Study of Automatic Matching Method about Bullet Head Traces Based on Similarity

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Abstract: In order to compare traces of bullet head, firstly the features of bullet head traces were extracted, piecewise quadratic function was mainly used for bullet-section curve fitting, a curved surface was composed by all bullet-section curves. Secondly, by the bullet curved surface of two sub-edges about two warheads, improved Euclidean distance was calculated to obtain the similarity of the two sub-edges. Circulation algorithm was designed to achieve automatic matching. Finally, this algorithm was used to make the similarity comparison about 22 bullets. Satisfactory results were obtained, so this model has some versatility.

Key words: Similarity, improved euclidean distance, circulation algorithm, automatic matching

INTRODUCTION

While bullets are fired, the warhead scratches of rifling marks were left on barrel. There are four barrel rifling of eight convex edges, divided into four main edges and four sub-edges. Therefore, the traces of the warhead are divided into four main ridge traces and four ridge sub-edges. Since bullet barrel is rotated only a small angle through the barrel, the scratches distribution is a slash shape (at an angle with the cylinder bus) and the main part of each piece traces are shown for different sizes and different shades of the lines.

In police practice, there is needed to judge whether two warheads the same gun fired according to the traces of the warhead. In the traditional method, through the visual observation of the microscope, the two lines on the type of warhead traces are compared to see whether the line thickness distribution is consistent. Traditional approach has two weaknesses: First, the efficiency is very low and the comparison to multiple warheads (e.g., dozens, hundreds) is almost impossible; second, warhead samples is not easy to preserve, prone to rust damage, etc.

This study attempts to improve the traditional method of comparison. Two features of depth and width about bullet and bullet curved surface were extracted to calculate the similarity of bullet. Circulation algorithm was designed to achieve automatic matching. And specific examples were chosen for model validation.

MODEL ASSUMPTIONS AND CONVENTIONS

- Correlation of scratches cladsh by different sub-edge is much lower than the correlation of scratches struck twice by same edge
- There is only one trough in the function image of the measurement error caused by the pan or rotate, that is means the least squares error function is monotonous respectively at its optimal value
- Barrel rifling is on the right of the max in the bullet marks
- All the observational data do not exist observational error

MODEL ESTABLISHMENT AND SOLUTION

Comparison feature extraction and data selection: In order to extract the main features of bullet scratches data from huge amounts of data, the following two aspects were mainly considered.

Extraction of width and depth of bullet: As bullets were caused by the barrel rifling, bullet caused by different rifling was different, mainly in the depth and width of the bullet. Define the minimum as the first minimum point at right of the maximum of bullet section; Absolute width w was horizontal distance of the first maxima on the right from the maximum value to the minimum value. Absolute depth h was vertical distance of the first minimum on the right from the maximum value to the minimum value. Fig. 1 is a simplified diagram of a section curved surface.

From Fig. 1, the horizontal axis was the X-axis and the vertical axis was the Z axis. By the graph, the bullet scratches had a maximum point A in projection of the X0Z surface, which was squeezed out of the barrel rifling, while bullet subjected to large impact, through the chamber (Li et al., 2011).
Fig. 1: Simplified diagram of bullet curved surface section

Fig. 2: Bullet curved surface section

**Extraction of bullet surface**: According to bullet curved surface, the warhead could be judged belongs to which gun, shown in Fig. 2.

From Fig. 2, the barrel rifling was right of the highest point. For different rifling, because of the degree of wear and bullet speed, the bullet marks were not the same, therefore, it was necessary to consider the curve at the right of the highest point. As the highest point of the curve to the right is more complex, so part of the image was need to intercept (Xu, 1998). The intercept method is as follows: At the highest point A corresponds to the X-axis coordinates, the distance of 0.4 mm was intercepted of his right (Translation error of the coordinate X, Y was eliminated). By the Image, the curve of 0.4mm right at the highest point could best reflect the features of bullet. Firstly, the minimum point B was found from the 0.1 mm range which was moved right by the highest point A corresponds to the X-axis coordinate. The point B is inside [0, 0.1 mm] and then the points D was obtained at the right position to 0.3 mm by moved. Then this curve was divided into AB, BD two paragraphs, quadratic curve was made to fit these two curves, then two smooth curves were obtained. (Xiao, 2007) And these two curves could also reflect the features of absolute width and absolute depth. For all y-values in XOZ?? section, a series of curves could be fitted by the above methods, the curved surface composed by those curves could directly reflect the features of bullet marks of warhead. So, if the feature similarity of four sub-edges about two warheads was higher, we could determine that the two warheads were issued by the same gun.

**Comparison scheme and steps**

**Step 1 data selection**: First, the main data which could reflect features of warhead scratches (bullet marks curved surface) was found from the huge amounts of data. The specific method was that: the X axis corresponding to the highest point in the mapping graphics of three-dimensional map was found in XOZ surface (A total of n surface). After shifting right 0.4 mm, X and Z axis coordinates were obtained corresponding to the point of 0.4mm. Since the X-axis coordinate was obtained at a certain step size (2.75 micron), so in this 0.4 mm distance, a total of m data was obtained (m<n). In summary, for a massive data of more than 40 million, a set of data can be extracted.

**Step 2 curve fitting**: Take one section of n, m data was segmented, one curve was divided into AB, BD two paragraphs and quadratic function was respectively fitted to these two curves, then curves Z1, Z2 were obtained.

