Using Mathematical Optimization Model to Discuss Different Dividends Tax Rate and Financing Capacity

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Abstract: Through mathematical optimization model, this study discusses the impact of different dividends tax rate on the firm's financing capacity. Using the liquidity and risk management framework under asymmetric information, the theoretical analysis shows that tax reduces the financing capacity of the firm and profit of the entrepreneur. However, different tax rate on dividends will improve the financing ability of entrepreneurs and then increase their utility.

Key words: Mathematical optimization, tax rate on dividends, different rate, financing capacity

INTRODUCTION

In the last few decades, an important change in the global tax policy is the tendency of integration of the corporate income tax and profit distribution tax for the tax structure of a country. This is because in most countries, the tax system for investors is a double taxation system before. That means the tax department could levy tax on dividends obtained by investors in addition to corporate income tax on profits (Overesch and Voeller, 2010). This is a manifestation of the classical tax system and its significant feature is that the discrete between corporate tax and personal income tax (Soerensen, 1995). Arlen and Weiss (1995) believe that the system is “unfair and inefficient” due to the following two reasons: Firstly, double taxation means unified tax on company profits without taking into account the income discrepancy of the shareholders, in other words, this system violates the principle of vertical equity of the tax; secondly, it violates the principle of tax neutrality and even reduces the economic efficiency. Practice has fully proved that direct consequence of “the dividend tax system” is the increase in tax burden of the equity financing and cost of capital (Chetty and Saez, 2010). Therefore, reforming the traditional dividend tax system, decreasing the cost of equity financing, optimizing the capital structure and improving economic efficiency become the goal long sought by the countries.

From the perspective of financing needs, study for the determinants of capital structure and optimal selection problem is one of the core issues of financial theory (Hackbarth and Mauer, 2012). The modern capital structure theory is based on the MM theory. The theory shows that in situation without taxes, incentives and information problems, the effect is the same regardless of the manner in financing (Modigliani and Miller, 1958). Jensen and Meckling (1976) believe that the changes of enterprise value depend on operating manager's behavior, especially its “perks” consumption which will reduce corporate value and the cost is not the same in different financing ways. Myers and Majluf (1984) inspected the influence of asymmetric information on capital structure and proposed the “pecking order” theory.

On the basis of the liquidity and risk management model under asymmetric information (Holmstrom and Tirole, 2000), this study investigates the effect of different tax rate on dividends over the firm's financing capacity using the mathematical optimization model.

MODEL AND ASSUMPTION

This study introduces different dividends tax rate on the basis of the liquidity and risk management model, in the contest of the variable-investment framework. We will give the basic assumptions in the following:

- **Participants**: An entrepreneur, investors and the government
- **Three periods**: Date 0 represents ex enter period; date 1 represents intermediate period; date 2 represents ex post period
- At date 0, the entrepreneur has a project requiring a variable investment Ie[0, +∞). The entrepreneur has initial wealth A. To implement the project the entrepreneur must borrow I-A from investors

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At date 1, the investment yields deterministic and verifiable income if $\geq 0$ and the firm is hit by a random liquidity shock. Continuation, though, requires reinvesting an amount $\rho I$, where $\rho$ is ex ante unknown and has cumulative distribution function $F(\theta)$ with density $f(\theta)$ on $[0, +\infty)$. The realization of $\rho$ is learned at date 1.

If the firm does not reinvest $\rho I$, then the firm is liquidated. The liquidation value is 0. If the firm reinvests $\rho I$, then the firm yields, at date 2, RI with probability $p$ and 0 with probability $1-p$.

The probability of success $p$ is affected by the effort degree of entrepreneur, however the effort degree of the entrepreneur is unobservable. Behaving yields probability $p = \phi I$ of success and no private benefit to the entrepreneur and misbehaving results in probability $p = \phi I$, $\rho I$, and private benefit $B \geq 0$. Let $\Delta p = \rho I - \phi I$.

The investment has positive NPV (net present value) if the entrepreneur behaves, but negative NPV otherwise.

Both the entrepreneur and investors are risk neutral. Investors behave competitively in the sense that the loan, if any, makes zero profit. The entrepreneur is protected by limited liability.

For notational simplicity, there is no time preference. The government takes different tax policies to the stock dividends based on the holding period. $T_g$ is the statutory tax rate for short-term income, $T_o$ is the statutory tax rate for long-term income. Where $\Delta T$ is the different tax rate on dividends.

To ensure the existence of equilibrium, for any cutoff of reinvestment $\rho^*$:

$$0 < k_c < B / \Delta p$$

We summarize the timing in Fig. 1.

