Fuzzy Control and Lyapunov-based Stability Control of the Three-phase Shunt Active Power Filter

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Abstract: When the load and system parameters are changing, the APF usually appears unstable and its exact mathematical model is quite difficult to determine. This study is to introduce Takagi Sugeno (TS) type fuzzy controller and Lyapunov-based stability theory into the Direct Current (DC) side voltage control and current control of the three-phase Active Power Filter (APF), aiming to guarantee the precision and stability of APF. Firstly, the TS fuzzy control is applied to voltage error and error rate at DC side to attain the source current amplitude of reference value, then the Lyapunov method is adopted to design the switching function of APF and the global stability of control system is ensured by insuring the derivative of energy-like Lyapunov function always negative. The parameters of TS fuzzy controller are optimized by using Adaptive Neuro-Fuzzy Inference System (ANFIS) in Matlab/Simulink. As the results of comparative tests show, the proposed method can guarantee the precision and stability of APF under the conditions of varying load and system parameters.

Key words: Active power filter (APF), takagi-sugeno fuzzy control, lyapunov method, stability control, adaptive neuro-fuzzy inference system

INTRODUCTION

As the expend use of electronics and other nonlinear loads, power quality problems occur, such as harmonic pollution and so on (Zhang et al., 2008). An important solution is applying the APF, which includes voltage control, reference current calculation and current control. Proportional-Integral (PI) control is usually applied to the voltage control. The current PI controller is easy to design and reliable when the exact mathematical model is easy to determine (Liu, 2004). But when the load and parameters are changing, as well as the model is nonlinear, it is difficult to determine the exact mathematical model (Saad and Zellouma, 2009). APF is in this situation, so the effectiveness using PI control is not so satisfied.

Fuzzy control doesn’t need the exact mathematical model of the subject. It adopts the lingual control rules directly, while the numerical results can be received through fuzzy sets and fuzzy logic reasoning and used as the control outputs (Shi and Hao, 2008). Mamdani type fuzzy controller and Takagi-Sugeno (TS) type fuzzy controller are often used, whose main difference is the output of the TS fuzzy controller is constant or linear function of the input parameters but without the defuzzifier. The greatest limitation of the Mamdani type fuzzy controller is that lots of fuzzy rules and parameters optimization are needed, which increases the complexity of the controller. TS fuzzy controller needs less fuzzy sets and fuzzy rules, which can compute faster.

The key point of TS fuzzy controller is to set up the TS model. As the Matlab application, the Adaptive Neuro-Fuzzy Inference System (ANFIS), which combines the neural network theory and TS fuzzy logic, is used to set up TS model through adaptive modeling method with mass data. Kumar et al. (2011) applied TS fuzzy controller to three-phase APF and Unified Power Quality Conditioner (UPQC). Hamadi et al. (2004) utilize ANFIS to optimize the parameters of TS fuzzy controller, whose control performance is better than the traditional PI controller. This study utilizes ANFIS to optimize the parameters of TS fuzzy controller and then adopts the TS fuzzy controller to the voltage control of the APF.

When the load is changing abruptly, the APF may be unstable, aggravating the compensation. Lyapunov stable control is used to guarantee the stability of the system. Lyapunov stable control has been used in the current control of the APF (Hou et al., 2010). When the load and system parameters are varying, in order to improve the
stability, this study utilizes the Lyapunov-based stability theory to design the current control strategy. Komurcuğil and Kukrer (2006), Wei et al. (2012) utilized the Lyapunov method to design the current controller of APF. While calculating the output quantity, the reference voltage was replaced with the steady value of DC side voltage. However, this study calculates using the steady value directly, which is more accuracy.

This study analyzes the theories of the control methods and done some tests to verify the effectiveness of the proposed methods. Firstly, the reference current calculation and voltage control are introduced. Secondly, the three-phase shunt APF is modeled. Thirdly, the nonlinear current control using Lyapunov stable control is explained. Fourthly, the DC side voltage ripple of APF is estimated. At last, the simulation is done using Matlab/Simulink in the case of changing load and system parameter, taking the AC side inductance parameter of APF for example. The mentioned methods are quite efficient.

