Importance Sampling Estimation on Call Dropping Probability for a Handover Scheme Under Poisson Assumption

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Abstract: The process of changing the channel associated with the current connection while a call is in progress is under consideration. The estimation of call dropping probability in handover process of a one dimensional traffic system in 3GPP LTE is discussed. The analytical probability of call dropping is deducted and then estimated by Monte Carlo simulation. As the call dropping probability is always very small, it can be considered as rare event. To reduce the sample size of simulation, importance sampling is used by increasing the probability of average coming rate for handover traffic and decreasing the probability of available channels. The simulation results suggest the sample size can be tremendously reduced by using importance sampling.

Key words: Handover, importance sampling, monte carlo, call dropping probability, poisson distribution

INTRODUCTION

Handover is the process of changing the channel (frequency, time slot, spreading code, or combination of them) associated with the current connection while a call is in progress (Lin et al., 2012). Usually, continuous service is achieved by supporting handover from one cell to another (Li et al., 2012). As shown in Fig. 1, it is often initiated either by crossing a cell boundary or by deterioration in quality of the signal in the current channel (Kwon et al., 2008; Zhang, 2010). In 3GPP Long Term Evolution (LTE), handover also called as cell reselection.

The handover process starts when the power received by the mobile station from a neighboring cell exceeds the power received from current cell by a certain amount, called handover threshold. If the target neighboring cell is heavy load and cannot allocate channel for the mobile station within certain time restriction, the call will be terminated and the handover attempt fails (3GPP, 1999).

Queueing priority schemes gives possibility to reduce the blocking probability of new calls, where the calls queuing in handover queues. Queueing priority channel assignment strategy is described by Tsvetanov et al. (2012). Analysis of a mobile cellular system with handover priority and hysteresis control is given by Radev and Radev (2010).

In nowadays broadband wireless networks such as 3GPP LTE, the Quality of Service (QoS) like call dropping probability is important and worth studying. Because all channels at target eNodeB are busy is very small, maybe less than $10^{-5}$, call dropping probability is always suggested as rare event. In such case the Monte Carlo simulation, which is implemented for probabilities not less than $10^{-2}$ will be very time-consumable and useless (Bucklew, 2004).

In this study, we design a novel simulation project by importance sampling to reduce the simulation runtime. The rest of this study is organized as follows. In section 2, we introduce the handover scheme. In section 3, the probability of call dropping is discussed. In section 4 and 5, we propose the simulation project by Monte Carlo and Importance sampling respective. The simulation results are shown in section 6. Finally conclusions are presented in section 7.

MODELING OF HANDOVER SCHEME AND PROBLEM STATEMENT

A simplified handover mechanism is shown in Fig. 2, where A is the channel array for BS and B is the handover

![handover_diagram.png](attachment:handover_diagram.png)

Fig. 1: Handover in cell edge

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For arbitrary time instant $k$ and channel $i$, the PDF of $a_i(k)$ can be modeled as 0-1 distribution, that is:

$$
\begin{align*}
\Pr[a_i(k) = 1] &= q \\
\Pr[a_i(k) = 0] &= 1 - q
\end{align*}
$$

In 3GPP LTE and LTE Advanced system, $N$ always equal to 100 or above and approximately satisfy with condition $N \rightarrow \infty$. Using Central Limit Theorem, $x(k)$ approximately follow Gaussian distribution:

$$
\Pr\{x(k) = n\} = \frac{1}{\sqrt{2\pi Nq(1-q)}} \exp\left[\frac{-(n-Nq)^2}{2Nq(1-q)}\right]
$$

The expectation and variance of $x(k)$ can be written as:

$$
E\{x(k)\} = NE\{a_i(k)\} = Nq
$$

$$
\text{Var}\{x(k)\} = N \text{Var}\{x(k)\} = Nq(1-q)
$$

If $s(k) \leq x(k)$, all the incoming handover traffic can be allocated without latency, the probability can be expressed as:

$$
p_s = \Pr\{s(k) \leq x(k)\} = \sum_{n=0}^{\infty} \frac{e^{-nq}}{n!} \frac{1}{\sqrt{2\pi Nq(1-q)}} \exp\left[-\frac{(n-Nq)^2}{2Nq(1-q)}\right]
$$

Else if $s(k) > x(k)$, some handover traffic need to be stored in Queue B temporarily waiting for next time allocation and the number is:

$$
\tau_i(k) = s(k) - x(k)
$$

The PDF of $\tau_i(k)$ can be expressed as:

$$
\Pr\{\tau_i(k) = m\} = \sum_{n=0}^{\infty} \frac{e^{-nq}}{n!(n+m)!} \frac{1}{\sqrt{2\pi Nq(1-q)}} \exp\left[-\frac{(n-Nq)^2}{2Nq(1-q)}\right]
$$

Assume the total available channel $x(k)$ is memoryless, that means $x(k+1)$ also follow Gaussian distribution:

$$
\Pr\{x(k+1) = n\} = \frac{1}{\sqrt{2\pi Nq(1-q)}} \exp\left[-\frac{(n-Nq)^2}{2Nq(1-q)}\right]
$$

ESTIMATING ON CALL DROPPING PROBABILITY

Without loss of generality, we assume $s(k)$ follow Poisson distribution, that is:

$$
\Pr\{s(k) = m\} = \frac{e^{-\lambda} \lambda^m}{m!}
$$

Here $\lambda$ is the mean of $s(k)$. 

