New Method for Fault Diagnosis Based on Dempster-shafer Theory

Hong Zhan and Jiwen Tan
School of Mechanical Engineering, Qingdao Technological University, Qingdao 266033, China

Abstract: A satisfying fault diagnosis result based on single-sensor is difficult to obtain. A multi-information fusion approach based on Dempster-shafer Theory (DST) was an effective method. The importance of multiple evidences is different. Although the combination rule of DST is widely used to track multiple cues. It is sometimes unreasonable because of the weight of evidence having not been well considered. A new method was proposed to weight evidences combined information entropy with consistency of judgment matrix in terms of different importance of evidences. Evidences were weighted and well adjusted. Evidences were fused based on combination formula of DST. Subjective and objective factors were well-considered during the process of fault diagnosis. Fusion results indicated the new method have better performance in dealing with combinations, it advanced reliability and decrease uncertainty of evidences. This new method had theoretical and practical application value.

Key words: Dempster-shafer theory, information entropy, judgment matrix consistency, multi-information fusion, fault diagnosis

INTRODUCTION

The result of fault diagnosis is very difficult to obtain considering the working environment influence, sensors’ sensitivity, information acquirement technology and so on. Obviously the traditional fault diagnosis method based on single evidence is insufficient. Many studies in literature suggest the use of multiple cues to increase the efficiency of fault diagnosis. Multiple targets fusing method has become one of the research issues. In order to satisfy the demand for reliability of fault diagnosis, DST has been widely used in this field because it is efficient to fuse (Smets and Kennes, 1994). But sometimes fault diagnosis result based on the classical DST is unreasonable (Zadeh, 1986). In practice, the method in signal processing, feature extraction and diagnosis is very complex. And the importance of evidences is very different to the fusion result as well. So, the result based on classical combination rule of DST has its drawback (Yager, 1987). Evidences should be weighted considering their different importance. In this study, a new method based on combination of information entropy with consistency ratio of judgment matrix is used to weight different evidences. Subjective and objective factors were well-considered during the fusion process.

BASES OF DST

Principle of dempster-shafer theory: The classical DST is a theory which based on frame of discernment denoted as $\Theta$ and power set denoted as $2^\Theta$. The frame of discernment is composed of $n$ exhaustive and exclusive hypotheses (Shafer, 1976). The Basic Probability Assignment (BPA) for all hypotheses must satisfy the following conditions:

$$0 \leq m(A_i) \leq 1$$
$$m(\emptyset) = 0$$
$$\sum_{A \in \Theta} m(A_i) = 1$$

Definitions such as belief function and plausibility function and so on have been discussed in related field studies conducted before (Shafer, 1976). It do not need to present in this study again.

DST is an efficient theory which offers appropriate aggregation method for BPAs. The combination rule of DST assumes the sources of evidences are independent. It can use the orthogonal sum to combine multiple BPAs (Smarandache and Dezert, 2006). The classical combination rule of DST is defined as:

$$m(A) = \frac{\sum_{A \supseteq B} m_i(B_j) m(A \cap B_j)}{1 - K}$$

$$K = \sum_{A \subseteq B \cap \emptyset} m_i(A) m_i(B) < 1$$
where, A and B are focal target elements. K denotes a measure of conflict between sources of evidences. The denominator 1-K is addressed normalization factor. The larger K is, the higher the conflict of sources between evidences is (George and Pal, 1996). If K = 1, the combination rule can not be use to fuse and the orthogonal sum does not exist as well. The combination rule has two properties: commutativity and associativity.

**A WEIGHT METHOD BASED ON COMBINATION OF INFORMATION ENTROPY WITH CONSISTENCY OF EVALUATION MATRIX**

Method in signal processing, feature extraction and diagnosis is different. So, the sources of evidence should be assign different weight. Weight method based on consistence of evaluation matrix concerns the subjective factors. But it doesn’t concern the quantity of information and the level of uncertainty. Subjective and objective factors should be well concerned during the combination process. For this purpose, a new alternative is presented for combination of basic belief assignments. This method concerns the influence to sources by information entropy and consistence of judgment matrix.

**Consistency ratio of judgment matrix:** The judgment matrix should be given by jury of critics. The eigenvalue \( \lambda_{max} \) of judgment matrix can be calculated by:

\[
\lambda_{max} = \frac{1}{n-1} \sum_{i=1}^{n} \left( Aw_i \right)^T \left( Aw_i \right)
\]

The consistency index \( C(A) \) is calculated by \( \lambda_{max} \), the formula is given by:

\[
C(A) = \frac{\lambda_{max} - n}{n-1}
\]

where, \( n \) is the size of judgment matrix.

