**Assumption 2**: The matrix:

\[
\frac{\partial F(x, u)}{\partial u}
\]

is no-singular, its sign is positive definite or negative definite.

Define the tracking error is:

\[
e_i = y_i - \hat{y}_i
\]

(3)

and \( e = [e_1, ..., e_n]^T \).

The time derivative of \( e \) is given by:

\[
\dot{e} = \dot{y}_1 - \dot{\hat{y}}_1 - [F(x, u) + \Delta(t)]
\]

(4)

Now, in order to facilitate the control system design for the system (2), the non-affine system (2) can be transformed into the affine system (4).

Where:

- \( F(x, u) = F(x, u^*) + \frac{\partial F(x, u)}{\partial u} u^*(x) + \frac{\partial^2 F(x, u)}{\partial u^2} u^*(x) + H(x, u) \)
- \( G(x) = \frac{\partial F(x, u)}{\partial u} u^*(x) + H(x, u) \)

\( H(x, u) \) is the higher order terms of the Taylor series expansion, \( u = u^*(x) \) is an unknown optimal control and an unknown smooth function minimizing the higher order terms.

In order to study the control design and stability analysis, the following important lemma and realistic assumption are considered.

**Lemma 1**: Any real matrix \( G(x) \in \mathbb{R}^{n \times n} \) with non-zero leading principal minors can be decomposed follows:

\[
G(x) = G(x) D T(x)
\]

(5)

where, \( G(x) \in \mathbb{R}^{n \times n} \) is a symmetric positive-definite matrix, \( D e^{\alpha x} \) is a diagonal matrix whose elements are \(+1\) or \(-1\) and \( T(x) \) is a unity upper triangular matrix. Moreover, the diagonal elements \( d_i \) of \( D \) are nothing else than the ratios of the signs of the leading principal minors of \( G(x) \).

**Proof of lemma 1**: See the proof in Ge and Wang (2002a, b).

**Assumption 3**:

- The sign of \( G(x) \) is unknown, but it must be positive-definite or negative-definite
- \( Ge(x) \) and

\[
\frac{d}{dt} G_i^*(x)
\]

are continuous functions

- \( \frac{\partial G_i(x)}{\partial y_j} = 0 \ \forall i = 1, ..., n, j = 1, ..., n \)

**Remark 1**:

- Assumption 3 is satisfied many affine physical systems, namely robotic systems and electric machines
- The required property on the partial derivatives of the control gain matrix ensure that

\[
\frac{d}{dt} G_i^*(x)
\]

does not depend on the system inputs.
Substituting Eq. 5 into the tracking error dynamics Eq. 3, it yields:

\[ e = y_t - F(x) - G_1(x)\Delta T(x)u(t) - H(x,u) - \Delta(t) \]  

Equation 6 can be arranged as follows:

\[ G_1(x)e - F(x,u) - Du - H_1(x,u,\Delta) \]  

Posing

- \( G_1(x) = G_1^T(x) \)
- \( F_1(x,u) = G_1(x)[G_2(x) - F(x)] - DT(x) - D\mu \)
- \( H_1(x,u,\Delta) = G_1(x)[H(x,u) + \Delta(t)] \)

Equation 7 can be written as:

\[ G_1(x)e - \frac{1}{2} G_1(x)e - F(x,u) + \frac{1}{2} G_1(x)e - Du - H_1(x,u,\Delta) \]

\[ = \alpha(\bar{x}) - Du - H_1(x,u,\Delta) \]  

Where:

\[ \alpha(\bar{x}) = [\alpha(1)(\bar{x}), ..., \alpha(n)(\bar{x})]^T - F(x,u) + \frac{1}{2} G_1(x)e \]

with \( \bar{x} = [x^T, u_1, ..., u_n]^T \). By carefully examining the expressions of \( F_1(x,u) \), \( \alpha(\bar{x}) \), the vectors \( \bar{z}_i \) can be determined as follows:

\[ \bar{z}_i = [x^T, u_1, ..., u_i]^T \]

\[ \bar{z}_i = [x^T, u_1, ..., u_n]^T \]

\[ \bar{z}_i = [x^T, u_1, ..., u_{i+1}]^T \]

\[ \bar{z}_i = x \]

It is clear from the property of the matrix \( D(x) - D \), that \( \bar{z}_i \) depends on control in puts \( u_1, ..., u_i \), \( \bar{z}_2 \) depends on control in puts \( u_1, ..., u_n \) and so on. The nonlinearity \( \alpha(\bar{x}) \) has an upper triangular control structure, allowing there by for algebraic loop free sequential determination of the control variables \( u_i \), for \( i = 1, 2, ..., n \).