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If \( r_{t}(k) < x(k+1) \), all the delayed incoming handover traffic can be allocated within latency time \( t = 1 \), the probability can be expressed as:

\[
p_1 = \Pr\{t(k) \leq x(k+1)\} = \Pr\{s(k) \leq x(k) + x(k+1)\} = \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \frac{e^{-\lambda v} \exp\left(-\frac{(n - N_q)^2 + (n - N_q)^2}{2N_q(1-q)}\right)}{m!} \cdot \frac{1}{\sqrt{2\pi N_q(1-q)}}
\]

(13)

Else if \( r_{t}(k) > x(k+1) \), some handover traffic need to be stored in Queue B again and the number is:

\[
r_t(k) = r_t(k) - x(k+1)
\]

(14)

The PDF of \( r_t(k) \) can be expressed as:

\[
\Pr\{r_t(k) = m\} = \sum_{n=0}^{\infty} \frac{e^{-\lambda v} \exp\left(-\frac{(n - N_q)^2}{2N_q(1-q)}\right)}{m!} \cdot \frac{1}{\sqrt{2\pi N_q(1-q)}}
\]

(15)

The probability of all the delayed incoming handover traffic can be allocated within latency time \( t = 2 \) is expressed as:

\[
p_2 = \Pr\{t(k) \leq x(k+2)\} = \Pr\{s(k) \leq x(k) + x(k+1) + x(k+2)\} = \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \sum_{n'=0}^{\infty} \frac{e^{-\lambda v} \exp\left(-\frac{(n - N_q)^2 + (n - N_q)^2}{2N_q(1-q)}\right)}{m!} \cdot \frac{1}{\sqrt{2\pi N_q(1-q)}}
\]

(16)

With the same way, the number of remaining incoming handover traffic waiting for the \( t \)-th allocation can be expressed as:

\[
\Pr\{r_t(k) = m\} = \sum_{n=0}^{\infty} \frac{e^{-\lambda v} \exp\left(-\frac{(n - N_q)^2}{2N_q(1-q)}\right)}{m!} \cdot \frac{1}{\sqrt{2\pi N_q(1-q)}}
\]

(17)

By substituting \( r_{t}(k) \) with \( r_{t}(k) \) and go on, the above equation can be expressed as:

\[
\Pr\{t(k) = m\} = \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} \frac{e^{-\lambda v} \exp\left(-\frac{(n - N_q)^2}{2N_q(1-q)}\right)}{m!} \cdot \frac{1}{\sqrt{2\pi N_q(1-q)}}
\]

(18)

And all the delayed incoming handover traffic can be allocated with latency time\( t \) can be expressed as:

\[
p_t = \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \frac{e^{-\lambda v} \exp\left(-\frac{(n - N_q)^2}{2N_q(1-q)}\right)}{m!} \cdot \frac{1}{\sqrt{2\pi N_q(1-q)}}
\]

(19)

The average latency time can be calculated as:

\[
E(t) = \sum_{t=1}^{\infty} \frac{\tau_t(k)}{p_t}
\]

(20)

We assume all the incoming handover traffic has the same waiting time tolerant threshold \( \alpha \), that means call dropping is occurred if and only if the waiting time is bigger than \( \alpha \). Then, the average probability of call dropping can be calculated as:

\[
p = E\left(\frac{r_t(k)}{s(k)}\right)
\]

(21)

The final question here is to solve \( p \) under given conditions (including but not limited to \( N, v, q \) and \( \alpha \)), but unfortunately the closed form solution is hard to achieve, we always use simulation to find the numeric results.

**MONTE CARLO SIMULATION**

By Monte Carlo method, the average call dropping probability can be estimated as:

\[
p = \frac{K}{L}
\]

(22)

Here, \( L \) is the total number of handover traffic (also called as sample size) and \( K \) is the number of dropping calls.