REFERENCE CURRENT CALCULATION AND VOLTAGE CONTROL

**Reference current calculation:** APF is to compensate harmonic current and to increase power factor. Define the reference APF current as $i^*_{\text{LH}} = i^*_{\text{LH}} + i^*_{\text{LH}} + i^*_{\text{LH}}$, while $i^*_{\text{LH}}$ presents the reference source current and $i^*_{\text{LH}}$ presents the load current. To attain the unity power factor, define the reference source current as

$$i^*_{\text{LH}} = I_m(t) \sin(\omega t)$$  \hspace{1cm} (1)

$$i^*_{\text{LH}} = I_m(t) \sin(\omega t - 2\pi/3)$$ \hspace{1cm} (2)

$$i^*_{\text{LH}} = I_m(t) \sin(\omega t + 2\pi/3)$$ \hspace{1cm} (3)

where, $I_m$ presents the amplitude of the reference source current and $\omega$ can be attained through PLL. As the fluctuation of DC side of APF can be shown the changing of source current amplitude, which can be attained through TS fuzzy control on the variation of DC side voltage (Wu and Jow, 1996).

**TS fuzzy control:** The characters of fuzzy controller lie on the chosen of fuzzifier and fuzzy rules but not the exact model of APF. Figure 1 shows the theoretical diagram of TS fuzzy controller, which is with double inputs and single output. The DC side voltage difference $e$ and its variation $e_1$ are fuzzified through D/F module. The reference source current amplitude $I_e$ can be achieved according to the output function $u - p e + q e_1 + k$. The core of the TS fuzzy controller is to construct the output function. Figure 3 shows the structure of TS fuzzy controller. From the first to fifth column are the input, fuzzifier, fuzzy rules, output functions and output. This design contains two inputs and one output. Each input is allocated to seven fuzzy sets.

The key of the TS fuzzy controller is to ascertain the parameters of the membership function and fuzzy rules. According to a large number of input and output data, through training and testing with Adaptive Neuro-Fuzzy Inference System (ANFIS) in Matlab/Simulink, the parameters of the TS fuzzy controller are optimized.

Figure 3 shows the membership functions of input values of APF DC side voltage difference, while Fig. 4 shows the membership functions of input values of APF DC side voltage difference variation. Seven fuzzy sets are chosen separately, whose responding membership functions are Gaussian function. Take Fig. 3 in account,
MODELING OF APF

The connection diagram of APF and power system is shown in Fig. 5, where C presents the DC side capacitor, L presents the filtering inductance, R presents the equivalent resistance. The current reference direction is shown in the figure, such as source current $i_{s(a,b,c)}$, load current $i_{l(a,b,c)}$, inverter output current $i_{r(a,b,c)}$ and DC side current $i_c$. $u_{ao(b,c)}$ presents the voltage of the Point of Common Coupling (PCC).

According to KCL and KVL, the switching functions of APF are as follows:

$$L_i\dot{u}_{i_h} + R_i u_i = u_{a_h} - \alpha_i u_a + u_{ao}$$  \hspace{1cm} (4)

$$L_i\dot{u}_{i_h} + R_i u_i = u_{a_h} - \alpha_i u_a + u_{ao}$$  \hspace{1cm} (5)

$$L_i\dot{u}_{i_h} + R_i u_i = u_{a_h} - \alpha_i u_a + u_{ao}$$  \hspace{1cm} (6)

$$C_{ii} = \alpha_i u_{i_h} + \alpha_i u_{i_h} + \alpha_i u_{i_h}$$  \hspace{1cm} (7)

$$u_{co} = \frac{1}{3} u_{ao} (\alpha_1 + \alpha_2 + \alpha_3)$$  \hspace{1cm} (8)