The value of consistency ratio \( C(R) \) can be used to check the consistence of judgment matrix. If \( C(R) \leq 0.1 \), the consistence of judgement matrix is satisfactory otherwise it is unacceptable. To obtain a judgement matrix with good consistence from the jury, judgement matrix should be reviewed and improved more and more by experts (Awasthi et al., 2008).

\[
C(R) = \frac{C(A)}{R_t}
\]

where, \( R_t \) is random consistence ratio. It can be obtained in the study of Awasthi et al. (2011).

**Information Entropy:** Herein hypotheses M information sources collected from multiple sensors. Let’s assume set \( U = \{ U_1, \cdot \cdot \cdot U_k \} \) have M evidences for one hypothesis. Where, \( m_i = (m_{1i}, \cdot \cdot \cdot m_{ki}) (i = 1, 2, \cdot \cdot \cdot M) \). According to the combination rule of DST which satisfies the condition:

\[
\sum_{i=1}^{M} m_{1i} = 1
\]

The information entropy of evidence \( i \) is given by:

\[
H_i = -\sum_{j=1}^{n} m_{ij} \log_2 m_{ij} \quad (i=1,2,\cdot \cdot \cdot M)
\]

In this study, the weight of different evidence is calculated by the following formula:

\[
w_i = \frac{(1-H_i)^0.1 \cdot \sum_{j=1}^{n} C(R_j)}{(m - \sum_{j=1}^{n} C(R_j))}
\]

The merits of this method is obvious to see. This formula combines the information entropy and consistence ratio of judgment matrix. Subjective and objective factors are well considered which can make the fusion result more reliable.

**WEIGHTED AVERAGE OF EVIDENCES**

The aim for weighted average of evidences is to make the decision-making process authoritative. The weighted average methods can be easily found in study (Zhu et al., 2002). The main steps in this section is given as follows:

\[
\bar{m} = \frac{\sum_{i=1}^{n} w_i m_i}{\sum_{i=1}^{n} w_i} = \left( \frac{\sum_{i=1}^{n} w_i m_{1i}}{\sum_{i=1}^{n} w_i}, \cdot \cdot \cdot \frac{\sum_{i=1}^{n} w_i m_{ki}}{\sum_{i=1}^{n} w_i} \right)
\]

where, \( \bar{m} \) is the average evidence. The bias arising between evidence \( i \) and average evidence \( \bar{m} \) in the proposition \( \{ U_i \} \) is given by:

\[
e_i = m_{1i} - \bar{m}
\]

So, the whole bias arising between \( m_i \) and \( \bar{m} \) in the proposition set is given as:

\[
e_i = (e_{i1}, \cdot \cdot \cdot e_{iM})
\]

Let’s assume:

\[
\delta = \frac{\sum_{i=1}^{n} (e_{i})^2}{n}
\]
where, $\delta_i$ represents the whole bias arising between $m_i$ and $\bar{m}$. Use $\delta_i$ to adjust $m_i$. Given $m'_i$ is the evidence after adjustment which is given by:

$$m'_i = (m_{n-\delta_i}\delta_i | q = 1, 2, \cdots N)$$ (15)

It can be seen from the previous reductions, the larger $\omega$, the closer $m_i$ to $\bar{m}$, the smaller $\delta_i$ is. Conversely, the smaller $\omega$, the further $m_i$ to $\bar{m}$, the larger $\delta_i$ is. Based on the previous reduction, this process is similar to discussion process of experts. Every expert revises his own view constantly. Finally experts generate coincident opinion relatively. The process reflects the authoritative of decision-making.

EXAMPLE BASED ON NEW METHOD

Experiment set-up: Fault diagnosis examples of motor rotor are given to illustrate the effectiveness of new method. In this study, vibration acceleration sensors and vibration displacement sensors are installed on sensitive sections of motor to collect signal. Multiple targets tracking method is used to diagnose the motor with vibration fault symptoms. The whole process of obtaining the BPAs of fault mode can easily be found in the study (Zhai et al., 2012). Let frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8 \}$ denote fault modes rotor unbalance, rotor misalignment, rotor clamping support loosening, oscillation of oil film, rotating stall and uncertainty, respectively. Six BPAs are given in Table 1.

