Numerical Analysis of Loan-to-value Ratio of Inventory Financing Based on Logistics Financial Innovation

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Abstract: Based on a kind of dynamic pledge loan under logistics financial innovation, this study adopts risk evaluation method of “corporate and debt”, considers the impact of time to capture and liquidity risk and establishes loan-to-value ratio decision model of inventory financing with randomly-fluctuate price. And then, takes copper for an example, confirms all model parameters and analyzes the impact of different factors on loan-to-value ratio. The research shows: these factors such as marking-to-market frequency, loan period, default probability, time to capture and liquidity risk have different impacts on loan-to-value ratio and the numerical analysis of their correlativity can provide scientific guidance for risk control practice of decision-makers of inventory financing.

Key words: Logistics finance, loan-to-value ratio, risk control

INTRODUCTION

As a fundamental type of logistic financing, inventory financing is a kind of practical chattel pledge with the participation of logistic firms. In the operation process of inventory financing, the borrowers offer their own inventory to banks and then the banks lend money to the borrowers and delegate logistics firms to store and monitor these inventory collateral. Recently the operation integration of logistics and finance becomes a new developing field which is paid close attention to by the finance circles and logistics circles and is getting more and more attention of various circles of society. In order to match with developing demand of inventory financing, banks need improve risk control of the operation. Especially, loan-to-value ratios discussed in this study, which is the ratio of the face value of debt to the collateral value pledged against the debt, is very important in inventory financing operation, but banks rely on the crude rules of thumb to determine the ratio completely in reality. The crude rules make banks not to determine appropriate loan-to-value ratios to control operation risk effectively. Therefore, the quantitative research on loan-to-value ratios may provide scientific approaches for bank and logistic enterprises to decide in inventory financing.

In theory, if the collateral is standard inventory commodity, such as aluminum, copper or petroleum, which can be exchanged in financial market, the quantitative analysis of loan-to-value ratios of inventory financing belongs to the research category of “the impact of collateral on credit risk exposures” and “the determination of loan-to-value ratios of collateral”. These researches may be divided into two kinds of approaches: structural form and reduced-form approaches. The structural form approach extends the Merton (1974) option-based framework and models credit risk as a short position in a put option. Based on merton’s framework, Stulz and Johnson (1985) firstly study the impact of collateralisation on the pricing of secured debt. Then, Jokivuolle and Peura (2003) present a model of collateral haircut determination for bank loans. In a word, in the structural form approach, the default of enterprises is assumed to be endogenous, which implies that the credit operation contract and the value of the collateral alone would push the enterprises to default. But actually, a wider variety of reasons which might be totally unrelated to the specific contract considered (e.g., a company might experience liquidity problems and file for bankruptcy) might make enterprises default and at the same time, enterprises also mightn’t default when the value of collateral is lower than the debt. Just considering these reasons, Cossin and Huang (2003) determine the default probability of enterprises exogenously and acquire the determination model of collateral haircut consistent with the level of risk tolerance of bank based on the reduced-form approach which is brought forward by Jarrow and Turnbull (1995) and Duffie and Singleton (1999).

The above researches on loan-to-value ratios are mainly focused on financial commodity. On the basis of these researches, the study focuses on the stock with
randomly-fluctuant price, considers risk estimation strategies of "mainbody+debt" and analyzes the loan-to-value ratio decision of the bank with downside risk constraint under dynamic pledge fashion.

**MODEL OF LOAN-TO-VALUE RATIO IN DYNAMIC PLEDGE FASHION**

If collateral is standard inventory commodity, their prices are assumed to follow geometric Brownian motions.

At the beginning of the loan contract, the bank asks the borrower for \( a_0 \) units of standard commodity as collateral which is consigned for logistics enterprises to store and whose spot price is \( B_0 \) and then determines a loan-to-value ratio \( \omega \) for each unit collateral. Thus, the model assumptions of inventory financing under dynamic pledge fashion are enumerated as follows:

**Assumption 1**: Price fluctuation of commodity collateral follows stochastic process of logarithmic normal distribution. The yield rate of compound interest of inventory follows normal distribution:

\[
\ln \left( \frac{B(t)}{B(t_1)} \right) - N \left( \left[ \mu_0 - \frac{1}{2} \sigma^2_0 \right] (t_2 - t_1), \sigma_0^2 (t_2 - t_1) \right)
\]

where:

\[
\sigma = \sigma_0 \mu_0 = \mu_0 - \frac{1}{2} \sigma_0^2 \mu_0
\]

is expected value of natural logarithm of inventory yield per unit time, \( \sigma \) is standard deviation of natural logarithm of inventory yield per unit time.