where, $\alpha_o$, $\alpha_i$ and $\alpha_1$ are the control output (duty ratio function) of APF. After dq transformation, the above equations can be written as followed:

$$L_i\dot{u}_{i_h} = -R_i u_i + \omega L_{i_h} + u_a - u_{ao} \alpha_3$$  \hspace{1cm} (9)

$$L_i\dot{u}_{i_h} = -R_i u_i - \omega L_{i_h} + u_a - u_{ao} \alpha_3$$  \hspace{1cm} (10)

$$C_{ii} = \frac{3}{2} (i_{a_1} \alpha_{a_1} + i_{a_2} \alpha_{a_2})$$  \hspace{1cm} (11)

Assuming that $\alpha_o = \tilde{a}_o + \Delta \alpha_o$ and $\Delta \alpha_o = \Delta \alpha_o$, where $\tilde{a}_o$ and $\tilde{a}_o$ are the steady values, while $\Delta \alpha_o$ and $\Delta \alpha_o$ are the fluctuating value. Assuming that $u_{ao} = \tilde{u}_{ao} + \Delta u_{ao}$, where $\tilde{u}_{ao}$ is the steady value, while $\Delta u_{ao}$ is the fluctuating value. The object of $i_{ah}$ and $i_{ah}$ are to tracing $I^*_{ah}$ and $I^*_{ah}$, while $\tilde{u}_{ao}$ is to tracing $U_{ao}$. Assume that $x_i = i_{ah} - i_{ah}$ and $x_j = u_{ao} - \tilde{U}_{ao}$. Equation 9-11 can be changed as:

$u_{ao} = 1.713e - 0.05922e - 3.419$

$u_{ao} = 1.085e - 0.00998e - 0.3386$

$u_{ao} = 1.518e + 1.6e - 8.25$
\[ Lx_t = -Rx_t + olx_t - \alpha_t \Delta x_t + u_{st} - \left(U_s + \Delta U_s\right) \alpha_t \quad (12) \]

\[ Lx_s = -Rx_s - olx_s - \alpha_s \Delta x_s - \left(U_s + \Delta U_s\right) \alpha_s \quad (13) \]

\[ Cx_s = \frac{3}{2} \left(x_s \alpha_s + x_s \alpha_s\right) - \frac{3}{2} \left(i_s \alpha_s + i_s \alpha_s\right) \quad (14) \]

Assume APF injects \( I^* \) then:

\[ Li_{ti} = -Ri_{ti} + ol\bar{i}_{ti} + u_{st} - \left(U_s + \Delta U_s\right) \bar{\alpha}_t \quad (15) \]

\[ Li_{ti} = -Ri_{ti} - ol\bar{i}_{ti} + u_{st} - \left(U_s + \Delta U_s\right) \bar{\alpha}_t \quad (16) \]

\[ C\Delta u_{st} = \frac{3}{2} \left(i_{st} \bar{\alpha}_t + i_{st} \bar{\alpha}_t\right) \quad (17) \]

Substitute (15-17) into Eq. 12-14 separately, Eq. 13-15 can be changed as followed:

\[ Lx_t = -Rx_t + olx_t - \bar{\alpha}_t x_t - \left(x_t - U_s + \Delta U_s\right) \alpha_t \quad (18) \]

\[ Lx_s = -Rx_s - ox_s - \bar{\alpha}_s x_s - \left(x_s + U_s + \Delta U_s\right) \alpha_s \quad (19) \]

\[ Cx_s = \frac{3}{2} \left(x_s \bar{\alpha}_s + x_s \bar{\alpha}_s\right) + \left(x_s + \bar{\alpha}_s\right) \Delta \alpha_s + \left(x_s + \bar{\alpha}_s\right) \Delta \alpha_s \quad (20) \]