**OPTIMAL MODEL**

Suppose that the financing contract between the entrepreneur and investors takes the following state-contingent form:

$$[I, \rho^*; (0, R_0, 0), (I - T_o) I, (I - T_o + \Delta T) I, R_0, 0)]$$

where, the contract specifies an investment level $I$; the contract specifies that $\rho^*$ is a cutoff of reinvestment: if $\rho < \rho^*$, the firm continues; if $\omega = \rho^*$, the firm liquidates; at date 1, the entrepreneur gets nothing and the short-term income $(1 - T_o) I$ will totally owned by investors; if the project success, the entrepreneur and investors, respectively get $R_0$ and $(1 - T_o + \Delta T) I I R_0$ in the case of continuation; if the project fail, both of them get 0.

The investment has positive NPV only when the entrepreneur behaves, so according to the contract, the entrepreneur’s optimization problem becomes:

$$\begin{align*}
\max_{R_0, I} & \quad F(\rho) p \Delta R - A \\
\text{s.t.} & \quad F(\rho) p \Delta R - F(\rho) p (I - T_o) I (1 - T_o + \Delta T) I R_0 \\
& \quad - R_0 \geq 1 + \int_{\rho^*}^{\rho} \rho F(\rho) d\rho - A
\end{align*}$$

where, $F(\rho) = \Pr (\rho \leq \rho^*)$ is the probability of continuation; the objective function is the entrepreneur’s net utility; the first constraint is the entrepreneur’s incentive compatibility constraint; the second constraint is the investors’ individual rationality constraint and:

$$\int_{\rho^*}^{\rho} \rho F(\rho) d\rho$$

represents their expected reinvestment. The first constraint can be simplified as:

$$R_0 \geq B I / \Delta p$$

Fig. 1: Figure of the timing
The second constraint can be simplified as:

$$R_a \leq k_a I + \frac{A}{F(\rho')p_a}$$  \hspace{1cm} (4)

**OPTIMAL CONTRACT**

We solve the optimal contract in three steps:

**Step 1:** Solve the optimal $R_a$ for a given cutoff of reinvestment $\rho'$ and investment level $I$

Figure 2 illustrates the “feasible contract set” of optimization problem 2. We can get that, from a geometric point of view, the “feasible contract set” of optimization problem 2 is constituted by the shaded area $\triangle EOF$ surrounded by constraints 3 and 4 and non-negative constraint.

First of all, it is easy to get these results. The intercept of the individual-rationality constraint line is $A/[F(\rho')p_a]$ and the slope is $k_a$, the intercept of the incentive-compatibility constraint line is 0 and the slope is $B/\Delta p$. From Eq. 1 we can see that $0<k_a<B/\Delta p$. So, the “feasible contract set” $\triangle EOF$ is not an empty set if $A>0$. That is, there will be a successful contract as long as the entrepreneur’s wealth is non-negative. Secondly, the objective function shows that it is the bigger the better for $R_a$. Therefore, the point $F$ constitutes the optimal contract and the optimal contract is $R_{a,*} = B I / \Delta p$.

**Step 2:** Find the optimal investment level $I$ for a given cutoff of reinvestment $\rho'$

Actually, take $R_{a,*}$ into the optimization problem 2 and it could be further simplified as that:

$$\max_{\rho'} m(\rho') I$$

$$\text{s.t. } m(\rho') I - F(\rho') p_a B I / \Delta p = -A$$

(5)

Where:

$$m(\rho') = (1 - T_0) r + F(\rho') p_a (1 - T_3 + \Delta T) R - (1 + \int_0^{\rho'} \rho f(\rho) d\rho)$$

is the margin per unit of investment. Solution of the optimization problem 5 is:

$$\rho' = \rho_{*,*} = \rho_*$$

$$\frac{1}{1 - k_a} - k(\rho')A, \frac{1}{1 - k_a} - m(\rho') k(\rho') A$$

And:

$$k(\rho') = \frac{1}{1 - k_a} + \int_0^{\rho'} \frac{\rho f(\rho) d\rho - F(\rho') p_a [0 - T_3 + \Delta T] R - B / \Delta p - 0 - T_0] r}$$

denotes the financing capacity.

**Step 3:** Consider the optimal cutoff of reinvestment $\rho'$, we may get the following three propositions.

**Proposition 1:** Both of the investment level and financing amount reach their maximum when $\rho' = \rho_{*,*}$ where $\rho_{*,*} = \rho_*(1 - T_3 + \Delta T) R / B / \Delta p$.