**Assumption 2**: Lending rate for this operation is denoted by constant \( R \), so the underlying loan value at \( t \) will be \( v_t = v_0 e^{\delta t} \) and the deposit rate \( r \) is defined to be risk-free rate.

**Assumption 3**: We assume that the maturity of the loan is \( T \), which is divided into \( K \) marking to market periods with interval \( \tau (\tau K = T) \). Moreover, at each marking to market time, margin call is assumed to happen and we will consider \( 0 \) as a trigger level for the margin call, i.e., as soon as the loaneable value of the collateral diverges from the underlying value, there is a margin call in order to reestablish equivalency.

**Assumption 4**: The constant default probability is assumed to be exogenous and independent of the inventory financing. If \( Q \) represents the annualized default probability, then \( \tau Q \) would represents the default probability in each period.

At the beginning of period \( k \) (\( k = 1, 2, 3, \ldots, K \)), the underlying value \( v_k e^{\delta (k-1) \tau} \) is the principal and interest of the loan, \( \alpha^{(k)} \) is the amount of collateral with unit price \( B_k \) and so we have the equivalency \( v_k e^{\delta (k-1) \tau} = \alpha \sigma_0 B_k \).

If the bank takes the dynamic pledge fashion of total value balance, at the end of period \( k \) of the loan contract, we face three situations:

- The underlying value is equal to the loaneable value of the collateral, i.e., \( v_k e^{\delta (k-1) \tau} = \omega \). In this situation, the borrower will neither deposit nor extract the collateral and the contract continues.
- The underlying value is smaller than the loaneable value of the collateral, i.e., \( v_k e^{\delta (k-1) \tau} < \omega \). In this situation, the borrower will receive an extra amount of collateral back such that:

\[
P(\text{loss} \geq L) = \sum_{i=1}^{K} \mathbb{Q}(\text{loss} \geq L) \mathbb{N} \left( \frac{D(\omega) - \mu_0 - \frac{1}{2} \sigma_0^2 \mu_0}{\sigma_0 \sqrt{\tau}} \right)
\]

and the contract continues.
- The underlying value is larger than the loaneable value of the collateral, i.e., \( v_k e^{\delta (k-1) \tau} > \omega \). In this situation, the margin call happens and the borrower is required to deposit more collateral such that \( v_k e^{\delta (k-1) \tau} = B_k \omega \). If the borrower defaults in this period, it will not be able to fulfill the requirement. The contract will stop and enter into a liquidation process and the bank will have a maximum loss of \( v_k e^{\delta (k-1) \tau} = B_k \omega (1 - \theta) \).

The maximum loss that the bank is willing to take is denoted by \( L \) which is the function of the loan amount \( v_0 \). We have \( L = v_0 \), where \( L \) is defined to be the degree of loan loss. If the bank takes the maximum loan-to-value ratio satisfying downside-risk constraint of the loan as the final one, we get the decision model of loan-to-value ratio:

\[
P(\text{loss} > L) < \beta, \quad \text{where} \quad \beta \quad \text{is the risk tolerance level of the loan, i.e., the bank requires that the probability of} \quad P(\text{loss} > L) < \beta.
\]

According to the basic assumptions, at \( k \) period of inventory financing, the probability that the bank considers is the joint probability of the event \( X_0 \) that the inventory's price fluctuation incurs a loss \( \delta s_0 \) more than \( L \) and the event \( Y_0 \) that the corporate defaults. The joint probability is \( P(k) = P(X_0 \cap Y_0) = P(X_0) P(Y_0) \).

According to the above assumption and analyses, we may get the following lemma and theorem.

**Lemma 1**: If the borrower's main body defaults, at \( k \) period, the probability that the price fluctuation of inventory incurs a loss \( \delta s_0 \) more than \( L \) is:

\[3354\]

\[ P(X_0) = \Phi \left( \frac{G(\omega) - (1 + \delta) \tau \left( \mu_0 - \frac{1}{2} \sigma_0^2 \right)}{\sigma_0 \sqrt{(1 + \delta) \tau}} \right) \]

The probability \( P(X_0) \) is the monotonic increasing function of \( \omega \) and:

\[ G(\omega) = \ln \left( \frac{\alpha e^{(\omega+\delta)\tau} - 1}{\omega x - e^{(\omega+\delta)\tau}} \right) \]  

(1)

