Periodic and Spiking Oscillations in Driving a Long Arm Camera

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Abstract: Image acquisition in complex environment is a difficult problem. To get a high quality image, cameras are designed to be active in tracking a moving object. When the design for the camera is unreasonable, the oscillation of the camera usually causes an unsteady image switch and thus affects the image definition as well. A driving system of long arm camera is presented in this study. In the situation of the rotor and the camera being designed in quite different physical scales, the full system could be transformed into two subsystems. The geometric singular perturbation method and the eigenvalue analysis are introduced to investigate the factors affecting the dynamical response of the system. The mechanisms of driving the camera into a small amplitude oscillation, a large amplitude oscillation or a spiking oscillation are explained from the aspect of manifold and verified via numerical simulations.

Key words: Image acquisition, long arm camera, fast-slow system, geometric singular perturbation method, spiking oscillation

INTRODUCTION

In the fields of information technology, the acquisition and processing of image is an essential technique. Abundant researchers and technicians have made efforts to stably track a certain object in complex environment, both in digital technology and in hardware components. Comaniciu et al. (2003) proposed a new approach toward target representation and localization. The feature histogram-based target representations in this approach were regularized by spatial masking with an isotropic kernel. In (Xiang, 2009), the tracking method of Mean Shift based on the objects color feature as well as an active camera was used. In this study, the driving system of an active camera with a long arm is taken into consideration. To get a high quality image when tracking a moving object, abnormal oscillations of the camera are undesired and should be avoided. That demands a strict design strategy.

In some special situations, the rotor and the camera are usually designed in quite different physical scales. Different physical scales may lead to multiple time scales and such kind of system is named as the singularly perturbed system or the fast-slow system. Because of the special structures, general perturbation methods (Nayfeh, 1973) are usually ineffective in dealing with the fast-slow system. An improved perturbation method known as geometric singular perturbation method was proposed by Jones (1995) from the view of geometry.

The driving system of long arm camera is basically a rigid-link flexible-joint system. It was firstly derived by Spong (1987) and then Chen and Shieh (1995) improved Spong’s model into a delay-coupled one. In this study, the time delay induced by the propagation of angular displacement along the long arm from the camera to the rotor is considered. The effect of the time delay on the system response provides guidance for a better control of the camera. This study is organized as follows. In section 2, a physical model of a long arm camera is established. The geometric singular perturbation method and the eigenvalue analysis are introduced to investigate the influence of the time delay on motion of the camera. In section 3, periodic oscillations with different amplitudes are discussed. The formation mechanism of an undesired oscillation, spiking oscillation, is proposed in section 4 and Section 5 contains some conclusions.

MODEL ANALYSIS

The driving system of long arm camera is shown in Fig. 1. The long arm is modeled as a linear torsion spring with stiffness $K$. The moment of inertia of the camera is denoted as $I$ and the moment of inertia of the rotor is denoted as $J$. $\theta_1(t)$ and $\theta_2(t)$ denote the angular displacement of the camera and the angular displacement of the rotor, respectively. Both of their angular references are the vertical axis. Suppose that there is a transformation delay denoted as $\tau$ in propagation or reaction process of two angular displacements, one gains the following governing equation:

\[ \begin{align*}
&\theta_1(t) = \theta_1(t), \\
&\theta_2(t) = \theta_2(t), \\
&\theta_3(t) = \theta_3(t)
\end{align*} \]

and the equilibrium manifold:

\[ \begin{align*}
&\left( \theta_1, \theta_2, \theta_3, p_2 \right) \in \mathbb{R}^4: \\
&\theta_1(t) = 0, \\
&\theta_2(t) = 0, \\
&\theta_3(t) = 0, \\
&-\beta \sin \theta_1(t) = 0
\end{align*} \]

The characteristic equation of the fast subsystem is given by:

\[ D(\lambda) = \lambda^2 + \epsilon^{3i} \alpha_1 \lambda + \epsilon^{3i} + \beta \cos \theta_1 = 0 \]

When \(1 + \beta \cos \theta_1 < 0\), considering that Eq. 4 has a pair of pure imaginary roots \(\lambda = \pm i \omega\) for \(\omega > 0\), separating the real and imaginary parts and considering that \(\sin^2 (\omega \tau) + \cos^2 (\omega \tau) = 1\), one obtains the critical boundaries of the stability of the slow manifold. In Fig. 2, the critical boundaries are displayed on the \( \tau - \theta_1 \) plane for \( \alpha_1 = 1 \), \( \beta = 3 \).

Once the time delay is determined, a horizontal line \( \tau = \tau_{\text{crit}} \) would cross the critical boundaries from the left to the right and separate the slow manifold into several segments. The stability of these segments would be discussed in following sections.