**Remark 2:** In the Eq. 8, the nonlinear functions \( \alpha(\bar{x}) \), \( \alpha_{i,j}(x,u,\Delta) \) are unknown, moreover \( H_1(x,u,\Delta) \) depends explicitly on the input \( u \), the control system design to asymptotically stabilize the dynamics is very difficult, so we will use an adaptive fuzzy system to approximate the unknown nonlinear function \( \alpha(\bar{x}) \) and a dynamic adaptive robust control to dynamically compensate for the effect of the uncertain nonlinearity \( H_1(x,u,\Delta) \).

**DESIGN OF THE FUZZY LOGIC SYSTEM**

Since the function \( \alpha(x) \) is unknown, the control system design to asymptotically stabilize the dynamics is very difficult. In this study, fuzzy systems are employed to approximate the equivalent controller. Using the product-inference rule, singleton fuzzifier and center-average defuzzifier then the output of the fuzzy system can be defined as:

\[ \alpha(x) = \sum_{i} \theta_i \mu_i(x) \]

where, \( \sum_{i} \mu_i(x) = 1 \). The number of total fuzzy rules, \( x \) is the input vector, \( \mu_i(x) \) is \( x_i \)'s membership of jth rule and \( \theta_i \) is the centroid of the jth consequent set. Equation 9 can be rewritten as following equation:

\[ \alpha(x) = \theta \hat{\psi}(x) \]

with:

\[ \theta = [\theta_1, ..., \theta_n]^T \]

and the fuzzy basis function can be written as:

\[ \hat{\psi}(x) = \frac{\sum_{i} \mu_i(x) \psi_i(x)}{\sum_{i} \mu_i(x)} \]

So, by applying the introduced fuzzy systems in Eq. 10 and \( N \) fuzzy rules:

Rule i: if \( x_i \) is \( F_i \) and ... and \( x_n \) is \( F_n \) then \( \alpha(x) \) is \( B_i \), \( i = 1, 2, ..., N \). Then \( \alpha(\bar{x}) \) can be approximated, over compact set \( \Omega_\varepsilon \), by the fuzzy system Eq. 10 as:

\[ \hat{\alpha}(\bar{x}, \theta) = \theta \hat{\psi}(\bar{x}) \]

Define the ideal parameters of \( \alpha \), as:

\[ \theta^* = \arg\min_{\theta} \| \alpha(\bar{x}) - \hat{\alpha}(\bar{x}, \theta) \|^2 \]

Note that the ideal parameter \( \theta^* \), is introduced only for the purposes of analysis and its value is not needed when implementing the controller. Define the parameter estimation error and the fuzzy approximation error as follows:

\[ \hat{\theta}_i = \theta - \theta^* \]

and

\[ \hat{\psi}(x) \]
\[ \varepsilon(z) = \theta(z) - \theta(z_0) \]  

(13)  

with \( \theta(z_0) = \theta(z_0) \).

As in literature (Boulkroune et al., 2008, 2010a, b, 2011, 2012), assumed the fuzzy approximation errors is bound for all \( z \in \Omega_0 \), i.e., \( |\varepsilon(z)| \leq \bar{\varepsilon}_0 \), \( \bar{\varepsilon}_0 \) is unknown constant.

Now, denote:

\[ \dot{\varepsilon}(z) = \frac{\partial}{\partial t} \varepsilon(z) \]  

(14)  

From above analysis, it yields:

\[ \dot{\varepsilon}(z) = \frac{\partial}{\partial t} \varepsilon(z) = \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \_ + \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) \]

Where:

\[ \dot{\varepsilon}(z) = \frac{\partial}{\partial t} \varepsilon(z) = \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) \]

and \( \dot{\varepsilon}_i = \dot{\varepsilon}_i - \dot{\varepsilon}_i \), for \( i = 1, \ldots, n \).

From Eq. 14 and Eq. 8 can be written as:

\[ \dot{\varepsilon}(z) = \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) \]

(15)  

Where:

\[ \dot{\varepsilon}_i = \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) - \frac{\partial}{\partial t} \varepsilon(z) \]

and \( \dot{\varepsilon}_i = \dot{\varepsilon}_i - \dot{\varepsilon}_i \), for \( i = 1, \ldots, n \).

For the system (1), the following fuzzy adaptive controller can be considered:

\[ u = D(\theta, z) + K \varepsilon + K \text{sign} \]  

(16)  

Where:

\[ D = D^2 - \text{diag}(d_1, \ldots, d_n) \]

After substituting the control law Eq. 16 into Eq. 15, it yields:

\[ \frac{d}{dt} \frac{1}{2} \varepsilon(z) = \frac{1}{2} \varepsilon(z) - \frac{1}{2} \varepsilon(z) - \frac{1}{2} \varepsilon(z) - \frac{1}{2} \varepsilon(z) \]

(17)  

Adaptation laws associated to the proposed controller Eq. 16 are given by:

\[ \dot{\varepsilon}_i = \gamma \varepsilon, \varepsilon_i(z) \text{ for } i = 1, \ldots, n \]

(18)  

where, \( \gamma \) is positive design constant.