**CURRENT CONTROL BASED ON LYAPUNOV METHOD**

According to the Lyapunov stable control theory, any linear and nonlinear system can be globally asymptotically stable; if the energetic function \( V(x) \) satisfies:

- \( V(0) = 0 \)
- When \( x+0 \), \( V(x) > 0 \) and \( V(x)<0 \)
- When \( |x| \to \infty \), \( V(x) \to \infty \)

According to the error of the compensate current and the DC side voltage, considering the influence of the system parameter varying, construct the energetic function \( V(x) \) and the differential \( V(x) \) as [10]:

\[ V(x) = \frac{1}{2} Lx_t^2 + \frac{1}{2} Lx_s^2 + \frac{1}{3} Cx_s^2 \quad (21) \]

\[ V(x) = -Rx_t^2 - Rx_s^2 + \left((x_t - U_s) x_t + i_{st} \bar{\alpha}_t\right) \Delta \alpha_t + \left((x_s - U_s) x_s + i_{st} \bar{\alpha}_t\right) \Delta \alpha_s \quad (22) \]

In order to ensure \( V(x)<0 \), define the as:

\[ \Delta \alpha_t = \beta_t \left((x_t - u_s) x_t + i_{st} \bar{\alpha}_t\right) \quad (23) \]

\[ \Delta \alpha_s = \beta_s \left((x_s - u_s) x_s + i_{st} \bar{\alpha}_t\right) \quad (24) \]

where, \( \beta_t \) is a constant and \( \beta_t < 0 \). Substitute \( x_t = i_{st}^* + x_t \) and \( x_s = \bar{i}_{st}^* + x_s \) into Eq. 16-17 and 24-25:

\[ \bar{\alpha}_t = \frac{-L_{st}^* - R_{st}^* + ol\bar{i}_{st} + u_{st}}{U_s + u_s - \bar{u}_s} \quad (25) \]

\[ \bar{\alpha}_s = \frac{-L_{st}^* - R_{st}^* + ol\bar{i}_{st} + u_{st}}{U_s + u_s - \bar{u}_s} \quad (26) \]

\[ \Delta \alpha_t = \beta_t \left(i_{st^2} u_s - (U_s + u_s - u_s) i_{st}^2\right) \quad (27) \]

\[ \Delta \alpha_s = \beta_s \left(i_{st^2} u_s - (U_s + u_s - u_s) i_{st}^2\right) \quad (28) \]

**Estimation of DC side voltage ripple:** Wei et al. (2012) simplified the control method based on Lyapunov stability theory, which neglect the influence of the DC side ripple without any explanation. Fang (2001) proposed an estimation method of the DC side capacitor voltage ripple based on current deposition. Define the APF DC side capacitor voltage fluctuation \( \delta = \frac{U_c}{u_{ac}} \) where \( U_c \) is the ripple voltage amplitude of \( u_{ac} \).

Assume the nonlinear load is three-phase rectifier, whose DC side is varying resistance load. Then the phase current of the AC side can be written as followed:

\[ i_s = \frac{2\sqrt{3}}{\rho} I_s \left[ \sin \omega t + \sum_{n=1,3,5} (-1)^{n+1} \frac{1}{n} \sin n\omega t\right] \quad (29) \]

where, \( I_s \) presents the DC side current of the rectifier load, then the fluctuation can be written as followed:

\[ \delta = \frac{U_{ac}}{u_{ac}} \frac{3\sqrt{2}E_1 + b_1 \omega L_1}{2\omega C_1} \quad (30) \]

where, \( E_1 \) presents the peak value of phase voltage, \( a_1 = -0.00951^*, b_1 = -0.03865^*(1^*) \), \( \tau = 2\sqrt{3}\beta_1/\rho \).