For the \( t \)-th sample, define 2-value variable \( Z_i \) as:

\[
Z_i = \begin{cases} 1, \text{the } t\text{-th sample dropped} \\ 0, \text{else} \end{cases}
\]

(23)

The average call dropping probability \( p \) also can be estimated as:

\[
\hat{p} = \frac{\sum Z_i}{L}
\]

(24)
The variance of $p$ can be estimated as:

$$\sigma_p^2 = \frac{1}{L} \cdot D \left( \frac{1}{L} \cdot \sum_{i=1}^{L} Z_i \right) = \frac{1}{L} \cdot D \left( \sum_{i=1}^{L} Z_i \right) \tag{25}$$

If all the samples are independent, $\sigma_p^2$ can be simplified as:

$$\sigma_p^2 = \frac{1}{L^2} \cdot L \cdot D(Z_i) = \frac{1}{L} \cdot p \cdot (1 - p) = \frac{p}{L} \tag{26}$$

The accuracy of $\hat{p}$ is always defined as:

$$\nu_p = \frac{\sigma_p}{p} \tag{27}$$

Then:

$$\nu_p = \frac{1}{\sqrt{\nu_p}} = \frac{1}{k} \tag{28}$$

In order to assure certain accuracy, for very small $p$, we need to run large number of $L$ to find enough $K$, which means large amount of simulation time.

**IMPROVED SIMULATION USING IMPORTANCE SAMPLING**

The most famous approach for rare event simulation is Importance Sampling. Importance sampling is connected with change the probability density distribution for increasing the frequency of appearance of more “significant” for simulation events.

The basic purpose of this simulation technique is to reduce dispersion or other estimation function, received as a result of computer simulation. During the simulation process is expected to receive samples proportional of their importance to expected results.

The importance sampling estimators can receive in advance given accuracy and in this way the simulation time can be shortened. For generation of significant sample is used limited number of independent variables with normal distribution. Then the conditional probability of appearance of rare event is changed with conditional probability of appearance less rare event with similar distribution.

In this research, the average call dropping probability $p$ also can be expressed as:

$$p = \frac{E \left[ \frac{r_{\alpha}(k)}{s(k)} \right]}{s(k)} \tag{29}$$

$$= \sum_{\alpha=0}^{\infty} \sum_{m=0}^{\infty} \frac{m^n}{n!} \Pr \{ r_{\alpha}(k) = m \} \Pr \{ s(k) = n \}$$

$$= \sum_{\alpha=0}^{\infty} \sum_{m=0}^{\infty} \frac{m^n}{n!} \cdot \exp \left[ \frac{1}{2} \cdot \frac{(n_1 - Nq)^2}{2Nq(1 - q)} \right] \cdot \frac{\alpha!}{\sum_{\alpha=0}^{\infty} \alpha!} \cdot \frac{(m + \sum_{\alpha=0}^{\infty} \alpha_n)!}{\prod_{\alpha=0}^{\infty} \alpha_n!}$$

$$= \sum_{\alpha=0}^{\infty} \sum_{m=0}^{\infty} f(m, n, N, \alpha, v, q)$$

Obviously bigger $v$ and smaller $q$ will derive bigger $p$. Let:

$$v = v^b$$

$$q = q^b \tag{30}$$

Here $b$ is a certain constant. By using importance sampling, $p$ can be re-expressed as:

$$p = \sum_{\alpha=0}^{\infty} \sum_{m=0}^{\infty} f(m, n, N, \alpha, v^b, q^b) \cdot \alpha(v, q) \tag{31}$$

Here $f(m, n, N, \alpha, v^b, q^b)$ is the importance sampling PDF, while:

$$\alpha(v, q) = \frac{f(m, n, N, \alpha, v^b, q^b)}{f(m, n, N, \alpha, v, q^b)} \tag{32}$$

is the weighting function.

**SIMULATION RESULTS**

The simulation parameter is chosen as $N = 100$ and $\alpha = 0$. The simulation results between call dropping probability $p$, average handover traffic coming rate $v$ and channel available probability $q$ is shown in Fig. 3. It can be seen $p$ increase with the increasing of $v$ and decreasing of $q$.

Define sample size reduction efficiency as:

$$\beta = \frac{L_{MC}}{L_{IS}} \tag{33}$$

where $L_{MC}$ and $L_{IS}$ are the least sample size to assure the simulation accuracy of $p$ is smaller than 10% for MC and importance sampling respective.

The numerical relations between $\beta$ and $v$, $q$ under different $b$ is shown in Fig. 4-6. It can be seen that:

- $\beta$ increases with the increasing of $q$
- $\beta$ increases with the decreasing of $v$
- $\beta$ increases with the increasing of $b$
CONCLUSION

A simulation approach using importance sampling for estimation of probabilistic parameters of handover call dropping probability at 3GPP LTE network is discussed in this study.

By increasing the probability of coming handover traffic and decreasing the probability of available channels, the probability of call dropping has been promoted, as well the simulation sample size has been reduced. Especially for small v large study q, the sample size reduction efficiency study β approaches a huge value.

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