The main result is summarized by the following theorem.

**Theorem 1:** Consider system (1), suppose that Assumption 1 is satisfied. Then the control law defined by Eq. 16 guarantees the following properties: (1) All signals in the closed loop system are bounded. (2) The tracking errors and their derivatives decrease asymptotically to zero i.e.:

\[ e_i(t) \rightarrow 0 \text{ as } t \rightarrow \infty \text{ for } i = 1, \ldots, n \]

**Proof of Theorem 1:** Since \( G_i(x) \) is symmetric positive-definite, the following Lyapunov function is defined:

\[ V = \frac{1}{2} \varepsilon(z) + \frac{1}{2} \varepsilon(z) \]

(19)  

where, \( \dot{\varepsilon}_i = \dot{\varepsilon}_i - \dot{\varepsilon}_i \), for \( i = 1, \ldots, n \).

The time derivative of \( V \) is given by:

\[ V = \frac{d}{dt} \frac{1}{2} \varepsilon(z) + \frac{1}{2} \varepsilon(z) \]

(20)  

with \( H(z) = 1 + ||z|| + ||z|| \), where, \( k^* = [k^*, \ldots, k^*] \) is an unknown constant vector.
Using Eq. 17-20 can be bounded as follows:
\[
\begin{align*}
\dot{V} & \leq e^T(z) - e^T(k_a)e^{-\varphi(z)} + e^T\varphi(x,u)k^* \\
& - \sum_{t=1}^{n} k_{x_i}^2 + \sum_{t=1}^{n} e^T\varphi(z) - \sum_{t=1}^{n} k_{x_i}^2\varphi(x,u) \\
& \leq -\sum_{t=1}^{n} k_{x_i}^2 < 0
\end{align*}
\]  
(21)

where, \(k_a = \text{diag}[k_{a_1}, ..., k_{a_n}], k_{a_i} > 0, i = 1, ..., n\), \(k_a\) is a adjustable constant, that ensures the following inequality Eq. 22 holds:
\[
e^T\varphi(z) + |\varphi|k^2\varphi(x,u) - e^T(k_a) \leq 0
\]  
(22)

From Eq. 21 and using the Barbalat’s lemma, It can be easily showed, \(e = 0\) when \(t = \infty\). Therefore, the tracking errors and their derivatives converge asymptotically to zero.

**SIMULATION STUDIES**

Here, simulation studies are carried out to show the effectiveness of the proposed adaptive fuzzy controller. we consider the tracking control problem of an academic non-affine uncertain system. The dynamic equations of this system are give by:
\[
\begin{align*}
x_1 &= x_1x_2 + u_1 + (1 + x_2)u_2 + \Delta_1(t) \\
x_2 &= (x_1 + x_2)^3 - 0.5u_1 + (2 + \sin x_2)u_2 + \Delta_2(t)
\end{align*}
\]  
(23)

where, \(x = [x_1, x_2]^T\) is the state vector of the system, \(u_1\) and \(u_2\) are the control inputs. \(\Delta_1(t)\) and \(\Delta_2(t)\) are external disturbances. These are considered to be square waves having an amplitude \(\pm 1\) with a period of \(2\pi(s)\). The control objective consists in allowing the system outputs \(x_1\) and \(x_2\) to track the desired trajectories \(x_{\text{d1}} = \sin(t)\) and \(x_{\text{d2}} = \sin(t)\), respectively.

The design parameters and initial conditions are chosen as follows:
\[
\begin{align*}
\gamma_1 &= \gamma_2 = 100, x_1(0) = 2, x_2(0) = 1, \quad k_1 = k_2 = 1, \\
\theta_1(0) &= \theta_2(0) = \text{null vector}
\end{align*}
\]

Simulation results in Fig. 1-3 are obtained by applying the design algorithm to the system (23). The time response of the state variables of the uncertain system are given in Fig. 1 and 2, a better tracking performance is achieved for the desired trajectories \(x_{\text{d1}}\) and \(x_{\text{d2}}\). Boundedness and smoothness of the controllers \(u_1\) and \(u_2\) is described in Fig. 3. The simulation results show that the proposed method is successful in control

**CONCLUSIONS**

An adaptive fuzzy tracking control for a class of non-affine uncertain systems has been presented. In the designing of fuzzy adaptive controller, the nature of the control gain matrix factorization has been used. Be design of a fuzzy adaptive law can guarantee the tracking error converges to zero and all of the closed-loop system signal
boundedness convergence. The simulation results show the effectiveness of the proposed controller.

ACKNOWLEDGMENTS

This research is jointly supported by the Foundation for the Natural Science Research in College of Anhui Province under Grant No. KJ2011Z357 and Scientific Research General Project of Huainan Normal University under Grant No. 2010QNL5. 2011LK79.

REFERENCES