In this study, a jury with 20 experts from relative research field give the judgment matrix as follows:

$$\begin{bmatrix} 1 & 4 & 3 & 3 & 4 \\ 4 & 1 & 1 & 1 & 4 \\ 3 & 1 & 1 & 1 & 3 \\ 3 & 1 & 1 & 1 & 3 \\ 4 & 1 & 1 & 1 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & 2 & 2 & 1 \\ 5 & 1 & 2 & 1 & 1 \\ 5 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & 2 & 2 & 1 \\ 5 & 1 & 2 & 1 & 1 \\ 5 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A_1 - A_2 - A_3$$

It can be seen from Table 1, it is difficult to obtain a reliable diagnosis result based on a single source. Conflicts arising between multiple cues and uncertainty make us to use multiple target tracking method. The orthogonal sum rule of DST makes the fusion result obvious in Table 2. BPA of $\theta_1$ is 0.85 and 0.87, respectively. Value of information entropy, consistency ratio and weight of evidences can be seen in Table 3. The weight values of evidences are expressed by Eq. 10. Information entropy and consistency ratio reflect subjective and objective factors respectively. The effectiveness of multiple cues fusion can advance the reality of fault diagnosis. It can be seen in Table 4.

Table 1: BPAs of multiple sensors

<table>
<thead>
<tr>
<th>$m_{g}^{ullet}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.78</td>
<td>0.02</td>
<td>0.03</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.28</td>
<td>0.35</td>
<td>0.19</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>0.22</td>
<td>0.28</td>
<td>0.15</td>
<td>0.14</td>
<td>0.11</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 2: Fusion results using DST

<table>
<thead>
<tr>
<th>$m_{g}^{ullet}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{g}^{ullet}$</td>
<td>0.09</td>
<td>0.85</td>
<td>0.02</td>
<td>0.01</td>
<td>0.020</td>
<td>0.010</td>
</tr>
<tr>
<td>$m_{g}^{ullet}$</td>
<td>0.08</td>
<td>0.87</td>
<td>0.02</td>
<td>0.01</td>
<td>0.016</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 3: Value of information entropy, consistency ratio and weight of evidences

<table>
<thead>
<tr>
<th>Evidence</th>
<th>Consistency ratio</th>
<th>Information entropy</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0468</td>
<td>1.1864</td>
<td>0.0500</td>
</tr>
<tr>
<td>2</td>
<td>0.0314</td>
<td>2.2203</td>
<td>0.4233</td>
</tr>
<tr>
<td>3</td>
<td>0.0478</td>
<td>2.4849</td>
<td>0.5266</td>
</tr>
</tbody>
</table>

Table 4: Conflict K of weighting average before and after

<table>
<thead>
<tr>
<th>Evidence</th>
<th>$K$ before weighting average</th>
<th>$K$ after weighting average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2</td>
<td>0.61</td>
<td>0.50</td>
</tr>
<tr>
<td>1 and 2 and 3</td>
<td>0.63</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 5: Fusion results after weighting average

<table>
<thead>
<tr>
<th>$m_{g}^{ullet}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{g}^{ullet}$</td>
<td>0.08</td>
<td>0.87</td>
<td>0.021</td>
<td>0.066</td>
<td>0.021</td>
<td>0.002</td>
</tr>
<tr>
<td>$m_{g}^{ullet}$</td>
<td>0.07</td>
<td>0.9</td>
<td>0.015</td>
<td>0.005</td>
<td>0.010</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Conflict K after weighting average is smaller. This is an important piece of information for us. BPA of fault mode $\theta_1$ reaches to 0.87 and 0.9, respectively in Table 5. Uncertainty falls to 0.002 and 0. From all these variations of figures, the effective of new method presented in this study is obvious to see.

CONCLUSION

In this article, a new method was proposed to weight different evidences. It based on the combination of information entropy with consistency ratio of judgment matrix. Subjective and objective factors are well considered in this process. During the process of weighting average, every expert revises his own view constantly. Which makes the process generates coincide opinion relatively. This process reflects the authoritative of decision-making. This new method has its value from theoretical and practical point of view. It could serve as a useful method to solve practical problem. It may appear as good enough in the fusion based on multiple cues.

ACKNOWLEDGMENT

This study was supported by the National Natural Science Foundation of China (No. 51075220) and basic research projects of QingDao city science and technology plan (No. 12-1-4-4-(3)-JCH).
REFERENCES