**PERIODIC OSCILLATIONS**

**Small amplitude oscillation:** In practical situations, a periodic oscillation is usually an ideal motion of the camera. An obvious reason is that when the camera moves in a stable period, the image tends to be smooth. The formation mechanism of a small amplitude oscillation is illustrated through the following example. In this example, the outside control torque is assumed to be \(u(t) = 0\). As shown in Fig. 3, a horizontal line \( \tau = 0.6 \) crosses the critical boundaries and intersects with the boundaries at two points. These intersection points are defined as the stability switch points of the slow manifold, separating the slow manifold into five segments, as shown in Fig. 4.

In Fig. 4, two Hopf bifurcation points \( H_l \) and \( H_r \) are obtained, indicating that the left equilibrium \( E_l \) and the right equilibrium \( E_r \) would bifurcate into two limit cycles. The multi-stability in Fig. 5 illustrates two small amplitude oscillations.
Fig. 3: Zoom of Fig. 2 for τ = 0.6

Fig. 4: Intersection of two manifolds when τ = 0.6

Fig. 5: Two small amplitude periodic oscillations

oscillations about two different equilibrium positions. The equilibrium positions of the periodic oscillation of the camera are determined by the initial conditions.

Large amplitude oscillation: As time delay increase to τ = 0.7, these two limit cycles expand to encounter with each other. As a result, a new limit cycle centering at the saddle point E₂ is formed. In fact, this new limit cycle represents a large amplitude periodic oscillation. The phase portraits are as follows in Fig. 6. It is easy to conclude that the oscillation amplitudes of those two former limit cycles are much smaller than the amplitude of this new one in Fig. 7. Thus, the camera is summarized to exhibit three kinds of periodic oscillations. Among them, one is a large amplitude oscillation around the second equilibrium position and the other two are small amplitude oscillations around the first and the third equilibrium positions.

The solution trajectories are illustrated in Fig. 8. The foregoing analysis shows that the trajectory may not be attracted to the left or the right small limit cycles even the equilibrium manifold intersects with two branches of the slow manifold. Instead, a larger amplitude periodic oscillation may arise, as long as the value of the time delay is appropriate. In this situation, the variation of the initial condition or the outside control makes no difference.
HOMOCLINIC ORBIT AND SPIKING

Recall the first and the second cases in the previous section. The geometric singular perturbation analysis and the numerical simulation results had stated clearly that in the case of $\tau = 0.6$, there existed two small limit cycles centering at the left and the right branches of the slow manifold. While in the case of $\tau = 0.7$, a big limit cycle encircling these two small limit cycles appeared. Intuitive deduction is that there should be a certain moment when the two small limit cycles expand to encounter with each other; namely, there exists a homoclinic orbit. Numerical analysis reveals that in this moment, the value of the time delay is near about 0.64. The simulation results are as follows in Fig. 9 and 10.

When control torque is $u(t) = 0$: Starting from an initial point (2, 3), the corresponding results are presented in Fig.9 and Fig.10. In this case, the movement of the camera is an approximate uniform periodic oscillation, but the switch of the oscillation direction is relatively sharp.

When control torque is $u(t) = a\dot{\theta}_1 \cos(\tau t) + b$, $a = -5$, $b = 15$: With the same method, a long arm camera under an inappropriate outside control torque is analyzed to possess no stable limit cycles. The trajectory of the system would switch between the homoclinic orbit and the tiny stable part of the slow manifold and thus forms a loop, as shown in Fig. 11. The system displays an abnormal behavior of the camera. This kind of motion includes a fast process of impulsive oscillation and a slow process of a nearly constant angular displacement. It is called the spiking
Fig. 12: Undesired spiking oscillation

oscillation. In the practical application of image acquisition, the spiking oscillation of the long camera is a big trouble and should be avoided.

CONCLUSION

A driving system of long arm camera is investigated in this study. A reasonable design of the driving system helps to get a high image definition and a steady image switch in the process of image acquisition. The purpose of this study is to clarify the factors affecting the design rationality. First of all, the effectiveness of the geometric singular perturbation method is proved in dealing with the long arm camera system with multiple time scales. Then, the stability analysis of the slow manifold explains the formation mechanism of the periodic oscillations and the spiking oscillation of the camera. Simulation results indicate that with the increase of the transformation delay, the long arm camera behaves from a small amplitude periodic oscillation to a large amplitude periodic oscillation. Between them, an approximate uniform periodic oscillation with a sharp switch of the oscillation direction occurs. Particularly, when an inappropriate outside control torque is attached to the arm, the extremely undesired spiking oscillation of the camera appears. The results in this study provide a guidance to suppress the undesired oscillation in the driving process of a long arm camera and to acquire a stable high quality image in the complex environment.

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REFERENCES