To calculate the max fluctuation of the APF DC side capacitor voltage, according to the simulation condition below, the max current effective value of the rectifier load DC side is calculated firstly, whose result is as followed:
\[ I_{\text{max}} = \frac{U_{\text{dc}}}{R_{\text{max}}} \times \frac{2}{2+0.5+0.5} = 171.6 \text{A} \]
\[ I' = \frac{2\sqrt{2}}{p} I_p = \frac{2\sqrt{2}}{p} I_{\text{max}} = 189.31 \text{A} \]

Then substitute the above calculation results into Eq. 30, the fluctuation can be achieved:

\[ \delta = 0.084\% \]

The fluctuation is quite small under the simulation conditions in this study. However, the fluctuation of DC side capacitor voltage is decided by many factors. When the load and system parameters are varying, the DC side capacitor voltage is influenced. Therefore, this study does not do any simplify.

**SIMULATION AND VERIFICATION**

To verify the effectiveness of the proposed method, three phase shunt APF are tested by the simulation of Matlab/Simulink. The connection diagram is as shown in Fig. 1. The experiment conditions are as followed. The grid line voltage is 380V, where the amplitude of phase voltage is \( E_\text{u} = 220\sqrt{2} \text{V} \). The grid frequency is 50 Hz. The values of the source inductance, source resistance and capacitor of the APF DC side voltage are \( L_s = 1 \mu \text{H} \), \( R_s = 0.5 \Omega \), \( C = 5 \text{ mF} \), separately. The value of reference DC side voltage is \( U_{\text{ref}} = 800 \text{ V} \), while the AC side resistance is \( R = 0.5 \). The simulation time is set to be 0.2 sec. While in the time interval 0-0.1 sec, the value of AC side filtering inductance is \( L = 0.2 \text{ mH} \). While in the time interval 0.1-0.2 sec, the value of AC side filtering inductance is \( L = 0.1 \text{ mH} \). The nonlinear load is modeled as three-phase rectifier load, whose DC side is connected with varying resistance load. In the interval 0-0.04, 0.04-0.08, 0.08-0.12, 0.12-0.16, 0.16-0.2 sec, the values of the resistance load are 10, 3.33, 10, 2, 10 \Omega.

When the values of nonlinear load and filtering inductive voltage are varying, the compensation conditions of the APF to the change of load and parameter value are shown in Table 1. Before compensation, the source voltage and current are distorted, especially to the source current, whose value of Total Harmonic Distortion (THD) is up to 25%. After compensation using the proposed method, the values of THD are decreased, especially to the source current, whose value of THD is reduced greatly.

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Compensation condition</th>
<th>THD of source voltage (%)</th>
<th>THD of source current (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-0.2 sec</td>
<td>Before compensation</td>
<td>4.55</td>
<td>3.31</td>
</tr>
<tr>
<td></td>
<td>After compensation</td>
<td>25.21</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Take phase A of the three-phase system for example, Fig. 6 shows the compensation of APF with the change of load and parameter, including the waveforms of source voltage, source current, load current, APF compensating current and APF DC side voltage. The amplitude of load current is varying abruptly, almost changing one time every 0.04 sec, whose THD is quite high, which is shown in Fig. 6(c). After compensation, the source current is sinusoidal like and almost in phase with the source voltage, which is shown in Fig. 6(a-b). The APF compensating current is varying greatly, whose value of THD is very high, which is shown in Fig. 6(d). The DC side voltage is quite near the reference voltage and it can rise up quickly to the reference voltage nearby even when the load changes abruptly, which is shown in Fig. 6(e).

**CONCLUSION**

As the exact mathematical model of the APF is quite difficult to determine, especially when the load and the parameters are changing. This study introduces the TS fuzzy controller to the voltage control of the APF, which can achieve better tracing result for adjusting the
parameters of output function according to the error and error changing rate of the DC side voltage. Applying the current control method based on Lyapunov stable theory, APF can work stable under the circumstance of changing of load and parameters. A globally stable control is possible and it is insensitive to the parameter uncertainties and robustness to the disturbances. Through testing with the Matlab/Simulink, the effectiveness of the proposed method is confirmed.

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