**Proof:** Actually:

$$\max_{\rho'} k = \min_{\rho'} 1 + \int_0^{\rho'} \rho f(\rho) d\rho - F(\rho') p_a - (1 - T_0) r$$

The first-order condition is:

$$\rho' f'(\rho') - f'(\rho') \rho_{*,*} = 0 \Rightarrow \rho' = \rho_{*,*}$$

**Proposition 2:** The entrepreneur’s margin per unit of investment reaches its maximum when $\rho' = \rho_{*,*}$, where $\rho_{*,*} = \rho_*(1 - T_3 + \Delta T) R$.

**Proof:** Because the margin per unit of investment:

$$m(\rho') = (1 - T_0) r + F(\rho') \rho_{*,*} - 1 - \int_0^{\rho'} \rho f(\rho) d\rho$$

The first-order condition is:

$$f'(\rho') \rho_{*,*} - f'(\rho') \rho' = 0 \Rightarrow \rho' = \rho_{*,*}$$

**Proposition 3:** If the threshold liquidity shock is equal to the expected unit cost of effective investment, the
entrepreneur’s welfare reaches its maximum and this optimal threshold lies between the expected per-unit-of-investment pledgeable income and income:

\[ c(\rho^*) = \rho^*, \hat{\rho}_0 < \rho^* < \hat{\rho}_1 \]

Where:

\[ c(\rho^*) = \frac{1 + \int_{\rho_0}^{\rho^*} \rho f(\rho) d\rho - (1 - T_o) \tau}{F(\rho^*)} \]

**Proof:** In fact:

\[ U'_i(\rho^*) = \frac{\hat{\rho}_0 - c(\rho^*)}{c(\rho^*) - \hat{\rho}_0} A \]

\[ \max_{\rho'} U'_i(\rho^*) = \min_{\rho'} c(\rho^*) \]

The first-order condition is:

\[ \int_{\rho_0}^{\rho^*} F(\rho^*) d\rho - (1 - T_o) \tau = 1 \]

so \( c(\rho^*) \) gets its minimum when \( \rho^* = \rho^* \) and:

\[ c(\rho^*) = \frac{1 + \int_{\rho_0}^{\rho^*} \rho f(\rho) d\rho - (1 - T_o) \tau}{F(\rho^*)} = \rho^* \]

Because of:

\[ U'_i(\rho^*) = \frac{\hat{\rho}_0 - \rho^*}{\rho^* - \hat{\rho}_0} A > 0 \]

we may get that:

\[ \hat{\rho}_0 < \rho^* < \hat{\rho}_1 \]

Figure 3 illustrates the conclusion of Proposition 1-3: The margin \( m(\rho^*) \) and the multiplier \( m(\rho^*) \) are both decreasing above \( \hat{\rho}_1 \) and both increasing below \( \hat{\rho}_0 \). That means if \( \rho^* > \hat{\rho}_1 \), the project could not be financed profitably and if \( \rho^* < \hat{\rho}_0 \), the borrowing capacity and the entrepreneur’s utility would be infinite. Therefore, the entrepreneur chooses a lower threshold in comparison to the ex post efficient one. Actually, one would want to continue if and only if this is ex post efficient that is, if and only if \( \rho^* \geq \hat{\rho}_1 \). Indeed, \( \rho^* = \hat{\rho}_1 \) maximizes the margin.

**COMPARATIVE STATIC ANALYSIS**

**Proposition 4:** The “first-best cutoff” of reinvestment has nothing to do with the different tax rate \( \Delta T \) and decreases with the tax rate \( T_o \):

\[ \frac{\partial \rho^*}{\partial \Delta T} = 0, \frac{\partial \rho^*}{\partial T_o} > 0 \]

**Proof:** From Eq. 6, we may get that \( \rho^* \) has nothing to do with \( \Delta T \) and:

\[ \rho^* F(\rho^*) - 1 + \int_{\rho_0}^{\rho^*} \rho f(\rho) d\rho - (1 - T_o) \tau = \rho^* \]

Taking partial derivative about \( T_o \) on both sides, we can get that:

\[ \rho^* f(\rho^*) \frac{\partial \rho^*}{\partial T_o} + F(\rho^*) \frac{\partial \rho^*}{\partial T_o} = \rho^* F(\rho^*) f(\rho^*) + \tau \]

that is:

\[ \frac{\partial \rho^*}{\partial T_o} = \frac{\tau}{F(\rho^*)} > 0 \]