**Theorem 1:** Given the impact of time to capture and liquidity risk, the loan-to-value ratio that the bank finally determines in dynamic pledge fashion follows:

\[ \sum_{t=1}^{\infty} (1-\gamma^t) \epsilon_t \Phi \left( \frac{G(\omega) - (1 + \delta) \tau \left( \mu_0 - \frac{1}{2} \sigma_0^2 \right)}{\sigma_0 \sqrt{(1 + \delta) \tau}} \right) = \beta \]  

(2)

**NUMERICAL ANALYSIS OF LOAN-TO-VALUE RATIO UNDER DYNAMIC PLEDGE FASHION**

According to the analysis of theorem 1, if collateral is standard inventory commodity such as copper which can be exchanged in futures market, their price fluctuation follows logarithm normal distribution.

By analyzing the price data of “Yangzi River 1” copper from March 17 of 2006 to March 23 of 2007 and by applying Kolmogorov-smirnov test to analyze whether logarithm data of return per week is normal distribution, we get Asymp. Sig. probability is 0.925, so logarithm data of return per week is seen to be normal distribution. Then, we may get the mean of logarithm data of return per week \( \mu_0 = 0.0051 \), Std. Deviation \( \sigma_0 = 0.058 \), so we get mean of logarithm data of return per year \( \mu = \mu_0 \times 50 = 0.255 \), Std. Deviation:

\[ \sigma = \sigma_0 \sqrt{50} = 0.41 \]

We suppose that a bank cooperates with a logistics company and determines to give a company pledge loan in March 36 of 2007. Based on the dynamic pledge practice, suppose \( \delta = 0 \) and \( \theta = 0 \) and parameter values are gotten as Table 1.

Keeping other parameters as Table 1, when \( K \) is set to be different values, we may observe the impact of marking-to-market frequency on loan-to-value ratio \( \omega \) as Table 2 based on Theorem 1.

From Table 2, we may obviously know that the loan-to-value ratio increases in marking-to-market frequency. This shows enhancing monitoring strength of the creditors can reduce the loan risk effectively, which is an important reason that the bank is willing to cooperate with large logistic enterprises. However, enhancing monitoring strength needs to increase monitoring cost and from Table 2, with \( k \) value increasing, the increasing extent of \( \omega \) is smaller and smaller, which shows that the bank, when cooperating with logistics company to monitor inventory financing, should keep the balance of monitoring cost and yield effectively.

Keeping other parameters as Table 1, when loan maturity \( T \) is set to be different values, we may observe the impact of loan maturity on loan-to-value ratio \( \omega \) as Fig. 1.

Keeping other parameters as Table 1, when the default probability \( Q \) is set to be different values, we may observe the impact of default probability on loan-to-value ratio \( \omega \) as Fig. 2.

Let \( \theta = 0.05 \) and other parameters are shown as Table 1. When the ratio of time to capture is set to be different values, we are able to observe the impact of the ratio on loan-to-value ratio as Fig. 3.

Let \( \delta = 0.50 \) and other parameters are shown as Table 1. When the ratio of liquidity risk \( \theta \) is set to be different values, we are able to observe the impact of the ratio on loan-to-value ratio as Fig. 4.

From Fig. 3 and Fig. 4, we obviously know that, the loan-to-value ratio increases with the ratio of time to capture \( \delta \) and the ratio of liquidity risk \( \theta \) decreasing, which is an important reason that the bank is willing to cooperate with logistics company. When the bank cooperates with logistics company, logistics company is
service to make the pledged inventory liquidated quickly, that is, to lower the value of $\delta$ and $\theta$, which will reduce the risk of inventory financing. In this case, the bank will enhance the loan-to-value ratio, give the loan with higher credit line and finally realize the “win-win” of the banks, logistics enterprises and the borrowers.

CONCLUSION

If collateral is stand This study analyzes the loan-to-value ratio decision of banks in dynamic pledge fashion and studies a kind of the main inventories in inventory pledge financing, i.e., copper. Based on the field investigation, we obtain the spot price data of “Yangzi River 1#” copper in spot market and deduce the price variance parameters demanded by loan-to-value ratio decision model. We suppose a borrower, pledge copper to apply for a loan of a bank and then, by designing other model parameters based on the pledge practice, we get the specific numerical value of loan-to-value ratios determined by the bank in dynamic pledge fashion and analyze the relationship of loan-to-value ratio and other important parameters. These results can provide a scientific foundation for three sides to participate in inventory pledge financing.

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