**Step 3 revaluations**: Since the part close to the point A in curve segment AB was not a good indicator to the bullet marks of warhead, so in the revaluations, the 20 step points at left of the minimum point B were taken as the approximate point of the curve AB and the 100 step points at right of the minimum point B were taken as the approximate point of the curve BD (Yang and Huang, 2008). In order to eliminate the translational error of X, Z axis, the origin of the position was moved to point B. Take an arbitrary surface, the coordinates corresponds to X-axis within 0.4 mm meet the above requirements were substituted into the fitting curve Z1, Z2, then a series of value were obtained for the processing of the origin move. Finally, Z1, Z2 were gotten to composite as Z' (total m'), total m' new group Z'.
**Step 4 define similarity:** In statistics, the similarity could be described by the distance and actually the similarity could be transformed the distance. If \( d_{ij} \) is a distance, the similarity is defined as:

\[
C_{ij} = \frac{1}{1 + d_{ij}}
\]

(Wang, 2003). The most common and intuitive distance was Euclidean distance:

\[
d_{ij} = \left[ \sum_{k=1}^{p} (x_{ik} - x_{jk})^2 \right]^{1/2}
\]

\( i = 1, 2, 3, \ldots, n \), \( j = 1, 2, 3, \ldots, p \)

But there is a drawback in Euclidean distance. The size of the distance was related to the unit of observation, i.e., different dimensions. So, before making comparison, the data was standardized to obtain:

\[
x_{ij} = \frac{x_{ij} - \bar{x}_j}{S_j}
\]

Where:

\[
\bar{x}_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij}
\]

and:

\[
S_j = \left[ \frac{1}{n-1} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2 \right]^{1/2}
\]

The standardized data was substituted into \( d_{ij} \), dimensionless distance could be obtained and then the similarity was \( C_{ij} \in [0, 1] \) (Wang, 2001).

**Step 6 the comparison method of bullet:** First, make for comparison two bullets bullet marks issued by two guns. Let the first bullet marks by one gun hit by a certain order of C1, C2, C3, C4, another bullet marks by another guns hit were C5, C6, C7, C8.

The comparison method was (Jiang, 1992):

- Compare the C1 and C5, followed by comparison of its sequence, C2 and C6, C3 and C7, C4 and C8 and four similarity values were obtained to form a row vector
- Compare the C1 and C6, the same sequence alignment, C2 and C7, C3 and C8, C4 and C5 and four similarity values were obtained to form a row vector
- Then compare the C1 and C7, empathy C2 and C8, C3 and C5, C4 and C6, respectively and four similarity values were obtained to form a row vector
- According to the above comparison method, C1 and C8, C2 and C5, C3 and C6, C4 and C7 for comparison and four similarity values were obtained to form a row vector

Thus a \( 4 \times 4 \) matrix \( A \) was obtained. As shown below:

\[
A = \begin{pmatrix}
C1C5 & C1C6 & C1C7 & C1C8 \\
C2C5 & C2C6 & C2C7 & C2C8 \\
C3C5 & C3C6 & C3C7 & C3C8 \\
C4C5 & C4C6 & C4C7 & C4C8
\end{pmatrix}
\]

Then, the minimum value of each line was removed and the similarity was the mean of remaining three values, \( \bar{C}_{ij} (j = 1, 2, 3, 4) \). At last, the similarity of two bullets was \( C = \max \bar{C}_{ij} \).

The complete flow scheme of comparison was shown in Fig. 3.

**EMPIRICAL ANALYSIS**

There were the measured data of sub-edges of 22 warheads from the 11 guns fired (each gun launch two warheads), the data file name was the 22 files beginning t and each contained four data sub-documents of sub-edges beginning with c. The c1, c2, c3, c4 in sub-file were the numbers of four sub-edges with one warhead in a fixed order (Liu, 2005). The data were sourced from the A title in Seventh Graduate Mathematical Modeling Contest.

**Calculation results:** According to the comparison scheme above, parts of the similarities between any two of the 22 bullet marks were calculated as shown in Table 1 below:
According to the similarity from high to low, the file number of the first five warheads in each warhead was given as follows (Zhou et al., 2013):

**Model testing:** In accordance with the above method, the data of the two bullet marks from one gun was given to derive matrix A:

$$A = \begin{pmatrix}
0.4388 & 0.2656 & 0.3949 & 0.4571 \\
0.4817 & 0.3360 & 0.4271 & 0.4527 \\
0.3786 & 0.2719 & 0.4630 & 0.4149 \\
0.4379 & 0.2406 & 0.4142 & 0.4399
\end{pmatrix}$$

The minimum values of each line were removed as 0.2656, 0.3360, 0.2719, 0.2406 and then the averages of the remaining three values were obtained as follows (Xie and Li, 2003):

$$\bar{C}_1 = 0.430, \bar{C}_2 = 0.445, \bar{C}_3 = 0.418, \bar{C}_4 = 0.431$$

Finally, the similarity of these two bullets from one gun was: $C = \max \bar{C}_j = 0.454$.

Then the results of comparison of two bullets from different guns were given as follows:

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\[
A = \begin{pmatrix}
0.3236 & 0.3875 & 0.2292 & 0.3122 \\
0.3384 & 0.4320 & 0.2911 & 0.3159 \\
0.3013 & 0.3868 & 0.2738 & 0.3269 \\
0.3349 & 0.4294 & 0.2861 & 0.2409
\end{pmatrix}
\]

The minimum values of each line were removed and then the averages of the remaining three values were obtained as follows:

\[
\bar{C}_1 = 0.341, \bar{C}_2 = 0.362, \bar{C}_3 = 0.338, \bar{C}_4 = 0.350
\]

Then the similarity of two bullets was:

\[
C = \max \bar{C}_j = 0.362
\]

In summary comparison, the similarity values of the two bullet marks from one gun was above 0.4 and the similarity values of the two bullet marks from different guns was less than 0.4. Therefore, by the above comparison, the comparison result for this algorithm was ideal.

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