**Proposition 5:** The financing multiplier \( k(\rho^*) \) is increasing with the increase of different tax rate \( \Delta T \) and decreasing with the tax rate \( T_o \):

\[ \frac{\partial k(\rho^*)}{\partial \Delta T} > 0, \frac{\partial k(\rho^*)}{\partial T_o} < 0 \]

**Proof:** First, we would prove the relationship between financing multiplier \( k(\rho^*) \) and different tax rate \( \Delta T \), because of:

\[ k(\rho^*) = \frac{1}{[0 + \int_{\rho_0}^{\rho^*} \rho f(\rho) d\rho - F(\rho^*)] \rho_h (1 - T_o + \Delta T/B + \Delta \rho - T_o)} \]

and the “first-best cutoff” of reinvestment has nothing to do with the different tax rate, it is easy to get that, k is increasing with the increase of $\Delta T$ that means:

$$\frac{\partial k(\rho^*)}{\partial \Delta T} > 0$$

Secondly, we may prove the relationship between financing multiplier $k(\rho^*)$ and tax rate $T_0$, by assuming that:

$$L(\rho^*) = 1/k(\rho^*)$$

We could find that:

$$\frac{\partial L(\rho^*)}{\partial T_0} = (\rho^* - \hat{\rho}) \frac{\partial \rho^*}{\partial T_0} f(\rho^*) + F(\rho^*) \rho_0 R + r$$

Since, $\rho^* > \hat{\rho}$ and $\frac{\partial L(\rho^*)}{\partial T_0} > 0$, we believe that $\frac{\partial k(\rho^*)}{\partial T_0} > 0$.

**Proposition 6:** The margin per unit of investment $m(\rho^*)$ is increasing with the increase of different tax rate $\Delta T$:

$$\frac{\partial m(\rho^*)}{\partial \Delta T} > 0$$

**Proof:** Because of:

$$m(\rho^*) = (1 - T_0) [r + F(\rho^*) \rho_0 (1 - T_0 + \Delta T) R - (1 + \int_0^{\rho^*} \rho f(\rho) d\rho)$$

and the “first-best cutoff” of reinvestment has nothing to do with the different tax rate, it is easy to get that, $m(\rho^*)$ is increasing with the increase of $\Delta T$.

**Proposition 7:** Increase in the tax rate $T_0$ has positive and negative effects on the margin per unit of investment $m(\rho^*)$. If $\frac{\partial m(\rho^*)}{\partial T_0} > 0$, then $\frac{\partial m(\rho^*)}{\partial T_0} > 0$. Otherwise $\frac{\partial m(\rho^*)}{\partial T_0} < 0$. Where $L = [r + F(\rho^*) \rho_0 R] / [F(\rho^*) (\hat{\rho} - \rho^*)].$

**Proof:** Since:

$$\frac{\partial m(\rho^*)}{\partial T_0} = f(\rho^*) (\hat{\rho} - \rho^*) \frac{\partial \rho^*}{\partial T_0} - r - F(\rho^*) \rho_0 R$$

So, proposition 7 is true.

**Proposition 8:** The entrepreneur’s utility $U^*_e$, is increasing with the increase of different tax rate $\Delta T$ and decreasing with the tax rate $T_0$:

$$\frac{\partial U^*_e}{\partial \Delta T} > 0, \frac{\partial U^*_e}{\partial T_0} < 0$$

**Proof:** Firstly, because of:

$$U^*_e(\rho^*) = m(\rho^*) k(\rho^*) A$$

In addition, we know that both $k(\rho^*)$ and $m(\rho^*)$ increase with the increase of different tax rate from Proposition 5-6. It is easy to get that the entrepreneur’s utility is increasing with the increase of different tax rate $\Delta T$. Secondly, we may prove the relationship between entrepreneur’s utility and tax rate $T_0$, from Eq. 7 we know that:

$$U^*_e = \frac{\rho^* - \hat{\rho}}{\rho^* - \hat{\rho}} A = \frac{[p_0 (1 - T_0 + \Delta T) R - \rho^*] A}{(1 - T_0 + \Delta T) R - B}$$

therefore:

$$\frac{\partial U^*_e}{\partial T_0} = \frac{(p_0 + \frac{\partial \rho^*}{\partial T_0} f(\hat{\rho} - \rho^*) A}{(\rho^* - \hat{\rho})^2}$$

and it is easy to find that $\frac{\partial \rho^*}{\partial T_0} < 0$ from proposition 4 and $\hat{\rho} < \rho^*$, so $\frac{\partial U^*_e}{\partial T_0} < 0$.

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